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Spreadsheet Conditional Formatting: An Untapped Resource for Mathematics Education

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Abstract
Electronic spreadsheets have been with us for almost 25 years. In that period, innovative educators have found many ways in which this ubiquitous class of software may be leveraged in support of learning, especially in mathematics and the physical sciences. Conditional formatting is a relatively recent feature of the modern graphical spreadsheet, and may be viewed as a generalization of the accounting procedure of indicating debits by red ink and other amounts in black. The paper discusses advances in technology-enabled pedagogy made possible by this seemingly trivial feature of the modern spreadsheet program.

Keywords
spreadsheet, conditional formatting, mathematics education

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1 Introduction

The modern spreadsheet program, as exemplified by Microsoft Excel XP, has a vast array of features which may be used to illustrate mathematical principles, facilitate the discovery of patterns, encourage conjecturing, and, in some cases, generate new knowledge. In this paper, we discuss the *conditional formatting* feature of Microsoft Excel. This is a relatively modern spreadsheet capability, first appearing in Microsoft Excel 97. Conditional formatting may be viewed as a generalization of the common accounting practice of rendering negative amounts (payments, debts) in red, while other quantities are shown in conventional black. In everyday colloquial English, we hear the expressions "in the red" and "in the black", which describe the state of being in debt and not in debt respectively. Excel 97 and its successors allow automatic formatting

eJSiE **1**(2):104-124

n	a[n] direct	a[n] recurrence	difference
1	0.2	0.2	0
2	0.68	0.68	0
3	1.736	1.736	0
4	3.944	3.944	0
5	8.40992	8.40992	0
6	17.235008	17.235008	0
7	34.3784576	34.3784576	0
8	67.2499328	67.2499328	0
9	129.6398136	129.6398136	0
10	247.0955599	247.0955599	3.12639E-13
11	466.7622403	466.7622403	7.95808E-13
12	875.3564047	875.3564047	1.7053E-12

Figure 1: Elementary example of conditional formatting (values)

of any cell, based on its current value. It is usually quite easy, therefore, to specify the formatting or highlighting of cells which satisfy some condition. For example, those which are not zero may be used in a difference column to highlight a possible discrepancy between sequences generated by recurrence and direct formula, as shown in Figures 1 and 2. Notice from Figure 1 that values generated by direct and recurrence formulas appear to be identical. That this facile conclusion is false is clear from the difference column of Figure 1. Since, with infinite precision arithmetic, both methods generate exactly the same sequence, this very simple but clear example affords an excellent opportunity for the teacher to point out the perils of floating-point arithmetic.

With a facility such as conditional formatting, the basic human ability of pattern recognition, including color perception, algorithms for which are so difficult for conventional binary computers ([31], [20]) may be exploited for educational gain. The authors routinely use conditional formatting in their mathematics and mathematics education classes to illustrate a wide range of mathematical concepts. More than this, conditional formatting may be used as the basis for certain problem solution techniques, or algorithms. In this paper, different instructional topics are selected to illustrate these ideas.

References to conditional formatting in the general spreadsheet literature, at least for any didactic use, are sparse indeed, however the very modern work of Neuwirth and Arganbright [28] has quite a good coverage.

2 Solving non-linear equations

The effectiveness of instructional computing in finding approximate solutions to algebraic equations that cannot be solved exactly has opened the gates for technology-enabled pedagogy in pre-calculus mathematics. Earlier, this pedagogy included the use of pro-

n	a[n] direct	a[n] recurrence	difference
1	=1.8^n - 1.6^n	=B2	=B2-C2
2	=1.8^n - 1.6^n	=B3	=B3-C3
3	=1.8^n - 1.6^n	=3.4*C3 - 2.88*C2	=B4-C4
4	=1.8^n - 1.6^n	=3.4*C4 - 2.88*C3	=B5-C5
5	=1.8^n - 1.6^n	=3.4*C5 - 2.88*C4	=B6-C6
6	=1.8^n - 1.6^n	=3.4*C6 - 2.88*C5	=B7-C7
7	=1.8^n - 1.6^n	=3.4*C7 - 2.88*C6	=B8-C8
8	=1.8^n - 1.6^n	=3.4*C8 - 2.88*C7	=B9-C9
9	=1.8^n - 1.6^n	=3.4*C9 - 2.88*C8	=B10-C10
10	=1.8^n - 1.6^n	=3.4*C10 - 2.88*C9	=B11-C11
11	=1.8^n - 1.6^n	=3.4*C11 - 2.88*C10	=B12-C12
12	=1.8^n - 1.6^n	=3.4*C12 - 2.88*C11	=B13-C13

Figure 2: Elementary example of conditional formatting (formulas)

gramming languages such as BASIC [12]; later the use of computer applications such as spreadsheets [25], [27], [24], [26] has come to prominence. In particular, Murphy [27], advocating the use of the method of iteration, has singled out "the spreadsheet approach" suggested by Levin & Abramovich [25] as an effective way of solving non-linear equations in one unknown through the method of iteration.

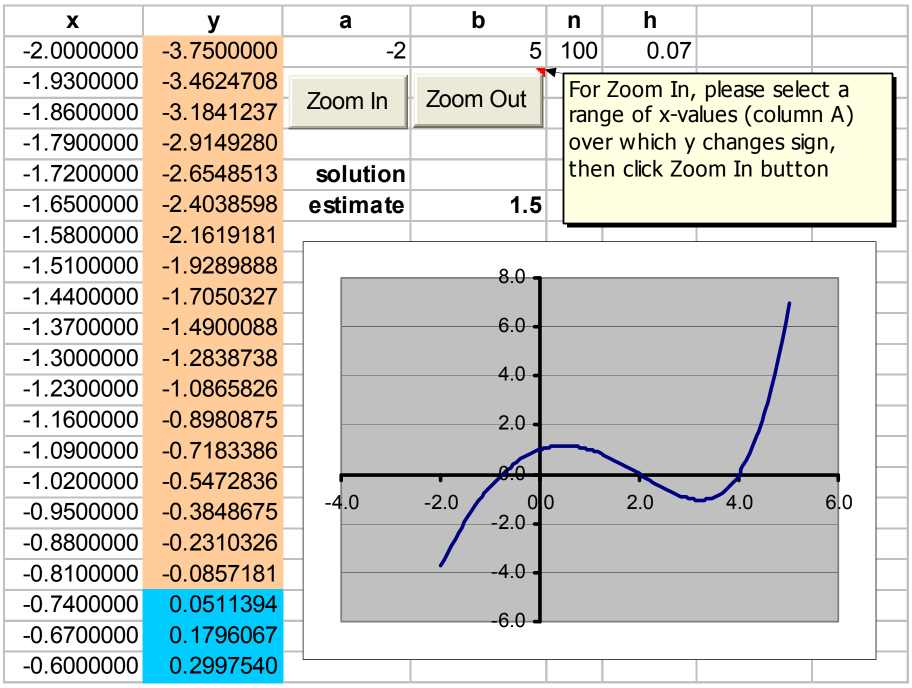
Another well-known method of solving the equation $f(x) = 0$ is by inspection of the graph of $y = f(x)$. The use of spreadsheet conditional formatting enhances this approach to solving non-linear equations in one unknown in a very simple way. As an example, consider equation 1.

