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Illustrating the Beats Phenomenon with Excel: the construction of meaning through experimentation

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Abstract

The work of students on a specific mathematical topic is described in terms of their initial lack of confidence to take the concept on board through to their final understanding of the topic; a change that was greatly supported by their use of spreadsheets. The outcome for the students parallels the findings of the study by Sutherland and Rojano which showed that, for Year 10-11 students 'pupils initially had difficulty accepting the idea of working with a general unknown' but that the 'spreadsheet environment supported pupils to move from thinking with the specific to the general'. In the mathematics education literature, the use of spreadsheets as a tool has an established level of acceptance both through research studies and policy documents, but there remains the need for further studies of its practical application both as a program that can enhance learning and as a tool with which students can express their understanding. This article is but one step in this direction, and adopts the pedagogical approach of Cobb where the students are given opportunities through social interaction in the classroom to develop their own understanding and representations of a concept.

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The work of students on a specific mathematical topic is described in terms of their initial lack of confidence to take the concept on board through to their final understanding of the topic; a change that was greatly supported by their use of spreadsheets. The outcome for the students parallels the findings of the study by Sutherland and Rojano [10] which showed that, for Year 10-11 students 'pupils initially had difficulty accepting the idea of working with a general unknown' but that the 'spreadsheet environment supported pupils to move from thinking with the specific to the general'. In the mathematics education literature, the use of spreadsheets as a tool has an established level of acceptance both through research studies (Abramovich. and Nabors [1] and Dugdale [5]) and policy documents (NCTM [7]), but there remains the need for further studies of its practical application both as a program that can enhance learning and as a tool with which students can express their understanding. This article is but one step in this direction, and adopts the pedagogical approach of Cobb [2] where the students are given opportunities through social interaction in the classroom to develop their own understanding and representations of a concept.

Beginning with an Assignment

As part of the Year 11-12 curriculum in mathematics, students begin to take on board a more sophisticated understanding of the circular or periodic functions than that which has been earlier presented to them in terms such as:

$$\sin(angle) = \frac{opposite}{hypotenuse}$$

They are coming to grips with the general definitions such as:

$$\sin(\theta) = \frac{y}{r}, \ \cos(\theta) = \frac{x}{r},$$

and are learning to extend the domain of the functions, first to [0, 2π] and then to other multiples of π. The more general form of a sine function:

$$f(x) = a \sin n(x - c)$$

introduces the concept of amplitude (*a*), frequency (*n*) and phase shift (*c*). The understanding of these concepts relies heavily on an appreciation of these parameters, and a typical class activity would be to set up a spreadsheet in which these parameters are controlled by scrollbars. The resulting spreadsheet provides an interactive environment in which such variations can be explored and explained by the students. The study of periodic functions at this level concludes with looking at the process of summing two or more sine curves, perhaps as a precursor to Fourier Series.

As an application of this, the musical phenomenon of beats that occur when two notes which vary by a small amount are played together. To encourage the students to explore this phenomenon, an assignment was set as shown in Figure 1. It was easy to find a student guitarist who was able to demonstrate the beats to the class, but a bit of a disappointment to discover that today's guitarist uses a simple electronic device that makes guitar tuning very straight forward. However, the students appeared to understand the concept and it seemed that they would find the assignment straightforward.

Assignment: The Beats Phenomenon

Part 1: Understanding the Problem

Discuss the question of beats with a guitarist and ask them to show you how it helps them to tune a guitar. Describe the 'beats' phenomenon in lay terms, using diagrams to illustrate your description.

Part 2: Devising a Plan

Set up a model of two vibrating guitar strings in Excel using parameters that enable you to investigate the 'beats' phenomenon around the Middle C frequency of 256 hertz.

Your model should enable you to graph the vibrations quite accurately, using 1° increments between 0° and 360°, and should include an automatic way of varying the frequency of the two strings.

Part 3: Carry out the Plan

Describe the formulae that you used in your model and select some representative graphs to support any conclusions that can be drawn from the spreadsheet.

