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Keywords

portfolio analysis, mean-Gini, mean-variance

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Abstract

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1 Introduction

The mean-variance approach, under which investment decisions are based on the mean and the variance of the probability distribution of investment returns, has revolutionized risk analysis and portfolio management since its introduction by Markowitz [15] over 50 years ago. To this date, the mean-variance approach still dominates finance textbooks at undergraduate and graduate levels. Levy and Markowitz [12] justify its use by showing that mean-variance optimization is equivalent to maximizing a second order Taylor-series approximation of expected utility functions. The degree of exactness depends on

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the shape of the probability distribution of returns and the nature of the utility function involved. From a theoretical perspective, the mean-variance approach is appropriate only when investment returns are normally distributed or investors' preferences can be characterized by quadratic functions.

As the assumption of quadratic utility is known to be problematic on theoretical grounds, normality of investment returns becomes necessary for the mean-variance approach to hold. The validity of the assumption of normality or even near normality, however, is questionable when applied to financial assets such as derivatives (which include various forms of options on stocks and other assets), stocks from emerging markets, and hedge funds. The effect of derivatives in altering the resulting portfolio return distributions is well recognized in the literature.[1] In the case of emerging market equities, Bekaert and Harvey [2], for example, reject normality for most of the return distributions in their sample. Hedge funds, each being a privately managed pool of investment capital and originally intended for wealthy investors, have become the hottest investment vehicle over the last decade. According to Ibbotson and Chen [7], in 1990, there were about 530 hedge funds managing about 50 billion dollars and, by the end of 2004, there were over 8,000 hedge funds with about one trillion dollars under management. It is well known that hedge fund returns display option-like payoff patterns that are not normally distributed.[2] Thus, the usual mean-variance framework is not expected to work well for all these financial assets.

As explained below, a mean-Gini framework is an attractive alternative. The Gini coefficient is a measure of inequality (or dispersion) that was first introduced to the economics profession almost a century ago. The name is in honour of Corrado Gini who developed this measure. The mean-Gini approach, with Gini treated as a risk measure for investments, was eventually introduced to the finance profession by Shalit and Yitzhaki [19] in 1984. Since then, it has become a highly regarded and popular investment tool. Its role as a viable alternative to the mean-variance approach has been acknowledged by investment practitioners as well.[3]

To illustrate what Gini measures in an investment context, suppose that an investment return can be characterized as a random draw from a probability distribution. To see whether the distribution is widely dispersed, we can take many random draws, a pair at a time, and observe the magnitude of the difference between each pair. For a widely dispersed distribution, the magnitude of that difference as observed from many repeated random draws would tend to be large. In contrast, for a narrowly dispersed distribution, the magnitude of that difference would tend to be small.

The Gini coefficient formally captures the expected value of that magnitude. Therefore, a mean-Gini framework is similar to a corresponding mean-variance framework in that it also uses two summary statistics — the mean and a measure of dispersion — to characterize the distribution of a risky prospect. With Gini being a measure of disper-

[1] See, for example, Cheung, Kwan, and Yip [4], Lien and Luo [13], and Shalit [18].

[2] See, for example, Fung and Hsieh [6].

[3] Recent examples of mean-Gini studies include Lien and Shaffer [14], Shaffer and Demaskey [17], and Shalit and Yitzhaki [20]. See these studies for other references. As Biglova, Ortobelli, Rachev, and Stoyanov [3] report, the mean-Gini approach is among the several major approaches in practice.

sion, the framework provides a necessary condition for stochastic dominance regardless of the probability distribution of returns.[4,5] Further, unlike the mean-variance results, the mean-Gini results will not involve situations where an investor ends up selecting an inferior investment among competing prospects. This is due to the fact that the restrictive assumption underlying the mean-variance framework (to ensure its validity) does not apply to mean-Gini. Hence, mean-Gini is potentially a better framework than mean-variance.