$$2^x - x^2 = 0 \tag{1}$$

One obvious solution of equation 1 is $x = 2$; another (slightly less obvious) solution is $x = 4$. Yet there exists a non-trivial solution $x \simeq -0.766$ (with an approximation to three decimal digits) of this equation. With the chart wizard facility in Excel, one may easily plot the function $f(x) = 2^x - x^2$ over a suitable interval, say $-2 \le x \le 5$. From this graph, it becomes clear that there exist at least three solutions; two of them are the above-mentioned integers 2 and 4. In order to obtain the solution between -1 and 0, one can use a graphical *zoom-in* process. By ensuring that the model in question is constructed in such a way as to easily change the plot interval, it is a very simple matter to *zoom-in* by changing a to -1 and b to 0.

Here, conditional formatting enters the picture. One can format the y column so that negatives, zero, and positive values of y have different colors. Consequently, one can easily identify a change of sign of the function $y = f(x)$, and then to choose appropriate new values for a and b, to effect another zoom-in operation. If one uses, say, 100 points to plot the function, it only takes a few zoom operations to obtain the solution (which may be taken as the mid-point of the current plot interval) to about six decimal digits.

An assignment was set by the second author for students of an elementary course in Visual Basic for Applications (VBA) to improve the model by writing VBA code to implement *zoom-in* and *zoom-out* buttons. This is shown in Figure 3. The idea in this model is just to select a range of x values (interval) over which y changes sign (color), then click on the button *Zoom In*. Clicking on *Zoom Out* just doubles the plot interval width, while maintaining the center of the interval. Figure 4 shows the state of the



x	y	a	b	n	h
-2.0000000	-3.7500000	-2	5	100	0.07
-1.9300000	-3.4624708				
-1.8600000	-3.1841237				
-1.7900000	-2.9149280				
-1.7200000	-2.6548513	solution			
-1.6500000	-2.4038598	estimate	1.5		
-1.5800000	-2.1619181				
-1.5100000	-1.9289888				
-1.4400000	-1.7050327				
-1.3700000	-1.4900088				
-1.3000000	-1.2838738				
-1.2300000	-1.0865826				
-1.1600000	-0.8980875				
-1.0900000	-0.7183386				
-1.0200000	-0.5472836				
-0.9500000	-0.3848675				
-0.8800000	-0.2310326				
-0.8100000	-0.0857181				
-0.7400000	0.0511394				
-0.6700000	0.1796067				
-0.6000000	0.2997540				

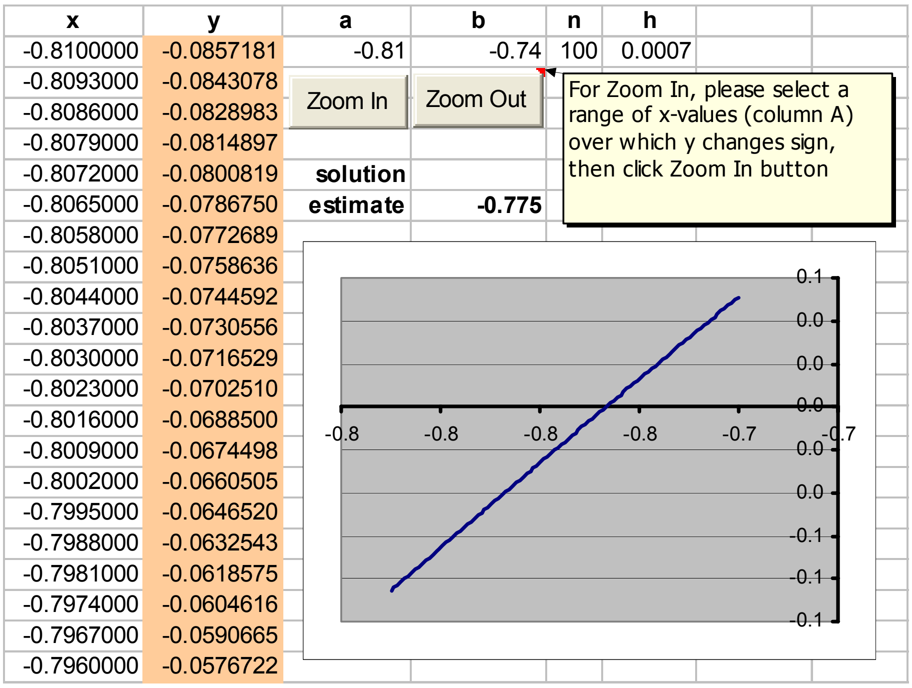
Figure 3: Solving $2^x - x^2 = 0$: before Zoom In

model after one click on the *Zoom In* button, and assumes that the user has selected a minimal range of x values (i.e., adjacent values) over which the function has changed sign. Further, from Figure 5, it can be easily seen that with just two *zoom-in* operations one can get a solution correct to several decimal places. This requires no keyboarding of values, but just two mouse clicks. Thus, there is minimal effort on the part of the student, but it is not a totally black box solution where the student sees nothing about how the problem is solved. We believe that this constitutes a good blend of theory and practice.

