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The spreadsheet as a tool for teaching set theory: Part 1 – an Excel lesson plan to help solve Sudokus

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Keywords

Sudoku, Excel, conditional formatting, names, set intersection, set partition

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The spreadsheet as a tool for teaching set theory: Part 1 – an Excel lesson plan to help solve Sudokus

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This paper is intended to be used in the classroom. It describes essentially every step of the construction of an Excel model to help solve Sudoku puzzles. For those up to moderate difficulty, it will usually solve the puzzle to completion. For the more difficult ones, it still provides a platform for decision support. The paper may be found useful for a lesson in which students, who, having some basic knowledge of Excel, are learning some of its lesser-known features, such as conditional formatting. It also generates a useful tool for working with Sudoku puzzles, from the very easiest right up to the ones often labelled as *fiendish* or *diabolical*. Fundamental mathematical concepts such as *set intersection*, *set partition* and *reduction of set partition to singletons* are very graphically illustrated by the present Excel model for Sudoku. Prominent spreadsheet concepts presented here are *conditional formatting*, *names*, *COUNTIF*, *CONCATENATE*. The paper is accompanied by a completed Excel model, constructed by using the steps described herein. No VBA code is employed; the whole thing is done with Excel formulas and conditional formatting.

Keywords: Sudoku, Excel, conditional formatting, names, set intersection, set partition.

1 Introduction and background for the teacher

Sudoku is a game that has achieved phenomenal global popularity over the past few years. It is syndicated in many newspapers around the world, as well as appearing on numerous websites and in puzzle books.

Many sources refer to it as a “numbers game”. This is only true in the sense that the symbols 1 through 9 are used as placeholders. There is (almost) no arithmetic involved at all, which has prompted some newspapers to claim that there is no mathematics in Sudoku. The symbols 1 through 9 could equally well be A through I, as all that is required is to have 9 different objects or symbols.

The claim that “Sudoku puzzles will not be solved by mathematics” is, of course, utter nonsense. Perhaps what such published claims mean is that there is no *arithmetic* involved. Even this is false, but perhaps it is almost true, as the only real arithmetic is usually in counting how many of each digit is present or absent in a given puzzle, which may be partially solved, or as originally posed.

Sudoku may be used as a beautiful illustration of elementary set theory. As such, it has great potential for investigation into this fundamental, foundational area of mathematics, even at elementary (primary) school level. The author’s 5-year old grandson can do Sudoku (but prefers to go and play). However, perhaps from grade 4 upwards, we submit that this material may be made accessible to the students, in a

formal school curriculum. It will also give them an invaluable skill for the future: a basic facility with spreadsheets. This is useful for any person, whether they end up as accountants or not. The spreadsheet is a fundamental and versatile tool, and in my view, the education of a school-leaver in the 21st century should not be considered complete without at least a basic knowledge of this ubiquitous class of software.

We present a series of steps to construct a Sudoku helper in Excel. In the simplest cases, it will solve the puzzle to completion. For more complicated Sudokus, the model gives decision support to the player. It computes sets of possible values, based on set intersections, and allows the player to work with “patterns” consisting of various subsets of possible values. In this way, the model may be used by teachers to develop several important themes: basics of set theory, including intersections and partitions, *reductio ad absurdum*, and in the most difficult Sudokus, binary search with backtracking. Such concepts are usually taught at tertiary level, but we argue that with the support of Excel, the simplicity of Sudoku, and finally the intrinsic fun and satisfaction of puzzle-solving, this work becomes accessible even at the level of elementary school (primary school).

Again, we emphasize that the intention is not to solve it completely (although for very simple cases, it will do this), but as a helper which does the “boring and tedious” bits. Then, the intellectual energy of the player may be released to look for patterns, and thus, to employ and develop higher-level thinking skills. For classrooms with Excel, and this should be pretty much all computer labs nowadays, even a grade 4 or 5 class may investigate basic mathematical concepts, establish knowledge in the most fundamental branch of mathematics, set theory, and have some fun at the same time.

We have plans to develop a more detailed account of how Sudoku can contribute to the mathematics classroom. Look for this article in a future edition of SIE! The version of Excel relevant to this paper is Excel 2003. Only a few steps need be modified in order to work with Excel 2007, and these are related to rearrangement of menu selections.

