August 2004

Constructing a spreadsheet simulation model of Chuck-a-luck: An application of logical functions

Fred J. Rispoli

Department of Mathematics and Computer Science Dowling College, Oakdale, NY, fred.rispoli@verizon.net

Follow this and additional works at: http://epublications.bond.edu.au/ejsie

Recommended Citation

Constructing a spreadsheet simulation model of Chuck-a-luck: An application of logical functions

Abstract

Chuck-a-luck is an inviting game played at carnivals in the Midwestern United States and England. A player picks a number from 1 to 6 and an operator rolls three dice. If the number chosen by the player comes up on all three dice, the player wins $3; if it comes up on two of the three dice, the player wins $2; if it comes up on one of the three dice, the player wins $1. Only if the chosen number does not appear at all does the player pay anything, which is just $1. We would like to know if Chuck-a-luck is a fair game, and what is its average payoff? In addition, we are interested in simulating this game using logical functions built into spreadsheet software as well as other computer programming languages.

Constructing a spreadsheet simulation model of Chuck-a-luck: An application of logical functions

Fred J. Rispoli

Department of Mathematics and Computer Science
Dowling College, Oakdale, NY

August 31, 2004

Background

Chuck-a-luck is an inviting game played at carnivals in the Midwestern United States and England. A player picks a number from 1 to 6 and an operator rolls three dice. If the number chosen by the player comes up on all three dice, the player wins $3; if it comes up on two of the three dice, the player wins $2; if it comes up on one of the three dice, the player wins $1. Only if the chosen number does not appear at all does the player pay anything, which is just $1. We would like to know if Chuck-a-luck is a fair game, and what is its average payoff? In addition, we are interested in simulating this game using logical functions built into spreadsheet software as well as other computer programming languages.

In the following exercises we will construct a simulation model of Chuck-a-luck using an Excel spreadsheet. The first exercise involves simulating dice. In Exercises 2, 3 and 4 the Chuck-a-luck model is developed and requires using the logical functions IF, AND, and OR, and the COUNTIF statistical function. The input to the model is a number from 1 to 6 which is chosen by the player. The output is the payoff for each game. Since the computation time required to simulate a game is miniscule, we can easily simulate 1,000 games in less than 1 second. We then analyze the simulation data using Excel formulas to compute summary statistics that estimate the average payoff of Chuck-a-luck. Exercises 5 and 6 require theoretical calculations of the probabilities of each payoff and the expected payoff of the game, which are compared to the simulation statistics to test for model accuracy. The last two exercises involve simulating other dice games using logical functions.

Before proceeding to the exercises we briefly review the functions needed. The IF function has the form

IF(logical statement that is either true or false, output if true, output if false).

For example, suppose that cell A5 contains the number 17. Then the Excel formula IF(A5<10,1,0) would return the value 0, since A5 < 10 is a false statement. We also note that the IF function can be nested which is needed in the simulation. For example, suppose that we want a cell with the following output:

return 2,   if A5 < 10;
return 1,   if 10 ≤ A5 < 20; and
return 0,   if 20 ≤ A5.

Then we could use the Excel formula IF(A5<10,2,IF(A5<20,1,0)).

The AND function has the form

$$\text{AND}(\text{statement } 1, \text{statement } 2, ... , \text{statement } n).$$

When all of the n statements are true, the return is TRUE, and otherwise the return is FALSE. The OR function has a similar form but the return is true if one or more of the statements are true, and is FALSE only when all n statements are false. To illustrate, suppose that A15 = 1, A16 = 2 and A17 = 3. The formula AND(A15=1,A16=2,A17=3) would return a TRUE, but AND(A15=1,A16 =1,A17=3) would return FALSE since now A16 = 1 is false. However, OR(A15=1,A16= 1,A17=3) would return TRUE. Once again, we note that the AND and OR functions can be nested. For example

AND(OR(A15=1,A16=1,A17=1),OR(A15=2,A16=2,A17=2),OR(A15=3,A16=3,A17=3))

would return TRUE.

The statistical COUNTIF function has the form COUNITF(cell range, criteria). For example, suppose that the column of cells from G1 to G75 contains data consisting of numbers from 1 to 10. If we want to count the number of times a 2 occurs, then we would use the formula COUNTIF(G1:G75,"=2"). Observe that the criteria should be in quotes.