Although mean-Gini is both conceptually and intuitively appealing and is well accepted by the academic finance profession,[6] it has not yet gained any textbook acceptance. This is in part due to the fact that its implementation in portfolio problems involving multiple assets is cumbersome. In contrast to what can be achieved within the mean-variance framework, which is well developed analytically, the corresponding mean-Gini solution methods are tedious, even for small-scale portfolio selection problems. As Okunev [16] explains, to determine an efficient mean-Gini portfolio without short sales (i.e., without allowing negative holdings of any assets) based on 100 assets with return data over 60 time periods requires solving a linear programming problem with as many as 3,640 variables and 1,772 constraints. Thus, the solution methods involved are much too complicated for coverage in courses of portfolio theory and investment analysis.

To bypass the above analytical complications, we propose here a simple spreadsheet-based approach for mean-Gini portfolio analysis. The analysis is particularly useful for investment situations where direct applications of the mean-variance approach are known to be inappropriate.[7] In what follows, Section 2 introduces the Gini coefficient and expresses it in an analytically convenient form. Section 3 shows, with an illustrative example, how the Gini coefficient can be estimated using Microsoft Excel^{TM} on

[4] If the cumulative distribution of asset X, defined in terms of wealth (or return), always lies to the *right* of the cumulative distribution of asset Y, then asset X is said to dominate asset Y according to first-degree stochastic dominance. In this case, the two cumulative distributions never cross each other. First-degree stochastic dominance applies to all increasing utility functions. If the dominance of one asset over another asset cannot be established by first-degree stochastic dominance, then we can try the second-degree stochastic dominance criterion. In order to use such a criterion, we require risk aversion of the investor. For a risk-averse investor, the increases in utility for constant changes in wealth (or return) decline as wealth (or return) increases. For any given investment outcome, if the accumulated area under the cumulative probability distribution of asset X is less than the accumulated area for asset Y, then asset X dominates asset Y according to second-degree stochastic dominance. This requirement of the two cumulative distributions ensures that the investor has a greater expected utility from investing in asset X than from investing in asset Y.

[5] The framework also provides a sufficient condition if the families of cumulative distributions intersect at most once. See Shalit and Yitzhaki [19] for the analytical correspondence between mean-Gini and stochastic dominance.

[6] Besides the studies referenced in footnote 3, see also Kolb and Okunev [8, 9].

[7] Just like the traditional mean-variance approach, the mean-Gini approach also relies on static optimization to reach its portfolio solutions. The lack of an analytically tractable dynamic version of the mean-Gini approach is its disadvantage as an investment tool. To this date, however, static mean-variance optimization is still routinely applied in practical investment settings. Thus, an alternative and easily implemented risk measure within a static framework, such as the Gini coefficient, which is free from the known problems associated with the variance of returns as a risk measure, is still practically relevant.

microcomputers. Section 4 extends the analysis to a portfolio setting to allow efficient mean-Gini portfolios to be constructed using available Excel tools. Finally, Section 5 provides some concluding remarks.

2 The Gini Coefficient

Following the approach of Shalit and Yitzhaki [19], let Z_1 and Z_2 be a pair of random returns drawn from a continuous probability distribution, where $f(z)$ and $F(z)$ are its probability density function and its cumulative probability density function, respectively. With the entire distribution extending over the range of $z = a$ to $z = b$, where $b > a$, we have

$$\int_a^b f(z)dz = \int_{z=a}^b dF(z) = 1 \tag{1}$$

and

$$F(z) = \int_a^z f(z)dz, \tag{2}$$

implying that $F(a) = 0$ and $F(b) = 1$. Then, with $E(\cdot)$ generally representing the expected value of a variable $(\cdot)$, the Gini coefficient is

$$\Gamma = \frac{1}{2} E\left(|Z_1 - Z_2|\right), \tag{3}$$

half of the expected value of the absolute difference between Z_1 and Z_2 given the above probability distribution.[8]