A useful extension of the activity described in this section is to explore whether or not the graph of any monotonic increasing exponential function would intersect the parabola $y = x^2$ at three distinct points. To this end, the spreadsheet can be modified enabling one to extend solving equations in one variable to those depending on a parameter. As an example, consider the equation

$$c^x - x^2 = 0 \tag{2}$$

Spreadsheet Conditional Formatting



x	y	a	b	n	h
-0.8100000	-0.0857181	-0.81	-0.74	100	0.0007
-0.8093000	-0.0843078				
-0.8086000	-0.0828983				
-0.8079000	-0.0814897				
-0.8072000	-0.0800819				
-0.8065000	-0.0786750				
-0.8058000	-0.0772689				
-0.8051000	-0.0758636				
-0.8044000	-0.0744592				
-0.8037000	-0.0730556				
-0.8030000	-0.0716529				
-0.8023000	-0.0702510				
-0.8016000	-0.0688500				
-0.8009000	-0.0674498				
-0.8002000	-0.0660505				
-0.7995000	-0.0646520				
-0.7988000	-0.0632543				
-0.7981000	-0.0618575				
-0.7974000	-0.0604616				
-0.7967000	-0.0590665				
-0.7960000	-0.0576722				

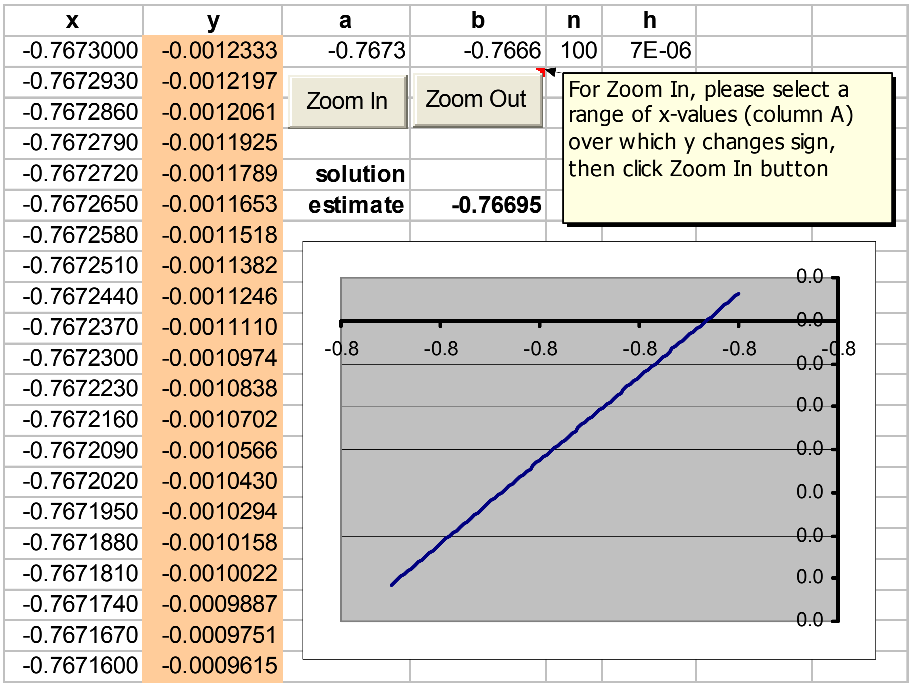
Figure 4: Solving $2^x - x^2 = 0$: after first Zoom In

with parameter $c > 1$. Figure 6 shows how equation 2 can be modeled on the spreadsheet both numerically and graphically with parameter c set as a slider-controlled discrete variable with any given increment. In particular, one can discover through visualization (Figure 6) that $c = 2.0813$ appears to be close to the maximum value that provides three solutions to equation 2.

3 Conditional formatting in teaching fundamentals of calculus

Among topics studied in elementary differential calculus, the topic dealing with infinite processes is of fundamental importance. The most basic concept that underpins such processes is that of limit. Mathematics education research spanning over the last twenty-five years indicates that the notion of limit, including its "epsilon-delta" notation, traditionally presents much conceptual difficulty for a great many students [32], [36], [11], [17]). It comes as no surprise that the "epsilon-delta" definition of limit is not only a real source of student misconceptions, but is, in many cases at least, glossed-over completely by the teacher.

The effectiveness of the use of computers in helping learners to overcome various misconceptions in calculus through visualization is well documented [37]. This includes the use of task-oriented computer programs, as well as the application of generic tools



x	y	a	b	n	h
-0.7673000	-0.0012333	-0.7673	-0.7666	100	7E-06
-0.7672930	-0.0012197				
-0.7672860	-0.0012061				
-0.7672790	-0.0011925				
-0.7672720	-0.0011789	solution			
-0.7672650	-0.0011653	estimate	-0.76695		
-0.7672580	-0.0011518				
-0.7672510	-0.0011382				
-0.7672440	-0.0011246				
-0.7672370	-0.0011110				
-0.7672300	-0.0010974				
-0.7672230	-0.0010838				
-0.7672160	-0.0010702				
-0.7672090	-0.0010566				
-0.7672020	-0.0010430				
-0.7671950	-0.0010294				
-0.7671880	-0.0010158				
-0.7671810	-0.0010022				
-0.7671740	-0.0009887				
-0.7671670	-0.0009751				
-0.7671600	-0.0009615				

Figure 5: Solving $2^x - x^2 = 0$: after second Zoom In

such as spreadsheets. More specifically, it has been shown how one can use a spreadsheet for visualizing limiting behavior of sequences and series [2], both through direct modeling and computerization of Bolzano-Cauchy principle of convergence [3].

In this section, the concept of *limit of a sequence* will be considered. The use of spreadsheet conditional formatting enables one to revisit this mathematically advanced territory and allow the software to assist in the understanding of the absolutely fundamental concept of limit. The approach, however, may be readily adapted to other limit concepts needed for beginning differential calculus.

Recall that a *sequence* of real numbers $\{a_n\}$ is said to have *limit* L as $n \to \infty$ if, given any $\epsilon > 0$, there exists $n_0 > 0$ such that $n > n_0 \Rightarrow |a_n - L| < \epsilon$. Given a particular sequence, and given the correct limit L, students may be asked to generate this sequence using a spreadsheet and find n_0 through conditional formatting. As an illustration, consider the sequence defined by equation 3.

$$a_n = \frac{8n - 3}{4n + 2} \tag{3}$$

By modeling this sequence on a spreadsheet, one can see that as n grows larger, the general term a_n gets closer and closer to 2; thus one may conjecture that $L = 2$ is the limit of the sequence $\{a_n\}$.