Exercises

1. The first step in building a simulation model of chuck-a-luck is to simulate the tossing of three dice. To simulate a number obtained from tossing a die we need to use the RAND() function which returns random number uniformly distributed between 0 and 1. This number must then be multiplied by 6, truncated to an integer using the INT function, and then we add 1.

Create Excel formulas that simulate the tossing of three dice. Place the output in cells C11, D11 and E11 in the worksheet labeled Exercises.

2. Next we determine the payoff using logical functions. Suppose that a player chooses a number which is located in cell C6. The simulated values obtained from the three dice tossed by the operator are located in C11, D11 and E11. Then the logic behind calculating the Payoff is illustrated by the following flow chart.
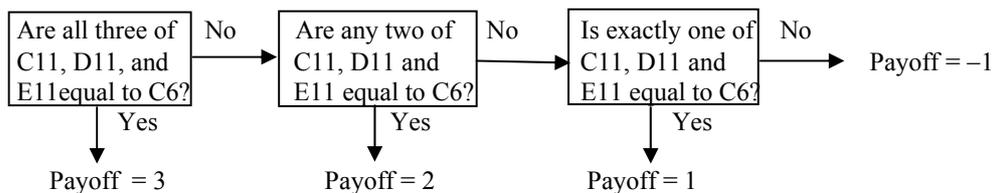
The functions needed to determine the payoff are the IF, the AND, and the OR logical functions reviewed in the Background Section. The idea is to first determine if C11 = C6, D11 = C6, and E11 = C6. If this is true, then we want the output to be 3. If this is false, then we use another if function to determine if one of the following is true:

$$C11 = C6 \text{ and } D11 = C6 \text{ and } E11 \neq C6, \text{ or}$$
$$C11 = C6 \text{ and } D11 \neq C6 \text{ and } E11 = C6, \text{ or}$$
$$C11 \neq C6 \text{ and } D11 = C6 \text{ and } E11 = C6.$$

If this condition is true, then the output value is 2. If it is false, then we use one more if function to determine if exactly one of C11, D11 or E11 equals C6. When this true, then return the value 1. If it is not true, then return the value −1.

Create Excel formulas that calculate the payoff using the simulated dice values in cells C11, D11 and E11, and the chosen number in cell C6. Place the output in cell G11 in the worksheet labeled Exercises. Once the Excel formula for the payoff is created you may simulated a game by simply pressing the F9 key.

3. The next step is to set up our model to repeat the simulation 1,000 times. Enter the numbers 2 and 3 in cells A12 and A13. Highlight the block of cells A11:A13. Then place

the mouse pointer in the bottom right corner of the highlighted block, click and drag the mouse down, filling in the column with game numbers. Stop at game number 1,000.

Before we copy the formula for the payoff, we must make the reference to cell C6 an absolute reference. Place dollar signs before and after C wherever C6 is used. Next highlight the cell range C11:G11. Copy and paste these stopping at game number 1,000.

4. (a) The data that we are interested in is located in cell range G11:G1010. Use the COUNTIF function to determine the probability of payoffs of $3, $2, $1 and −$1. Place these probabilities in column J.
(b) Use the average function to determine the average payoff. Place the average in cell J22.

5. Use counting principles to determine the probabilities of winning $3, $2, $1 and $−1, and determine the expected payoff for chuck-a-luck. Place your answers in column L.

6. Compare the answers obtained in Exercises 4 and 5.

7. Consider the following game. A player rolls one die. If the outcome is 3 or 5, the player wins $2. If the outcome is 1, 2, 4 or 6 the player must pay $1. Construct a simulation of this game. Place your formulas in cells P4 and Q4 in the Exercises worksheet. Test your simulation by typing the F9 key repeatedly to see if you are obtaining the correct output.

8. Consider the following game. A player rolls two dice. If the outcome on both dice is the same (i.e. doubles) the player wins $5. If the sum of the two dice is 7 or 11, the player wins $3. If neither of these occurs, the player must pay $1. Construct a simulation of this game. Place your formulas in cells P12, Q12 and R12 in the Exercises worksheet. Test your simulation by typing the F9 key repeatedly to see if you are obtaining the correct output.