As expected values involving an absolute difference are difficult to work with, we now follow the same approach as in Dorfman [5] to express the Gini coefficient in an equivalent but analytically more convenient form. A crucial algebraic expression used in that study is[9]

$$|Z_1 - Z_2| = Z_1 + Z_2 - 2 \ \min(Z_1, Z_2), \tag{4}$$

where $\min(Z_1, Z_2)$ represents the smaller case between Z_1 and Z_2. Accordingly, we have

$$\Gamma = \frac{1}{2} \left\{E(Z_1) + E(Z_2) - 2E\left[\min(Z_1, Z_2)\right]\right\}, \tag{5}$$

where

$$E(Z_1) = E(Z_2) = \int_a^b zf(z)dz = \int_{z=a}^b zdF(z) \tag{6}$$

[8] Some authors simply define Γ as $E\left(|Z_1 - Z_2|\right)$ and some others, especially those using the Gini coefficient to study income inequality of the economy, define Γ as $\frac{1}{2\mu} E\left(|Z_1 - Z_2|\right)$, where μ is the mean of the distribution. We choose to stay with the form $\frac{1}{2} E\left(|Z_1 - Z_2|\right)$ in this pedagogic note, as it is the form commonly adopted for mean-Gini portfolio analysis in the finance literature. See, for example, Shalit and Yitzhaki [19, 20].

[9] If $Z_1 \leq Z_2$, then, with $\min(Z_1, Z_2) = Z_1$, the right hand side of the expression becomes $Z_2 - Z_1$, which is non-negative. If $Z_1 > Z_2$ instead, then, with $\min(Z_1, Z_2) = Z_2$, the right hand side of the expression becomes $Z_1 - Z_2$, which is positive. Thus, in both cases, the right hand side of the expression is the magnitude of the difference between Z_1 and Z_2, which is $|Z_1 - Z_2|$.

is the mean of the distribution, μ. Before we can work with the Gini coefficient, however, we still have to find an explicit expression for $E\left[\min(Z_1, Z_2)\right]$ in terms of $f(z)$ or $F(z)$.

For this task, consider an arbitrary value of z from the distribution $f(z)$ or, equivalently, the cumulative distribution $F(z)$. The probability that "both Z_1 and Z_2 are greater than z" and the probability that "at least one of Z_1 and Z_2 is not greater than z" must sum to 1 as the two probabilities cover all potential outcomes for each given value of z. With

$$\Pr(Z_1 \leq z) = \Pr(Z_2 \leq z) = F(z), \tag{7}$$

the probability that "both Z_1 and Z_2 are greater than z" is

$$\Pr(Z_1 > z) \times \Pr(Z_2 > z) = [1 - F(z)]^2. \tag{8}$$

Noting that the probability that "at least one of Z_1 and Z_2 is not greater than z" is the same as the probability that "the minimum of Z_1 and Z_2 is not greater than z," we have

$$\Pr[\min(Z_1, Z_2) \leq z] = 1 - \Pr(Z_1 > z) \times \Pr(Z_2 > z) = 1 - [1 - F(z)]^2. \tag{9}$$

The probability $\Pr[\min(Z_1, Z_2) \leq z]$ can be viewed as the value of the cumulative distribution function $G(y)$ of the random variable $y = \min(Z_1, Z_2)$, at $y = z$. As this random variable extends over the range of $y = a$ to $y = b$, it follows that

$$E\left[\min(Z_1, Z_2)\right] = \int_{y=a}^{b} y dG(y), \tag{10}$$

which is analytically equivalent to

$$\int_{z=a}^{b} z dG(z) = \int_{z=a}^{b} z d\left\{1 - [1 - F(z)]^2\right\} = 2\int_{z=a}^{b} z[1 - F(z)]dF(z). \tag{11}$$

The Gini coefficient, therefore, becomes

$$\Gamma = \mu - 2\int_{z=a}^{b} z[1 - F(z)]dF(z) = 2\int_{z=a}^{b} z\left[F(z) - \frac{1}{2}\right]dF(z). \tag{12}$$