The next step is to use the model shown in Figure 7 in order to find n_0 given the value of ϵ. Let $\epsilon = 0.01$. Conditional formatting may be used to identify the first element

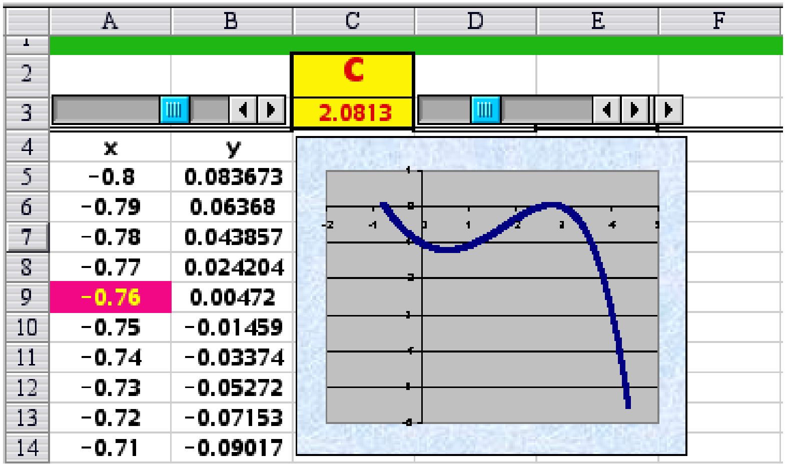


Figure 6: Enhanced $f(x) = 0$ model with slider-controlled parameter

of the sequence satisfying $|a_n - 2| < \epsilon$. Because

$$\left|\frac{8n-3}{4n+2} - 2\right| = \left|\frac{8n-3-2(4n+2)}{4n+2}\right| = \frac{7}{4n+2} \tag{4}$$

the inequality $7/(4n+2) < 0.01$ holds true for $n > 174.5$; thus the least n_0 would be 175.

Spreadsheets can also be used to illustrate graphically the limit of a sequence by the usual horizontal bands of width 2ϵ and centered on the limit L. By relating the conditional-format-identified n_0 to the definition of limit and to the graph, the teacher has a wealth of materials with which to support and motivate the algebraic approach of determining n_0, given ϵ, an outline of which is given by the sentence which embeds equation 4. A pedagogically sound extension of this activity is to show that a given value for a claimed limit is incorrect. That is, to show that there exists at least one $\epsilon > 0$ for which it is not possible to find n_0 with the property required by the above definition of limit. Consider the sequence of equation 3. In order to demonstrate that $L = 1$ is not its limit, one only has to produce a single value of ϵ for which the inequality $|(8n-3)/(4n+2)-1| < \epsilon$ does not hold true for infinitely many values of n. Observing the results of modeling of the (monotonically increasing) sequence 3 within the spreadsheet shown in Figure 7 enables one to recognize that its terms always differ from the claimed limit, $L = 1$, by at least $1/6$. This is almost the entire proof that $L = 1$ is *not* the limit of sequence 3. The challenge, of course, is to make students sufficiently aware that this counterexample (i.e., setting $\epsilon = 1/6$) is enough to complete the proof.

n	a[n]	abs(a[n] - limit)	limit	epsilon
1	0.833333333	1.166666667	2	0.1
2	1.3	0.7		
3	1.5	0.5		
4	1.611111111	0.388888889		
5	1.681818182	0.318181818		
6	1.730769231	0.269230769		
7	1.766666667	0.233333333		
8	1.794117647	0.205882353		
9	1.815789474	0.184210526		
10	1.833333333	0.166666667		
11	1.847826087	0.152173913		
12	1.86	0.14		
13	1.87037037	0.12962963		
14	1.879310345	0.120689655		
15	1.887096774	0.112903226		
16	1.893939394	0.106060606		
17	1.9	0.1		
18	1.905405405	0.094594595		
19	1.91025641	0.08974359		

Figure 7: Limit of a sequence in Excel

4 Modular arithmetic

In an invited lecture to ICME-7, van Lint [38], addressing the question "What is discrete mathematics and how should we teach it?" mentioned elementary number theory as one of the most important topics in the study of discrete structures at the tertiary level. Indeed, as teaching experience at Bond University of the second author indicates, for discrete mathematics classes at that level it is becoming increasingly important to include at least some coverage of the elementary theory of numbers. Many IT applications exist; perhaps most notable are the applications to mathematical cryptography, and in particular to Rivest-Shamir-Adleman (RSA; see, for example, [19]) and similar public key systems. The mathematics behind this system is simple enough to be presented in a first-year tertiary discrete mathematics class. In order to support the RSA investigations, one or two lectures are conducted, with supporting workshop and tutorial, on modular arithmetic and simultaneous linear congruences. This has been done with success [35], and students have made unsolicited comments on the practical utility of the material.

The solution of a set of simultaneous linear congruences may be regarded as the intersection of several arithmetic sequences. This concept can be modeled very neatly by making use of spreadsheet conditional formatting. It is worth noting that while spreadsheet-based modeling techniques are used for illustrative purposes, the students are also required to demonstrate mastery of an algebraic approach to solving simul-

	A	B	C	D	E	F	G
1		4	0		5	7	11
2		13		0	3	6	2
3		8	1	1	8	13	13
4		13	2	2	13	20	24
5		13	3	3	18	27	35
6		13	4	4	23	34	46
7		18	5	5	28	41	57

Figure 8: Simultaneous linear congruences

taneous linear congruences. The following example illustrates the point. Solve the simultaneous linear congruences 5–7.

$$x \equiv 3 \pmod{5} \tag{5}$$
$$x \equiv 6 \pmod{7} \tag{6}$$
$$x \equiv 2 \pmod{11} \tag{7}$$

The problem is to find non-negative integers satisfying all three congruences. Note that each such congruence defines an arithmetic sequence with common difference being the modulus. This enables one to generate numerical solutions to each of the congruences within a spreadsheet pictured in Figure 8 either through recursive or closed-form definition of an arithmetic sequence (columns `E`, `F`, `G` with corresponding modulo numbers defined in cells `E1`, `F1`, `G1`). The next step is to arrange so generated terms of the three sequences in increasing order (column `B`) and find the smallest number that is listed there three times. The `MODE` function applied to sufficiently large range beginning from cell `B3` displays the number 13 as the first solution in cell `B2`. The use of the `LOOKUP` function (defined in cell `B1`) enables the spreadsheet to generate positional rank (defined in column `C`, beginning cell `C3`) of the third 13 in column `B`. By changing the seed value in cell `C3` to 5 (positional rank of the third 13 plus one) allows one to eliminate from column `B` all numbers less or equal to 13, thus interactively display the next (in terms of numerical order) solution of the three simultaneous congruences. This process continues allowing one to generate solutions of the congruences one by one made visible through spreadsheet conditional formatting. A fundamental tool for working in modular arithmetic are the addition and multiplication tables. Such tables are easily constructed in a spreadsheet, and conditional formatting may be used to identify various patterns in the tables. As the slider is moved, interesting color patterns appear, disappear, and sometimes reappear. Students can be asked to explain this behavior. For example,