2 Rules of the Game

The game consists of a 9 x 9 grid of cells, in turn split into nine 3 x 3 “blocks”. The rules are very simple: some numbers are given, and you have to fill-in the grid by finding the rest. Every row, column and block must contain the numbers 1 to 9. That’s it! It *sounds* simple, and certainly, some Sudoku puzzles are rather easy. But some are fiendishly difficult. We hope that, by building this Sudoku decision-support tool in Excel, you will have some fun. At the same time, you will learn quite a bit about spreadsheet formulas and modelling, and about the mathematical concept of a *set*. You will learn other spreadsheet esoterica too, such as *Conditional Formatting*. This is a very powerful facility available in the modern spreadsheet program, and makes it a lot easier to spot patterns. Let’s begin.

3 Workbook / Worksheet Setup

1. Open Excel and create a New Workbook: File > New > Blank Workbook.
2. Rename Worksheet 1 "Sudoku": Right click "Sheet 1" and go to "Rename". Type in "Sudoku".
3. Delete remaining worksheets, as you only require one for this exercise: Right click sheet 2 and select "Delete". Do the same for sheet 3.
4. Save the file: File > Save As. Give it the name "Sudoku". Click "Save".
5. Select the whole worksheet: Click on the top left square where the column and row reference names meet (i.e. to the left of "A" and above "1". The whole worksheet will turn blue. With the worksheet selected, do the following:
6. Set the font to Arial, size 14.
7. Centre all cell contents vertically and horizontally (Format > Cells > Alignment). Set both drop-down boxes to "Centre". Click OK.
8. Reset the column and row width and height to 30 pixels. Click between rows 1 and 2 and readjust the height to 30 pixels. Because the entire sheet is selected, all rows will be resized. Do the same with the columns. You should now have a grid of perfect squares.
9. Reset the width of columns A to K. Select them by clicking and dragging over the letters A to J (you will know they are selected as the columns will be blue and the column references, black. Click between A and B and resize the column to 40 pixels.

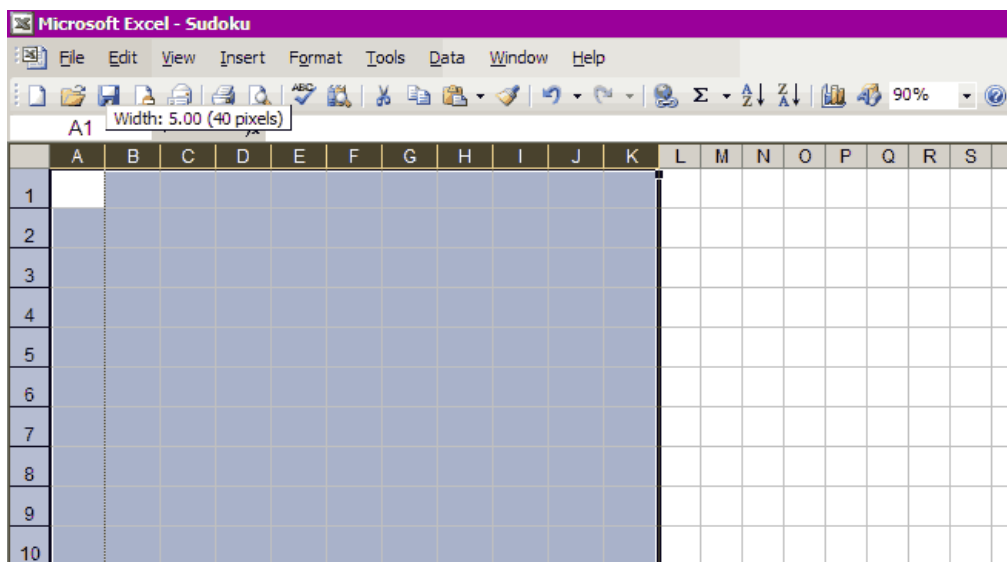


Figure 1

You are now ready to start creating your game!

4 Creating the “GAME” Grid

10. Click on cell B3 and drag across and down to cell J11 (Use the *name box* as a guide to which cells are selected; stop when it says 9R x 9C. Whilst cells B3:J11 are selected create an outline grid using the border button in the formatting toolbar (if it is not visible select View > Toolbars > Formatting). Now select cells B3 to D5 and create a bold border around them. While these 9 cells are still selected, drag the fill handle of D5 down to D11. Also while the 27 cells are still selected, drag the fill handle of D11 across to J11. You should end up with a grid like that looks like Figure 4.