Given that

$$E[F(z)] = \int_{z=a}^{b} F(z)dF(z) = \frac{1}{2}\left\{[F(b)]^2 - [F(a)]^2\right\} = \frac{1}{2}, \tag{13}$$

$$\int_{z=a}^{b} \left\{F(z) - E[F(z)]\right\} dF(z) = 0, \tag{14}$$

and

$$\int_{z=a}^{b} E(z)\left\{F(z) - E[F(z)]\right\} dF(z) = E(z)\int_{z=a}^{b} \left\{F(z) - E[F(z)]\right\} dF(z) = 0, \tag{15}$$

equation (12) can be rewritten as

$$\Gamma = 2 \int_{z=a}^{b} [z - E(z)] \{F(z) - E[F(z)]\} \, dF(z). \tag{16}$$

As the covariance of two random variables is the expected value of the product of their deviations from the corresponding means, this expression is equivalent to

$$\Gamma = 2 \ \mathrm{cov}\left[z, F(z)\right]. \tag{17}$$

Here, drawing on Dorfman [5] and Shalit and Yitzhaki [19], we have shown, in more analytical detail for pedagogic purposes, that the Gini coefficient of a random variable is twice the covariance of the variable and its cumulative distribution.[10]

3 Estimation of the Gini Coefficient

Now that we have expressed the Gini coefficient in terms of a covariance, its empirical estimation is greatly simplified as there is no need to take the absolute difference between two random draws from a distribution. Further, as Lerman and Yitzhaki [11] explain intuitively, the Gini coefficient as expressed in equation (17) is proportional to the covariance between the observed values of the variable z and the ranks of these observed values when they are sorted in an ascending order. The idea is that, with the individual ranks being $1, 2, \ldots, T$ for T observations of the variable, the cumulative distribution corresponding to the observation with a rank of t is t/T. For example, in the case of 100 observations of z, the 25th, 50th, and 75th lowest values of the 100 observations correspond to the cumulation distributions of 0.25, 0.50, and 0.75, respectively. The following is an illustration of how the Gini coefficient can be estimated by using Excel based on this intuitive idea.

Suppose that we have 12 monthly returns from an asset, stored under the heading "Return" in cells `C4:C15` of an Excel worksheet as shown in Figure 1.[11] With the 12 returns sorted in an ascending order, we assign a rank of 1 to the lowest of the returns, a rank of 2 to the next lowest, and so on, and indicate the corresponding ranks in cells `D4:D15`. If we explicitly sort the return data in an ascending order, they will appear as -0.030, -0.005, -0.001, $0.004, \ldots,$ 0.040, and the sorted returns will have individual ranks of $1, 2, 3, 4, \ldots, 12$, respectively. To simplify the task, however, we rank the returns in cells `C4:C15` by using the `RANK` worksheet function for cells `D4:D15` instead. For example, the return 0.010 in cell `C4` has a rank of 6 among the 12 cells in `C4:C15`. This return is the 6th lowest among the 12 returns. A rank of 6 is obtained by using the

[10] In additional to stating the Gini measure in terms of the expectation of an absolute difference [equation (3)] or a covariance [equation (17)], many economics students relate the Gini measure to the Area between the Lorenz curve and the diagonal line representing perfect equality. For these three equivalent representations and the relationship between stochastic dominance and the mean-preserving spread, see Atkinson [1].

[11] Notice that the reliance on only 12 data points for parameter estimation is not a good idea in practice. What is shown here is just an illustration of the method involved.

cell formula "=RANK(C4,C$4:C$15,1)" for cell D4, where the use of C$4:C$15 instead of C4:C15 allows the cell formula in D4 to be copied and then pasted to D4:D15. The order indicator "1" used in the RANK worksheet function can also be other integers except zero, as a "0" will lead to rankings in a descending order instead.

	B	C	D	E	F	G
2						
3	Month	Return	Rank			
4	1	0.010	6	D4=RANK(C4,C$4:C$15,1)		
5	2	0.020	9	(Copy and paste D4 to D4:D15.)		
6	3	0.015	8			
7	4	-0.030	1			
8	5	0.005	5			
9	6	0.022	10			
10	7	-0.005	2			
11	8	0.040	12			
12	9	0.030	11			
13	10	0.012	7			
14	11	0.004	4			
15	12	-0.001	3			
16						
17		Gini				
18		0.010288				
19		C18=2*COVAR(C4:C15,D4:D15)/(COUNT(C4:C15)-1)				
20						