1. Why there is no zero product for prime modulus?

10

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	0	2	4	6	8
3	3	6	9	2	5	8	1	4	7
4	4	8	2	6	0	4	8	2	6
5	5	0	5	0	5	0	5	0	5
6	6	2	8	4	0	6	2	8	4
7	7	4	1	8	5	2	9	6	3
8	8	6	4	2	0	8	6	4	2
9	9	8	7	6	5	4	3	2	1

Figure 9: Modulo 10 multiplication table

2. Why there are highly regular patterns created by the highlight of zero products otherwise, and interesting ones for those being doubles of primes?

3. Why does each element in a prime modulus system have a multiplicative inverse?

4. Does the latter property hold true for other modulus systems? Why or why not?

A suitable Excel model, with conditional formatting set to identify zeroes and ones, plus a slider control to alter the multiplication is shown in Figures 9 and 10.

5 Ramanujan's taxicab numbers

Srinivasa Ramanujan was an Indian mathematician who collaborated with G. H. Hardy in the early part of the twentieth century [22]. Ramanujan was visited in hospital by Hardy, who had traveled in taxi number 1729, and remarked that 1729 was a rather boring number. According to the story, Ramanujan immediately countered that 1729 was in fact a very interesting number, being the smallest positive integer which is the sum of two perfect cubes in two different ways [21]. Indeed, $1729 = 10^3 + 9^3 = 12^3 + 1^3$. Using a spreadsheet, one can find all such "taxicab numbers" within a specified range; for example those numbers less than 5000 that are expressible as the sum of two cubes in two different ways.

To this end, one constructs a 20 by 20 table (Figure 11), so as to generate ordered pairs (p, q) where $1 \leq p \leq 20$ and $1 \leq q \leq 20$, defines the names `p` and `q`, and then fills each cell in the table with the expression $p^3 + q^3$. After defining the name `table` to refer to the block of cells in which the formula `=p*p*p + q*q*q` has been replicated, a cell outside the table is used to house the formula `=MODE(table)`. Upon selecting the table, one uses conditional formatting to highlight cells matching the mode. It can be seen that 1729 is highlighted four times. Because of the symmetry of the problem, it is recognized

11

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	2	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	2	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

Figure 10: Modulo 11 multiplication table

p\q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	9	28	65	126	217	344	513	730	1001	1332	1729	2198	2745	3376	4097
2	9	16	35	72	133	224	351	520	737	1008	1339	1736	2205	2752	3383	4104
3	28	35	54	91	152	243	370	539	756	1027	1358	1755	2224	2771	3402	4123
4	65	72	91	128	189	280	407	576	793	1064	1395	1792	2261	2808	3439	4160
5	126	133	152	189	250	341	468	637	854	1125	1456	1853	2322	2869	3500	4221
6	217	224	243	280	341	432	559	728	945	1216	1547	1944	2413	2960	3591	4312
7	344	351	370	407	468	559	686	855	1072	1343	1674	2071	2540	3087	3718	4439
8	513	520	539	576	637	728	855	1024	1241	1512	1843	2240	2709	3256	3887	4608
9	730	737	756	793	854	945	1072	1241	1458	1729	2060	2457	2926	3473	4104	4825
10	1001	1008	1027	1064	1125	1216	1343	1512	1729	2000	2331	2728	3197	3744	4375	5096
11	1332	1339	1358	1395	1456	1547	1674	1843	2060	2331	2662	3059	3528	4075	4706	5427
12	1729	1736	1755	1792	1853	1944	2071	2240	2457	2728	3059	3456	3925	4472	5103	5824
13	2198	2205	2224	2261	2322	2413	2540	2709	2926	3197	3528	3925	4394	4941	5572	6293
14	2745	2752	2771	2808	2869	2960	3087	3256	3473	3744	4075	4472	4941	5488	6119	6840
15	3376	3383	3402	3439	3500	3591	3718	3887	4104	4375	4706	5103	5572	6119	6750	7471
16	4097	4104	4123	4160	4221	4312	4439	4608	4825	5096	5427	5824	6293	6840	7471	8192

Figure 11: Ramanujan taxicab numbers showing the first solution, 1729

p\q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	9	28	65	126	217	344	513	730	1001	1332	1729	2198	2745	3376	4097
2	9	16	35	72	133	224	351	520	737	1008	1339	1736	2205	2752	3383	4104
3	28	35	54	91	152	243	370	539	756	1027	1358	1755	2224	2771	3402	4123
4	65	72	91	128	189	280	407	576	793	1064	1395	1792	2261	2808	3439	4160
5	126	133	152	189	250	341	468	637	854	1125	1456	1853	2322	2869	3500	4221
6	217	224	243	280	341	432	559	728	945	1216	1547	1944	2413	2960	3591	4312
7	344	351	370	407	468	559	686	855	1072	1343	1674	2071	2540	3087	3718	4439
8	513	520	539	576	637	728	855	1024	1241	1512	1843	2240	2709	3256	3887	4608
9	730	737	756	793	854	945	1072	1241	1458	1729	2060	2457	2926	3473	4104	4825
10	1001	1008	1027	1064	1125	1216	1343	1512	1729	2000	2331	2728	3197	3744	4375	5096
11	1332	1339	1358	1395	1456	1547	1674	1843	2060	2331	2662	3059	3528	4075	4706	5427
12		1736	1755	1792	1853	1944	2071	2240	2457	2728	3059	3456	3925	4472	5103	5824
13	2198	2205	2224	2261	2322	2413	2540	2709	2926	3197	3528	3925	4394	4941	5572	6293
14	2745	2752	2771	2808	2869	2960	3087	3256	3473	3744	4075	4472	4941	5488	6119	6840
15	3376	3383	3402	3439	3500	3591	3718	3887	4104	4375	4706	5103	5572	6119	6750	7471
16	4097	4104	4123	4160	4221	4312	4439	4608	4825	5096	5427	5824	6293	6840	7471	8192

Figure 12: Ramanujan taxicab numbers showing the second solution, 4104

that the table has some redundancy, however it is easier to construct a square table than a triangular one. To find other numbers satisfying the "taxicab" condition, just delete the contents of one of the cells with 1729; in Figure 12, this has been done in the column in which $q = 1$. The next solution, 4104, will automatically be highlighted by the conditional formatting condition (Figure 12). It seems that if a dataset is multimodal, Excel will find the smallest mode, although this is not stated in Microsoft's online help for Excel XP. Additional detail may be found elsewhere [34]. Similar techniques can be applied to enhance learning environments dealing with Pythagorean triples and partition of numbers into two squares [4].