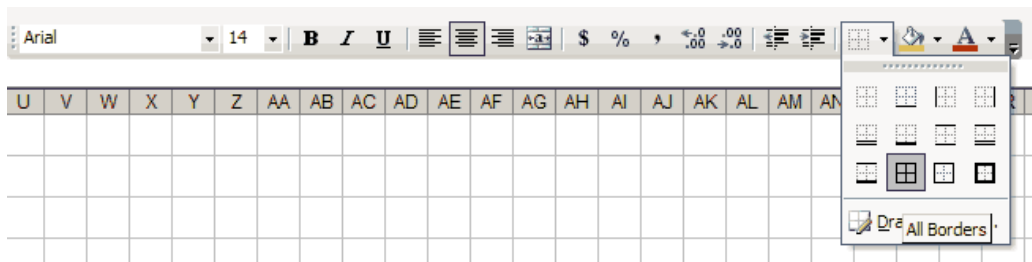


Figure 2

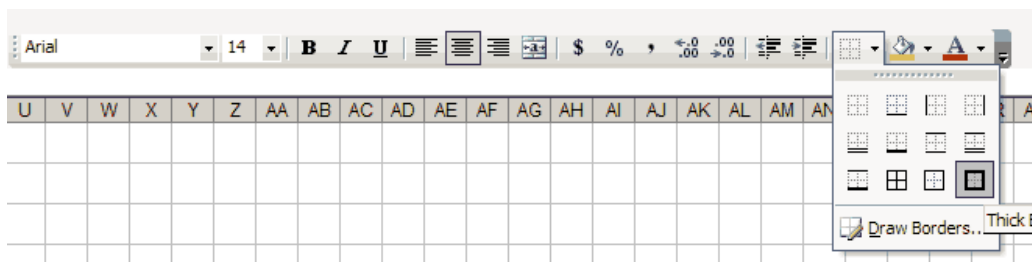


Figure 3

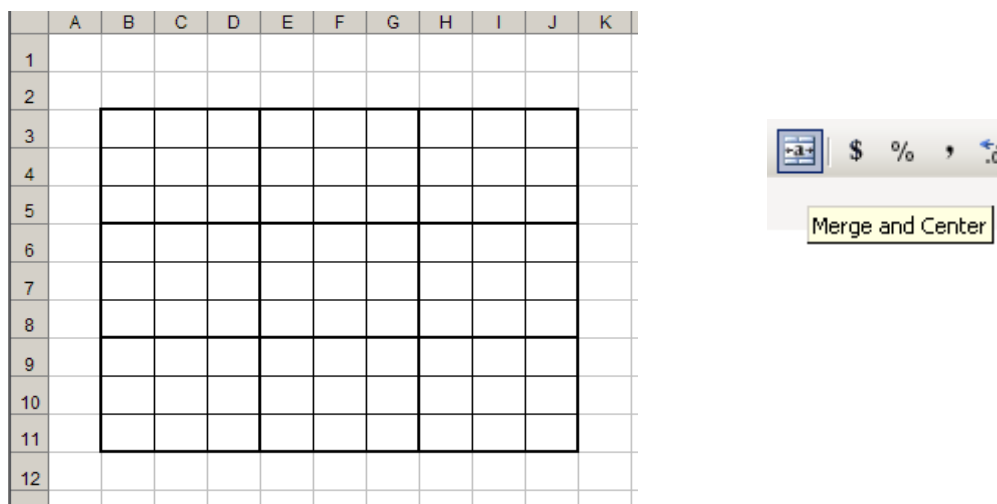


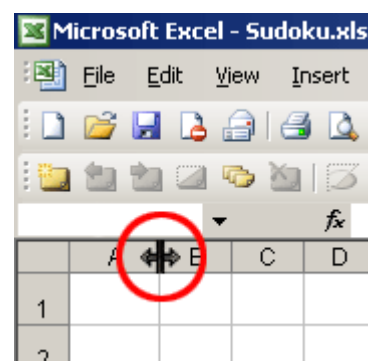
Figure 4

11. Select cells B1 to J1 and click the “Merge and Centre” button in the Formatting Toolbar. Type “Game” and make the font Bold. See figure 5.