Figure 1: An illustrative example of the estimation of the Gini coefficient using *Excel Solver.*

With all the returns sorted in an ascending order, the individual ranks when divided by the number of returns (which is 12 in this example) are the corresponding cumulative probabilities.[12] In order to estimate the Gini coefficient, we use the COVAR worksheet function, with the cell formula "=2*COVAR(C4:C15,D4:D15)/(COUNT(C4:C15)-1)" where the COUNT worksheet function provides the number of observations involved.[13] The

[12] In case of a tie, the RANK worksheet function assigns the same rank to the numbers involved To illustrate, if the four lowest numbers in the example were $-0.030, -0.005$, -0.005, and 0.004, instead, their ranks would have been $1, 2, 2$, and 4, respectively. Therefore, the individual rank when divided by the number of cases does not always capture exactly the cumulative distribution. If a sizeable number of realized returns is used to estimate the Gini coefficient, the problem due to a tie in the return observations should not be serious. In order to bypass the problem entirely, however, we can add an infinitesimally small random number (by using the RAND worksheet function, along with a small multiplicative constant) to each return to break any possible tie.

[13] Although there are separate worksheet functions in Excel for sample and population variances, only one worksheet function, COVAR, which is intended for population covariances, is available there for all covariance cases. In practical applications of mean-Gini analysis, as the number of observations to capture the return distribution tends to be large, any potential understatement of the sample covariance caused by using COVAR is likely inconsequential. In this illustrative example, where there are only 12 monthly return observations, an adjustment to the COVAR result seems necessary. The use of the denominator "COUNT(C4:C15)-1" instead of "COUNT(C4:C15)" in the cell formula is for this purpose. When

Gini coefficient thus obtained is 0.0102879. What this value means is that, if pairs of random draws are to be generated repeatedly from a distribution of monthly returns, for which the sample estimate of the distribution is based on the 12 observations of monthly returns as shown in cells `C4:C15`, the corresponding estimate of half of the expected absolute difference between a pair of monthly returns is about 1.03%. The greater dispersion of the return distribution, the greater is the expected absolute difference.

4 Mean-Gini Portfolio Selection without Short Sales

The idea of using the Gini coefficient to measure risk extends to portfolios of assets as well. Let $R_1, R_2, \ldots, R_n$ be the random returns of n assets. The random return of a portfolio p, which is a linear combination of these individual random returns, is

$$R_p = \sum_{i=1}^{n} x_i R_i, \tag{18}$$

where $x_1, x_2, \ldots, x_n$ are the portfolio weights (i.e., the proportions of investment funds as allocated to the individual assets) satisfying the conditions of

$$\sum_{i=1}^{n} x_i = 1 \tag{19}$$

and

$$x_i \geq 0, \text{ for } i = 1, 2, \ldots, n. \tag{20}$$

These conditions ensure that the available investment funds be fully allocated among the assets considered and that short sales of assets be disallowed. Further, let $\mu_i = E(R_i)$ be the expected return of asset i, for $i = 1, 2, \ldots, n$. The portfolio's expected return is

$$\mu_p = E(R_p) = \sum_{i=1}^{n} x_i \mu_i, \tag{21}$$

which is a linear combination of individual expected returns.

In the same manner as how equation (17) is derived, the portfolio's Gini coefficient can be written as

$$\Gamma_p = 2 \text{ cov}[R_p, F(R_p)], \tag{22}$$

where $F(R_p)$ is the cumulative probability distribution of the portfolio returns. The idea is that, for any given set of portfolio weights, as the random return of a portfolio is a linear combination of the random returns of individual assets in the portfolio with

comparing different investment prospects based on a common number of realized return observations, however, the choice of particular denominator in the cell formula has the effect of scaling all computed values of their Gini coefficients in the same manner. Such scaling does not affect the relative riskiness of the individual investment prospects.

known probability distributions, the portfolio's Gini coefficient can be established from the implied probability distribution of portfolio returns.