Part 4: Interpret the Results

Show your spreadsheet to a guitarist and make a record of their comments on the insight that the graphs give into the 'beats' phenomenon.

Figure 1: The Beats Assignment

Initial Feedback

Being unaccustomed to report writing, or to describing mathematical activity in everyday language, the early feedback from the class was that this was a very difficult assignment and they had a real problem with understanding what was being asked for. The temptation was to hold a briefing session in which clear indications of what was required could be given. However, research findings such as Cobb [3], Costa and Liebmann [4], Gergen [6] and Von Glaserfeld [11] suggest that the teachers' role is not to define the understanding, but to create the environment in which understanding can be constructed by the students themselves, particularly in a context in which interaction and discussion amongst themselves takes place. As a result, the students were encouraged to spend time developing the Excel model and discussing amongst themselves what would constitute a successful answer. Early comments such as:

> "We don't understand what the graphs are showing."

> "What's actually happening when you add two sine curves together?"

showed that the class had some way to go in order to construct meaning from the activity.

Changing attitude – improving outcomes

As the discussion continued, and the sophistication of the spreadsheets increased, the levels of understanding increased noticeably. With increased understanding also came an improved commitment to the task, because the students were beginning to take control.

It was expected that the students would produce spreadsheets along the lines specified in the assignment, and the example provided with this article shows that this was achieved. For instance, the following graph, which has a domain of $[0, 2\pi]$ and a range of $[-2.5, 2.5]$ shows the beats effect when a note of 256 hertz is added to another note of frequency 253 hertz.
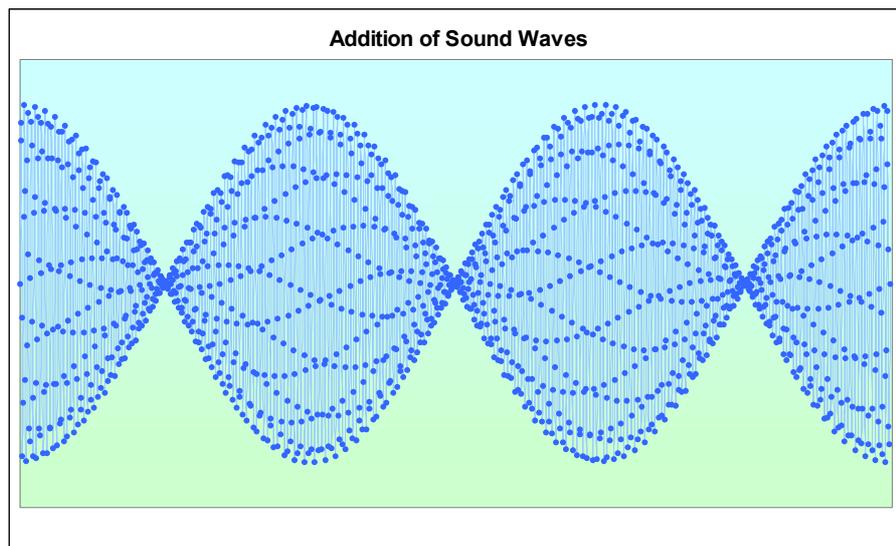
Figure 2: Addition of Sound Waves

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The graph of Figure 2 clearly shows 3 beats per second and through the use of a scrollbar, the spreadsheet that this student produced shows similar effects with frequencies other than 253. There is also the interesting interference pattern produced by the data point markers, but producing an explanation of this wasn't taken up by the students.