6 Designing environments for younger learners

So far, the use of conditional formatting has been discussed in connection with the use of spreadsheets in undergraduate mathematics courses. The material of the next three sections stems from the first author's experience in using a spreadsheet in preparing teachers, in particular those learning to teach mathematics at the elementary level. The use of spreadsheets with younger children was advocated by the National Council of Teachers of Mathematics [29] and success of such use was reported in a number of publications over the past decade [16], [8], [9], [14], [15], [7], [5]. This section will show how conditional formatting allows for the simultaneous use of two pedagogically sound features of a spreadsheet–the use of macros and worksheet protection.

To this end, note that by protecting a worksheet one prevents accidental changes to cells filled with formulas structuring the worksheet. A macro is a series of commands and functions that can be run whenever one needs to perform a task repeatedly. An example of task based on the use of macros in connection with using a spreadsheet as a manipulative-computational environment was described elsewhere [6]. It shows how the use of an action-oriented macro turns a spreadsheet into a scaffolding device

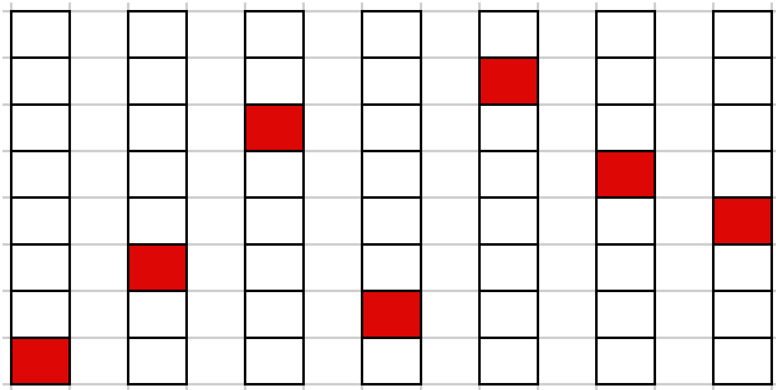


Figure 13: A seven-day map

[39] that supports one's thinking in trial and error form in a problem-solving setting. Both features are extremely beneficial when younger children are spreadsheet users; yet macros cannot be run directly on a protected worksheet. A way around such an incompatibility of the two features is to apply conditional formatting to cells which otherwise contain formulas, thus providing support of trial and error thinking through an appropriate semiotic feedback. Such formulas can be hidden within the conditional formatting dialogue box; thus allowing one to use this feature in place of worksheet protection.

As an example, consider the following "real-life" situation used as context for one of the assignments for the course "Using Spreadsheets in Teaching School Mathematics" designed for prospective teachers of mathematics and taught by the first author at the State University of New York [1].

> Once upon a time Hilton built an 8-story hotel in New York. The hotel turned out to be a very special one: every night a monster visited one floor there. The monster started with the 1^{st} (ground) floor, on the next night he skipped one floor and visited the 3^{rd} floor, then he skipped two floors and visited the 6^{th} floor, then he emerged on the 2^{nd} floor (by skipping floors 7, 8, and 1), then he skipped four floors to visit the 7^{th} floor. Then he continued going up and down by skipping five, six, seven and so on number of floors. Figure 13 shows the map of the monster's behavior during the first seven days.

Whereas the task for students in the course is to construct a manipulative-computational environment for the creation of such a map, the task for younger children is to extend this map to another two (or more) weeks by shading through macros appropriate cells in 8-cell towers prepared in advance by a teacher. A hidden didactic idea behind this task is to make young learners intuitively realize that skipping, say, 14 floors in an 8-story tower is equivalent to skipping `MOD(14,8)=6` floors and in doing so to build foundation for future learning of modular systems. Through this process, the children can make mistakes in developing the map of the monster's behavior. In order to provide an interactive feedback on their actions with the goal to help correcting possible mistakes (and thus *learn* from mistakes) one can create manipulative-computational environment that controls student's mathematical action. More specifically, if a student applies a macro to a wrong floor, the spreadsheet interactively fills with red color a cell immediately beneath the corresponding tower.

One strategy of establishing a hot-link between iconic and numeric notations of a spreadsheet is the use of binary code. For example, the far-left tower (a set of eight cells) in Figure 13 can be associated with an 8-vector $(0, 0, 0, 0, 0, 0, 0, 1)$, each element of which is a result of applying the Excel function `=COUNT` to a corresponding cell from the array. In turn, each such vector can first be viewed as a binary form (with one and only one non-zero face value) and then translated into decimal form (which, apparently, is a power of two) by using floor's number diminished by one as an exponent for base two. In such a way, the map pictured in Figure 13 can be associated with the seven powers of two: $2^0 = 1, 2^2 = 4, 2^5 = 32, 2^1 = 2, 2^6 = 64, 2^4 = 16$, and $2^3 = 8$. Note that the floor numbers in Figure 13 are congruent to the first seven triangular numbers modulo eight. This pattern continues for all triangular numbers but multiples of eight that correspond to the top (eighth) floor.

This makes it possible to generate a correct numerical map and interactively control the development of an iconic map by a student so that any incorrect shading results in the appearance of a red color underneath a corresponding grid through conditional formatting. In such a way, even if one accidentally hits a cell in which a warning of error may appear, the mechanism responsible for such a warning would not be deleted.