5 Naming Rows, Columns and Blocks

12. Click on A3 and type “row1”. Now click on the fill handle of A3 and drag down to cell A11. The cells should now auto fill and read “row1” to “row9”. Resize column A as required if the contents does not fit. To do this, double click between A and B (Figure 6). Alternatively, just make the font smaller, say 8pt.

	A	B	C	D	E	F	G	H	I	J	K	L
1		GAME										
2		col1	col2	col3	col4	col5	col6	col7	col8	col9		
3	row1											
4	row2											
5	row3											
6	row4											
7	row5											
8	row6											
9	row7											
10	row8											
11	row9											
12												



Figures 5 and 6

13. Click on B2 and type “col1”. Now click on the right corner of the cell and drag your mouse across to cell J2. The cells should now auto fill and read “col1” to “col9”. To make these fit, reduce the font to 8pt.
14. Select Cells A2 to J11 and go Insert > Name > Create. Make sure that “Top row” and “Left column” are ticked and press OK. Excel will create names based on the row and column headings. To see what it has done, click in the name box. You will see a list of 18 names. Click on one of them and Excel will highlight that named range. These names are extremely important for us, as we need to continually refer to them in the formulas we are going to build. Use of names makes it so much easier to construct a correct Sudoku model.
15. We have defined our 9 rows and 9 columns. It remains to define the 9 blocks. Select cells B3 to D5 and type “block1” into the Name Box. Press “Enter”. **Important note: remember to hit “Enter” after each one, otherwise it won’t work.** Repeat the process on cells E3 to G5 and name that selection “block2”. Continue doing this for the remainder of the grid. The order of these names is indicated by the labels in Figure 6. You should have 9 “block” names in total. A visit to the name box now will reveal that we have 27 names defined: 9 rows, 9 columns, and 9 blocks.

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		GAME								
		col1	col2	col3	col4	col5	col6	col7	col8	col9
row1										
row2		block1			block2			block3		
row3										
row4										
row5		block4			block5			block6		
row6										
row7										
row8		block7			block8			block9		
row9										

Figure 6

16. Add a gentle pastel colour to the column and row names. Format > Cells > Patterns. See figure 8.

6 Creating a checking system for possible numbers

17. The next step involves finding cells in which the number “1” could possibly go according to the given puzzle in the “Game” grid. We need to write a formula to check if the number “1” can be put. Select Cells B3 to J11. Go to Edit > Copy. Click in cell M3 and go Edit > Paste. Your spreadsheet should look something like Figure 8.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1		GAME																					
2		col1	col2	col3	col4	col5	col6	col7	col8	col9													
3	row1																						
4	row2																						
5	row3																						
6	row4																						
7	row5																						
8	row6																						
9	row7																						
10	row8																						
11	row9																						
12																							
13																							

Figure 7

18. Type 1 in cell L2. Also, name it "one" by typing this name into the name box.
 19. Insert row and column numbers as shown in Figure 9. Choose a pleasing colour scheme. This grid is an image of our game, and the plan is to get Excel to figure out where a "1" can go. We develop a formula which will show us all the places where a "1" could possibly go in the main GAME grid.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	
1		GAME																					
2		col1	col2	col3	col4	col5	col6	col7	col8	col9		1	1	2	3	4	5	6	7	8	9		
3	row1											1											
4	row2											2											
5	row3											3											
6	row4											4											
7	row5											5											
8	row6											6											
9	row7											7											
10	row8											8											
11	row9											9											
12																							

Figure 8

20. But first, enter the following puzzle into your "Game" grid. We use this to test if our "possible 1s" formula works.

	A	B	C	D	E	F	G	H	I	J	K
1		GAME									
2		col1	col2	col3	col4	col5	col6	col7	col8	col9	
3	row1	1		7		5			9	6	
4	row2	8				3					
5	row3	4					9	5		8	
6	row4	7	6						2	3	
7	row5	3		4	8			1		9	
8	row6			1	9	6		4			
9	row7	5				8	2		7	4	
10	row8	6						3			
11	row9			2	3	4					5
12											

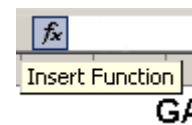


Figure 9

21. Now, a "1" can go in a cell only if it is not already in the same row or the same column or the same block. Our strategy is as follows: Pick a cell, say the top left one (M3). We get Excel to count the number of ones in the same row,

column, and block. If all of these are zero, the “1” must be one of the “possibles” for that cell. We start with counting how many ones there are in row1. Select cell M3 and click on the “Insert Function” button to the left of the formula bar. Next to “Or select a Category” use the drop-down list to select “Statistical”. See Figure 11. Navigate your way to “COUNTIF”. Click the OK button.