An equivalent form of equation (22), as reported in Shalit and Yitzhaki [19], [20], is

$$\Gamma_p = 2 \text{ cov}\left[\left(\sum_{i=1}^{n} x_i R_i\right), F(R_p)\right] = 2\sum_{i=1}^{n} x_i \text{ cov}[R_i, F(R_p)]. \tag{23}$$

This equivalent form indicates that the portfolio's Gini coefficient is twice the weighted average of the covariances between the individual asset returns and the portfolio's cumulative distribution. With $x_1, x_2, \ldots, x_n$ imbedded in $F(R_p)$, an analytical search for the set of portfolio weights that minimizes Γ_p subject to constraints (19)–(21) is a formidable task. However, given the currently available spreadsheet tools, such as the *Excel Solver* (which is an optimization tool developed by Frontline Systems, Inc.), mean-Gini portfolio construction is straightforward. The example below is an illustration for a three-asset case, where asset 1 is the same asset as considered earlier.

For simplicity, let us assume that the sample average return $\overline{R}_i$ is a good estimate of the expected return μ_i, for each asset i. The idea of using the Excel Solver is that, once we arbitrarily assign an initial set of portfolio weights, we can compute the portfolio returns from all realized returns of the individual assets. With the realized portfolio returns explicitly ranked, the estimated cumulative distribution of returns for that set of portfolio weights is revealed. Then, the portfolio Gini can be computed as well. The *Excel Solver* allows us to search numerically for the set of portfolio weights that minimizes the portfolio Gini under the constraints that the portfolio weights be summed to one and the sample average of the portfolio returns be equal to a specified value.

In the example, we specify the sample average of the portfolio returns to be $\overline{R}_p = 0.015000$ and arbitrarily set the initial portfolio weights to be $x_1 = x_2 = x_3 = 1/3$.[14] The worksheet involved is shown in Figures 2 and 3, where all worksheet functions and operations involved are explicitly indicated. Figure 2 shows how the worksheet is set up. The 12 observations of random returns of the three assets are entered in cells `D10:D21`, `E10:E21`, and `F10:F21`. Their mean returns, as computed using the `AVERAGE` worksheet function, are stored in cells `D24:F24`. Using the initial portfolio weights in cells `D32:F32` (copied from cells `D29:F29`), we compute in cell `H35` the portfolio mean return via an intermediate step in cells `D35:F35`.

The same portfolio weights in cells `D32:F32` allow us to compute, for each of the 12 observations, the corresponding portfolio return via an intermediate step in cells `D42:F53`. The 12 portfolio returns are stored in cells `H42:H53`, and their corresponding ranks are stored in cells `I42:I53`. The portfolio Gini, as stored in cell H24, is twice the covariance between the portfolio returns (in cells `H42:H53`) and their corresponding

[14] Like the various cases of mean-variance portfolio selection in Kwan [10] using the same spreadsheet tools, the final portfolio results are robust regardless of the initial portfolio weights attempted. Therefore, the choice of the initial portfolio weights can be arbitrary. Notice also that, for greater precision of the numerical results, options are available in the *Excel Solver* regarding search methods and convergence criteria.