The student responsible for Figure 2, and others along with him, was not satisfied with the graph as a demonstration of the beats phenomenon. This graph shows the effect, but does not help to explain it. To find a fuller explanation, he took a magnifying glass to the first few waves, producing a very different graph, but one that held the key to the explanation. In Figure 3, the domain is reduced to [0, 0.2] rather than the [0, 6.3] range of Figure 2. The sin $256x$ + sin mx curve (yellow) is shown when $m = 240$, and both the sin $256x$ (dark blue) and sin mx (magenta) curves are incorporated into the graph. With the lower frequency of m = 240, the way in which the two beats go out of step is now clearly visible, and by the time 0.2 is reached, the two graphs are almost directly opposed to each other, and the addition graph has been reduced to a vibration of very small amplitude.
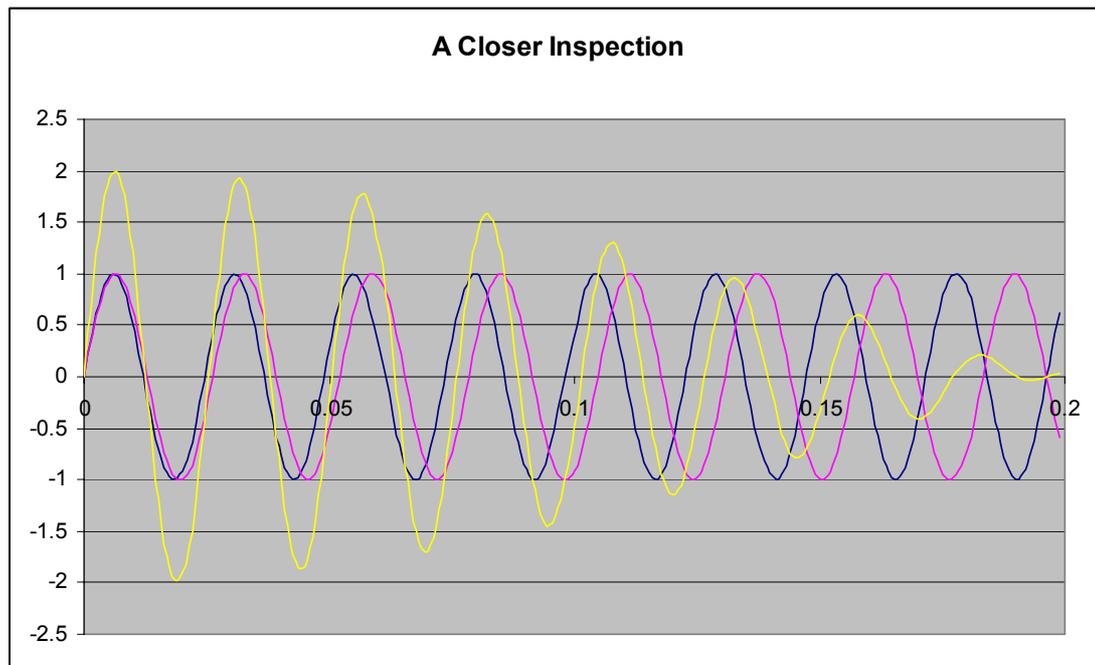
Figure 3: A closer inspection

In class discussion, the students claimed that Figure 3 gave a clear reason why the beat phenomenon occurred, while Figure 2 was useful in showing how many such beats occurred per second as the compared wave got closer and closer to the 256 hertz wave. At this stage of the course, the students were not familiar with trigonometric formulae, such as:

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \tag{1}$$

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However, later in the course, we were able to revisit Figure 2 and see how the cosine component of Equation 1 determines the profile of the graph and controls the number of beats per second. Given that each peak of the graph in Figure 2 sounds as a beat, a formula for the number of beats per second can be given as $|m - 256|$.

Conclusion

Through discussion and experimentation, the students reached a high level of understanding of the beats phenomenon. They also showed themselves capable of finding their own way to demonstrate a concept using a graphical representation. It is hoped that other teachers will find similar uses for spreadsheets. In a series of practical articles, O'Sullivan ([8] and [9]) points to the range of possibilities that interactivity within spreadsheets can support. Perhaps reports of the type given here are what is needed to promote the increase in usage of spreadsheets in the classroom, as they show a successful outcome achieved by real students who initially had a genuine problem of understanding and confidence.

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