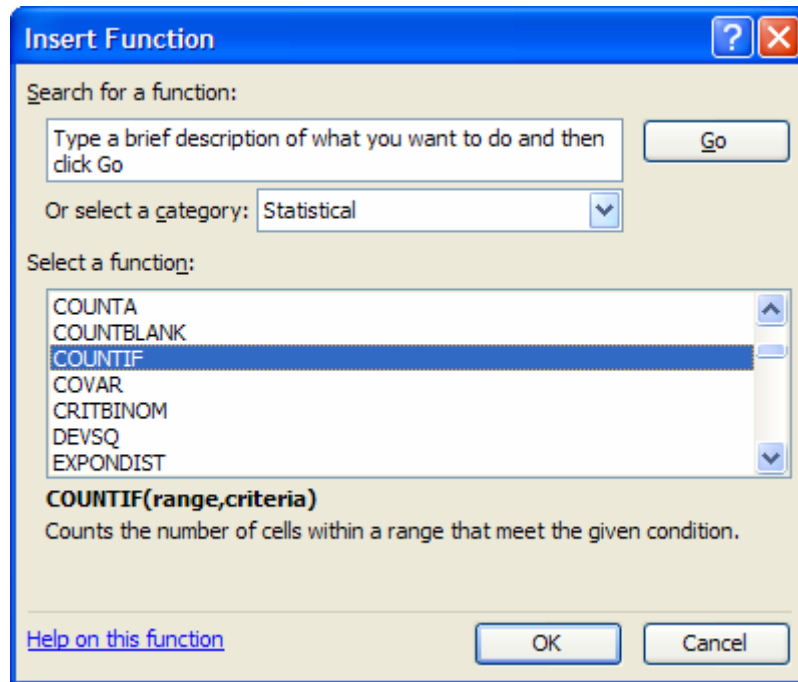


Figure 10

22. Click the white box after the word “Range” and then select cells B3 to J3 (i.e. “row1”). Press TAB. Click on cell L2 (i.e. “one”) and press enter. Your dialog box should look like Figure 12. Press “OK”. You should now have a “1” in cell M3. Why?