	C	D	E	F	G	H	I	J	K	L
6	Asset Labels									
7		1	2	3						
8										
9	Input Data: Time Series of Monthly Returns									
10	1	0.010	0.020	0.041						
11	2	0.020	0.040	0.034						
12	3	0.015	-0.010	0.001						
13	4	-0.030	0.025	0.020						
14	5	0.005	0.015	0.050						
15	6	0.022	0.011	-0.030						
16	7	-0.005	0.003	0.042						
17	8	0.040	0.004	0.010						
18	9	0.030	0.022	0.022						
19	10	0.012	0.012	-0.020						
20	11	0.004	0.050	0.028						
21	12	-0.001	-0.020	0.017						
22										
23	Mean Returns					Portfolio Gini				
24		0.0101667	0.0143333	0.0179167		0.00702778				
25		D24=AVERAGE(D10:D21)				H24=2*COVAR(H42:H53,I42:I53)/(COUNT(H42:H53)-1)				
26		(Copy and paste D24 to D24:F24.)								
27										
28	Initial Portfolio Weights									
29		0.3333333	0.3333333	0.3333333	(Copy and paste D29:F29 to D32:F32.)					
30										
31	Portfolio Weights					Sum of Portfolio Weights				
32		0.3333333	0.3333333	0.3333333		0.99999999	H32=SUM(D32:F32)			
33										
34	Portfolio Weights Multiplied by Mean Returns					Portfolio Mean Return				
35		0.0033889	0.0047778	0.0059722		0.014139	H35=SUM(D35:F35)			
36		D35=D32*D24								
37		(Copy and paste D35 to D35:F35.)				Required Portfolio Mean Return				
38						0.015000				
39										
40	Time Series of Portfolio Weights					Time Series of Monthly Portfolio				
41	Multiplied by Monthly Returns					Returns and Corresponding Ranks				
42	1	0.003333	0.006667	0.013667		0.023667	9			
43	2	0.006667	0.013333	0.011333		0.031333	12			
44	3	0.005000	-0.003333	0.000333		0.002000	4			
45	4	-0.010000	0.008333	0.006667		0.005000	5			
46	5	0.001667	0.005000	0.016667		0.023333	8			
47	6	0.007333	0.003667	-0.010000		0.001000	2			
48	7	-0.001667	0.001000	0.014000		0.013333	6			
49	8	0.013333	0.001333	0.003333		0.018000	7			
50	9	0.010000	0.007333	0.007333		0.024667	10			
51	10	0.004000	0.004000	-0.006667		0.001333	3			
52	11	0.001333	0.016667	0.009333		0.027333	11			
53	12	-0.000333	-0.006667	0.005667		-0.001333	1			
54		D42=D$32*D10				H42=SUM(D42:F42); I42=RANK(H42,H$42:H$53,1)				
55		(Copy and paste D42 to D42:F53.)				(Copy and paste H42 to H42:H53; I42 to I42:I53)				

Figure 2: An illustrative example of Mean-Gini portfolio analysis using *Excel Solver*: initialization of the worksheet.

ranks (in cells `I42:I53`), divided by `[COUNT(H42:H53)-1]` (which is 11 in the example). Clearly, the initial portfolio weights cannot be the final results as the corresponding portfolio mean return in cell `H35`, 0.014139, deviates from the required value of 0.015000 in cell `H38`.

To use the Excel Solver to search for the set of optimal portfolio weights, we open the Solver dialog box from the menu bar. We specify the target cell, which is cell `H24` in the example. This cell is to be minimized. In the dialog box, we also indicate which cells to change in the search of the solution. Here, they are the three cells containing the individual portfolio weights (cells `D32:F32`). We also specify all the conditions to be satisfied. Specifically, we want these three cells to be non-negative, the cell containing the sum of all portfolio weights (cell `H32`) to be equal to 1, and the cell containing the portfolio mean return (cell `H35`) to be equal to the cell containing its required value (cell `H38`). The results of the numerical search are given Figure 3. This is the same worksheet in Figure 2 after the completion of the task by the *Excel Solver*.

Figures 2 and 3 show the numerical search for the efficient mean-Gini portfolio corresponding to $\overline{R}_p = 0.015000$ only. By using the same approach over different specified values of the portfolio mean return, over the range of 0.010167 to 0.017917 (the lowest to the highest sample mean among the three assets considered), we can construct the entire efficient frontier — the set of all efficient mean-Gini portfolios — without short sales. The procedure involved in each case will be the same as what is shown in Figures 2 and 3.

It is worth noting that the analysis as described above can be implemented on other spreadsheet products as well. The software Solver is available for use in *Lotus 1-2-3*^{TM} and *Quattro Pro*^{TM} as well. The standard versions of *Solver*, which can accommodate optimization problems with as many as 200 decision variables is already suitable for most mean-Gini portfolio selection problems in practice. Larger-scale problems, however, do require the use of enhanced versions of the same software.