Note that the above context can be extended to include k-types of monsters, each of which starts from the first floor and skips kn floors on the n-th night. While the case $k = 1$ (i.e., the one mentioned above) corresponds to what may be referred to as triangular monster, the cases $k = 2, 3, 4$ correspond, respectively, to square, pentagonal, and hexagonal monsters. Through this activity, polygonal numbers can be introduced in a whimsical content, oriented to younger learners. The unity of the context and spreadsheet conditional formatting becomes a useful educational resource in building a foundation for learning significant mathematical ideas.

7 Developing meta-context through conditional formatting

An important component of mathematics teacher education program is providing prospective teachers with experience in formulating and discovering a solution to one's own problem. This experience, when used appropriately, has great potential to influence mathematics taught in schools [18]. It has been argued continuously that problem posing is an informed extension of problem solving [23], [13], [33] and thus the two intellectual activities are closely related. Whereas the role of technology in problem solving is well understood, the role of technology in problem posing is a less explored domain.

This section shows how spreadsheet conditional formatting can enhance a unified

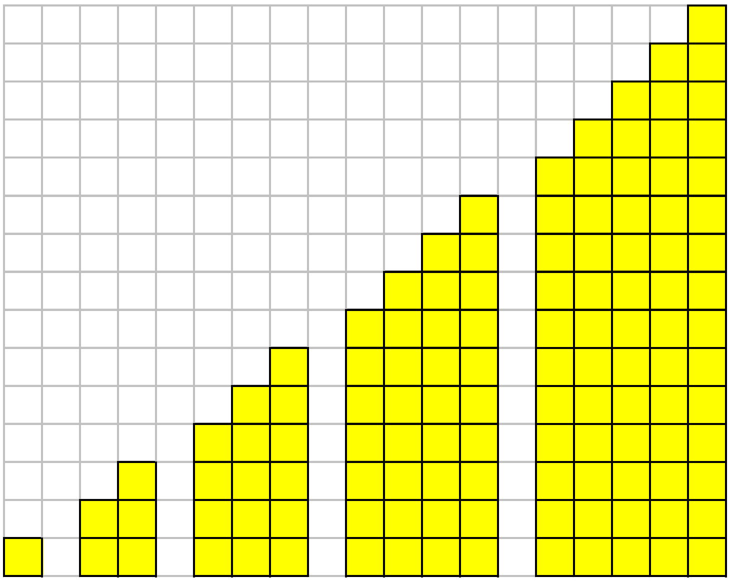


Figure 14: A blueprint of Jeremy's towers

pedagogy of problem posing and solving through creating meta-context [6] that supports the variation of syntactic and conceptual structures of a problem situation [23]. To begin, consider the following hands-on assignment designed for elementary pre-teachers enrolled in a problem-solving course.

> Jeremy is making towers from blocks according to the pattern of Figure 14. How many blocks does he need in order to build: (i) the 5^{th} tower in this pattern;(ii) the 10^{th} tower in this pattern; (iii) the n-th tower in this pattern.

While the answers to the above questions can be obtained by using a simple drawing (Figure 14) and are, respectively, $65, 505$, and $n(n^2+1)/2$, one can see that *Jeremy's problem* may be extended to allow for each tower's width to be two, three, four, etc. blocks greater than the previous one (Figure 15). In other words, in such extended situations, the widths of the towers in each set are arithmetic sequences with differences equal, respectively, to two, three, four, etc. Each new problem situation requires a separate blueprint as its meta-context. This brings about the need for the development of a computational environment capable of producing blueprints for any value of the difference of an arithmetic sequence involved. Once such an environment is developed, spreadsheet conditional formatting can come into play allowing one to generate such blueprints by changing just one parameter, namely, the difference, at the click of a mouse. Note that in a decontextualized form, *Jeremy's problem* is equivalent to that of arranging counting numbers in the groups

$$(1), (2,3), (4,5,6), (7,8,9,10), (11,12,13,14,15), \ldots,$$

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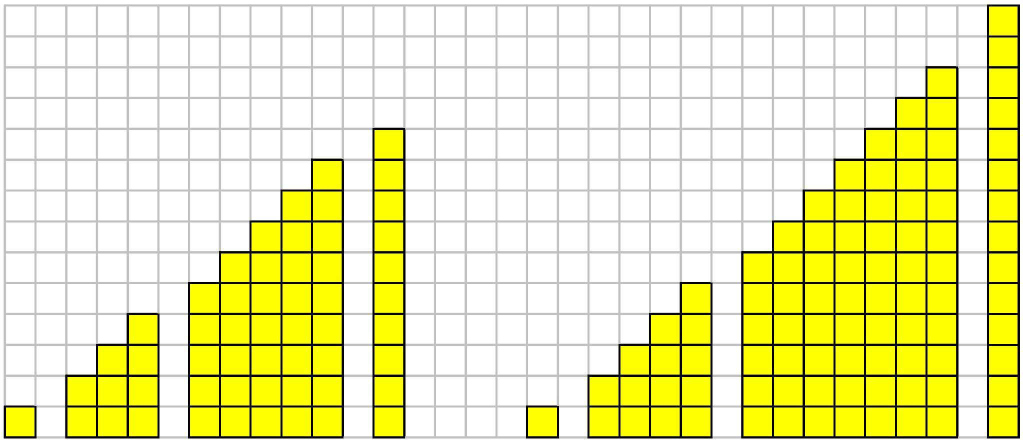

Figure 15: Extended blueprints

and then finding the sums of numbers in the 5^{th}, 10^{th}, and n-th group. Furthermore, the extended situations can be decontextualized in a similar way. Classroom observations over a number of years of teaching mathematical problem solving to elementary pre-teachers provided strong evidence that while the teachers are capable of recognizing a connection between contextualized and decontextualized versions of the problem, they fail to comprehend the latter when it is taught in a pure numeric context. Therefore, the worth of content-bounded mathematics didactics implies the importance of creating environments that support visual enhancement of such a didactics. Furthermore, Polya's famous heuristic advice [30] of using a simpler problem as a vicarious thinking tool points to the importance of developing context-bounded skills in problem formulating (or, better, re-formulating) among pre-teachers of mathematics. Such skills can be technologically enhanced through computerizing the production of meta-context involved.