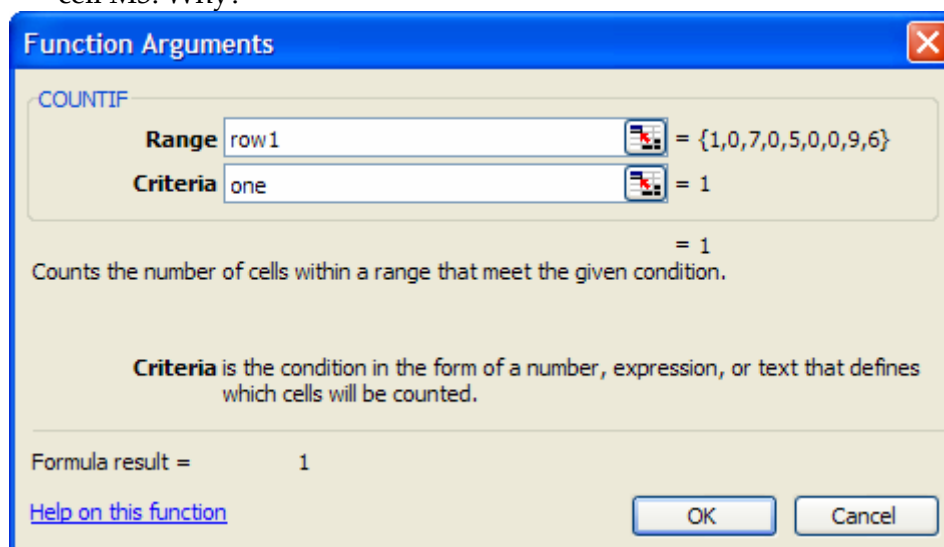


Figure 11

23. We now have the number of ones in row1. But this on its own is not enough. *Idea: what if we do the same for col1 and block1 (COUNTIF) and add them up?* If this number is zero, that means that “1” does not appear in row, col1, or block1. Therefore, it is eligible to go in the current cell. So, in summary, we need to amend our formula to add up the three “COUNTIF” functions. We also need to see if this addition comes to zero. If it does, then we want to display a “1” in the cell, otherwise nothing. Amend the formula as shown in Figure 13. *Save your work.*

	A	B	C	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V		
1				GAME																		
2				col1	col2	col3	col4	col5	col6	col7	col8	col9	1	1	2	3	4	5	6	7	8	9
3	row1	1		7		5			9	6			1									
4	row2	8				3							2									
5	row3	4					9	5		8			3									
6	row4	7	6							2	3		4									
7	row5	3		4	8				1		9		5									
8	row6			1	9	6		4					6									
9	row7	5				8	2		7	4			7									
10	row8	6							3				8									
11	row9			2	3	4					5		9									

Figure 12

24. Now we need to copy the formula to the remaining cells of our 9 x 9 grid. There are various ways to do this. Probably the simplest way, and one which also preserves our border formatting is as follows. Click in M3 and copy its contents to the clipboard (CTRL-C or copy button). Then select the entire 9 x 9 grid (M3:U11). Now right-click the mouse somewhere on the selection and go Paste Special. Check the Formulas radio button and hit OK. All 81 cells of the grid will now contain the same formula as M3.
25. Some changes to the formulas are now in order. The original formula in M3 referred to row1, col1 and block1, but this is only correct for M3. The formula for N10, for example, should make reference to row8, col2, block7. Now, we don't want to have to go to all of the 80 remaining cells and change each one separately! We get Excel to do most of the repetitive work, and have to do just a little repetition ourselves. Here are the steps.
26. Select the range M4:U4. All formulas here should refer to row2. Go CTRL-H to get the Find & Replace dialog box. In “find what” type “row1”, and in “replace with” type “row2”. Leave out the quotation marks. Click “Replace All” and Excel will report 9 replacements. *Do not close the Find & Replace*

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- dialog*. Now select the next row (M5:U5) and replace “row1” by “row3”. You will notice that “row1” is still there so you don’t have to retype it. Continue in this fashion until row9 is done. Close the Find and Replace dialog box. *Save your work*.
27. Now do a similar thing with the column references. Column 1 (M3:M11) is already correct, so start with col2 (N3:N11). Find and replace “col1” with “col2”. Continue as described in the previous step until all nine columns are correct. *Save your work*.
 28. Finally, we must get the block references correct. Still working with our “ones” grid of 9 x 9, select the cells in our “ones grid” corresponding to block2 of our game (block1 is already correct). This is the range P3:R5. Now do find & Replace “block1” with “block2”. Keep going until all nine block references are correct. *Save your work*.
 29. You should now observe several ones in our “ones” grid. Look at the pattern of ones carefully. They should represent the cells where ones could go in the game, with a few exceptions. Can you see where these are? *Hint: A one cannot go in a cell which is already occupied*. We’ll fix this problem later.
 30. Now that we have a grid which (almost) tells us where ones can go. We need the same information for the rest of the numbers 2 up to 9. To get this is quite easy. The first step is to copy the “ones” grid: select the range L2:U11 (the grid with row and column labels plus the “1” in the top left corner). Copy this range to the clipboard.
 31. Now we generate the remaining eight grids. Figure 14 gives a suggested layout. Note that there is a gap of two rows/columns between each 9 x 9 grid. This is intentional, and will be used later if you continue on to the advanced section.