5 Concluding Remarks

We intend to achieve two objectives in this pedagogic study. First, by showing in detail that the Gini coefficient can be written in terms of a covariance, we make the sophisticated mean-Gini approach more accessible to finance students while recognizing that familiarity with integral calculus and statistical concepts of probability distributions and cumulative probability distributions is still needed in order to understand more fully the analytical derivations involved. Second, with a numerical example, we illustrate that, although mean-Gini portfolio analysis appears to be more difficult than the mean-variance approach, it can still be performed numerically using currently available spreadsheet tools.

As investment assets are becoming more sophisticated, so are their return distributions. The mean-Gini approach, which is consistent with stochastic dominance criteria for decisions under risk, is ideal for portfolio analysis for a great variety of financial assets. They include various derivatives, emerging market equities, and hedge funds,

	C	D	E	F	G	H	I	J	K	L
6	Asset Labels									
7		1	2	3						
8										
9	Input Data: Time Series of Monthly Returns									
10	1	0.010	0.020	0.041		Solver Parameters:				
11	2	0.020	0.040	0.034		Target Cell: H24; Min				
12	3	0.015	-0.010	0.001		Changing Cells: D32:F32				
13	4	-0.030	0.025	0.020		Constraints: D32:F32>=0, H32=1, H35=H38				
14	5	0.005	0.015	0.050						
15	6	0.022	0.011	-0.030						
16	7	-0.005	0.003	0.042						
17	8	0.040	0.004	0.010						
18	9	0.030	0.022	0.022						
19	10	0.012	0.012	-0.020						
20	11	0.004	0.050	0.028						
21	12	-0.001	-0.020	0.017						
22										
23	Mean Returns					Portfolio Gini				
24		0.0101667	0.0143333	0.0179167		0.0081				
25		D24=AVERAGE(D10:D21)				H24=2*COVAR(H42:H53,I42:I53)/(COUNT(H42:H53)-1)				
26		(Copy and paste D24 to D24:F24.)								
27										
28	Initial Portfolio Weights									
29		0.3333333	0.3333333	0.3333333						
30										
31	Portfolio Weights					Sum of Portfolio Weights				
32		0.255814	0.2606814	0.4835046		1	H32=SUM(D32:F32)			
33										
34	Portfolio Weights Multiplied by Mean Returns					Portfolio Mean Return				
35		0.0026008	0.0037364	0.0086628		0.015000	H35=SUM(D35:F35)			
36		D35=D32*D24								
37		(Copy and paste D35 to D35:F35.)				Required Portfolio Mean Return				
38						0.015000				
39										
40	Time Series of Portfolio Weights					Time Series of Monthly Portfolio				
41	Multiplied by Monthly Returns					Returns and Corresponding Ranks				
42	1	0.002558	0.005214	0.019824		0.027595	10			
43	2	0.005116	0.010427	0.016439		0.031983	12			
44	3	0.003837	-0.002607	0.000484		0.001714	3			
45	4	-0.007674	0.006517	0.009670		0.008513	5			
46	5	0.001279	0.003910	0.024175		0.029365	11			
47	6	0.005628	0.002867	-0.014505		-0.006010	1			
48	7	-0.001279	0.000782	0.020307		0.019810	7			
49	8	0.010233	0.001043	0.004835		0.016110	6			
50	9	0.007674	0.005735	0.010637		0.024047	8			
51	10	0.003070	0.003128	-0.009670		-0.003472	2			
52	11	0.001023	0.013034	0.013538		0.027595	9			
53	12	-0.000256	-0.005214	0.008220		0.002750	4			
54		D42=D$32*D10				H42=SUM(D42:F42); I42=RANK(H42,H$42:H$53,1)				
55		(Copy and paste D42 to D42:F53.)				(Copy and paste H42 to H42:H53; I42 to I42:I53)				

Figure 3: An illustrative example of Mean-Gini portfolio analysis using *Excel Solver*: the final results.

where mean-variance portfolio analysis is not expected to be appropriate because of its strict distributional requirements on asset returns. With the mean-Gini approach better understood and more readily accessible, we hope that this pedagogic study can generate further teaching interests in applying mean-Gini to portfolio analysis and risk management involving non-traditional financial assets.

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