The spreadsheet pictured in Figure 16 represents such a computer-generated meta-context. It shows consecutive natural numbers defined in row 1 beginning from cell `D1` which are used as mediators in generating other sequences. In the particular case of unit difference, the subsequence $2, 5, 9, 14, \ldots$ can be associated with the gaps between the towers. Because each term of the latter sequence is greater than the corresponding triangular number by its positional rank, an integer N belongs to this sequence provided that the equation $n(n+3)/2 = N$ has an integer solution. For that, the combination $\left(\sqrt{9+8N}-3\right)/2$ should be an integer. This criterion can be generalized to include any difference between the widths of consecutive towers and then computerized using a spreadsheet.

Spreadsheet conditional formatting comes into play when one highlights the rectangular enclosure in which towers are located, opens the conditional formatting dialogue box, enters a formula that identifies cells in the enclosure filled with positive integers, and chooses an appropriate font color and the outline option so as, respectively, not to display numerals and to outline cells that belong to a tower.

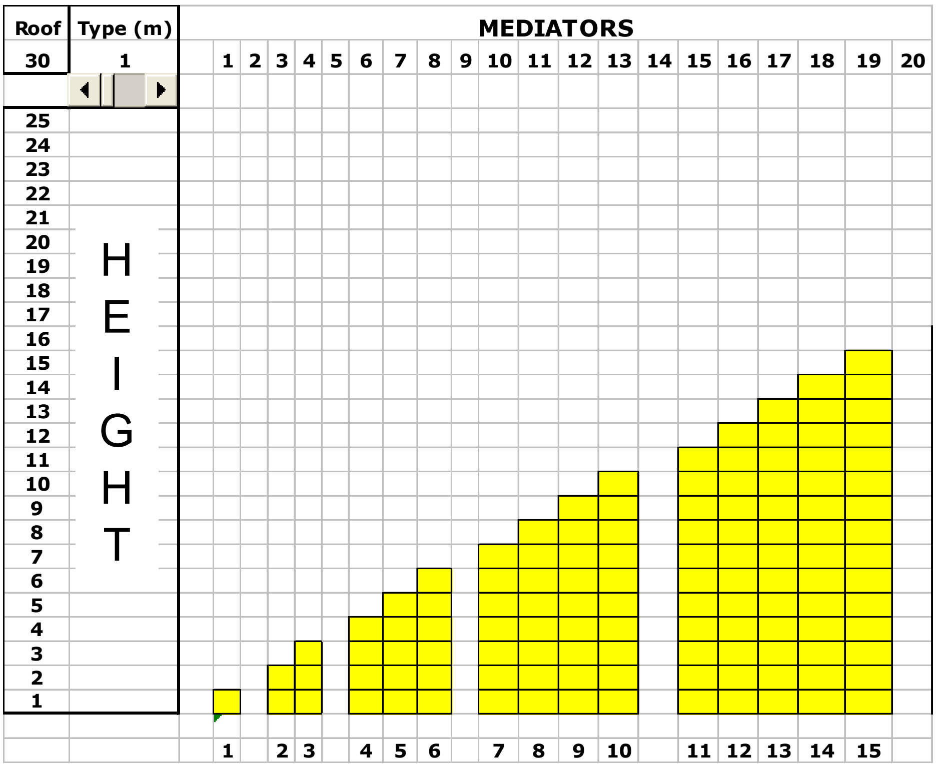

Figure 16: Jeremy's towers generated through conditional formatting

To conclude this section, note that other problems can stem from *Jeremy's problem* to allow for new mathematical concepts to be used in instruction. For example, by formatting the spreadsheet pictured in Figure 16 so as to display numbers hidden in the cells of the towers and in those beneath the towers, one can come across different polygonal numbers and see their distribution among the natural number sequence. In such an environment one can formulate and solve problems including, for example, the summation of numbers that belong to the same floor in a set of towers. In particular, the importance of triangular numbers as building blocks in exploring various numeric and geometric patterns can be highlighted.

Another type of problem may deal with the concept of geometric probability: *If one of Jeremy's towers is put into the circumscribed rectangular enclosure and a dart is thrown into it, what is the probability that the tower would not be hit?* Apparently, this inquiry is equivalent to *Jeremy's problem*. Conditional formatting has great potential to enhance the presentation and formulating of problems having such a geometric nature.

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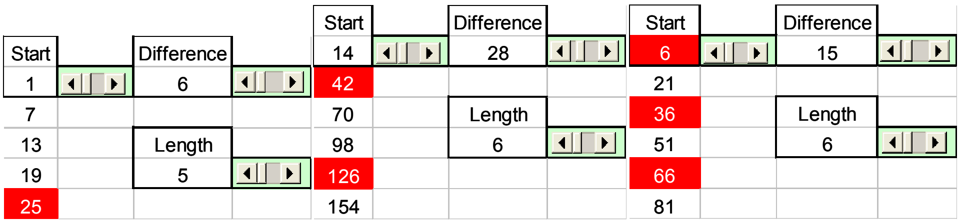

Figure 17: Three arithmetic sequences with a common property

8 Conditional formatting as a window on unsolved problems

Conditional formatting not only enhances problem solving, but also it can create an environment from which, quite unexpectedly, challenging problems can emerge. So, in introducing to pre-teachers of mathematics basic techniques associated with the use of spreadsheets, several computational environments were created, among them a generator of an integer arithmetic sequence of a variable size. Then conditional formatting was introduced as a tool allowing a spreadsheet to identify those terms that are multiples of its length. For example, a five-term arithmetic sequence $1, 7, 13, 19, 25$ has one multiple of five, a six-term arithmetic sequence $14, 42, \ldots, 154$ has two multiples of six, and six-term arithmetic sequence $6, 21, \ldots, 81$ has three multiples of six (Figure 17).

Proceeding from the three examples, one may conjecture that any n-term integer arithmetic sequence contains at least one multiple of n. Yet, the following three-term sequence $5, 8, 11$ defies this conjecture. Apparently, many sufficient (but not necessary) conditions can be formulated in terms of parameters of an arithmetic sequence that guarantee the above-mentioned phenomenon. The authors wonder: *Is it possible to formulate such conditions that are both sufficient and necessary?*

9 Conclusion

The authors have identified just a few examples in this paper, but the possibilities for educational applications of conditional formatting would seem to be very great indeed. It is hoped that the ideas expressed in this paper will serve at a catalyst for further research and investigation into the use of conditional formatting for mathematics education. Conditional formatting certainly appears to be a very much under-used feature of Excel; certainly for educational purposes. In a recent survey of the use of spreadsheets in education [10], it was found that almost no published examples of this potentially very powerful and useful facility exist.

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