SUDOKU IN EXCEL

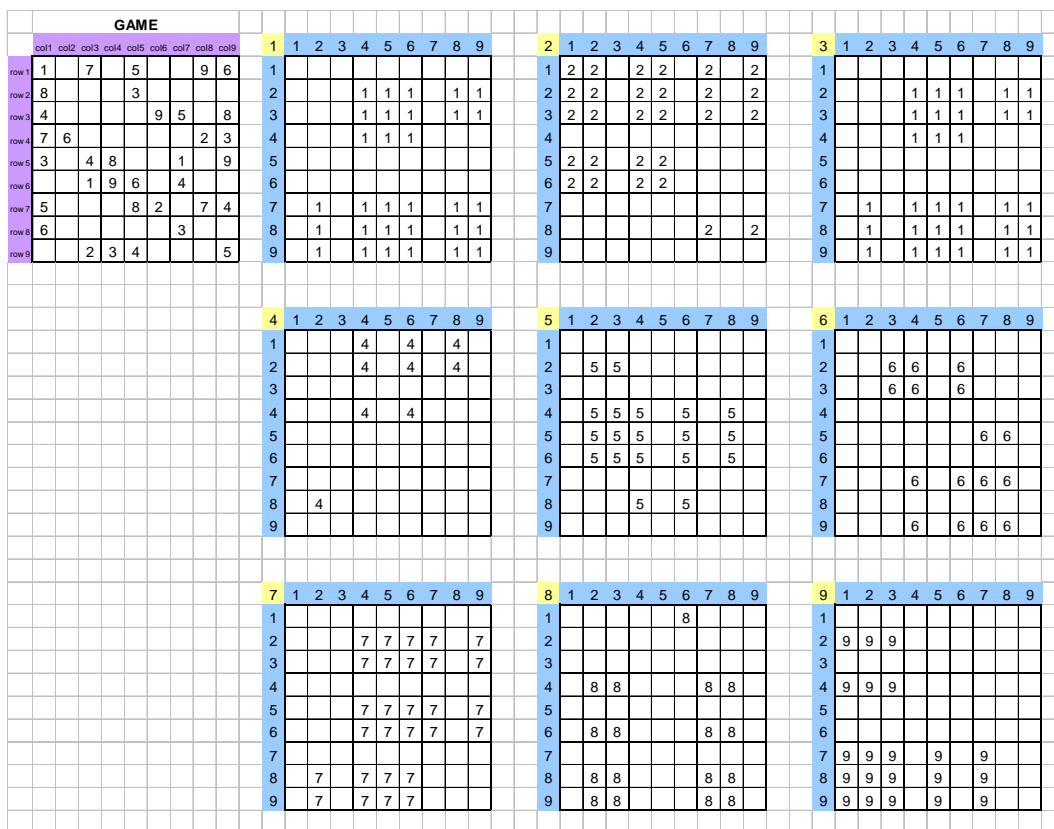


Figure 13

32. You should now have 9 grids making reference to the cell named “one” (cell L2). We now select the grid that we want to give us all possible locations of the “2” values. Firstly, assuming you have the layout of Figure 14, select the top left cell of the new grid which will locate where *twos* will go (this should be cell X2), go to the name box, and name it “two” (without the quotes). Also put the value 2 in this cell.
33. The final step in making our “grid of twos” (which will currently be a grid of ones) is simply to replace all the ones with twos in the 81 formulas. As each formula makes reference to “one” four times, this will involve $81 \times 4 = 324$ replacements! We think you’ll agree that it’s good that Excel does this with ease, and we don’t have to do them one-by-one. Brief steps: select the 9×9 grid (without the row and column labels). CTRL-H and replace “one” by “two”. Excel should report 324 replacements. You can leave this dialog open and do the replacements on the other 7 grids. Most of the work is now done. Don’t forget to replace the numbers at the top left of each grid, and create names “three” up to “nine”. *Save your work.*
34. We now have pretty much all the information we need to see just what are the possible numbers in each vacant cell of the GAME square are. We now create our final grid, which we’ll call POSSIBLES. Create it by just putting

borders on cells just below our GAME grid (see Figure 15). The simplest way to do this is to select the original game grid (B3:J11), CTRL-C, then click in B15, then go Paste Special, Formats. Name it POSSIBLES.

35. We now need to consolidate our 9 “possibility grids” into our new POSSIBLES grid. This is a great help in solving the puzzle. The most important idea is to use the CONCATENATE function to string together all the (potentially 9) possible for each cell. Then there are some small adjustments we need to make afterwards. First step: click in the top left cell of POSSIBLES grid. This should be the cell B15. Then Insert Function CONCATENATE. It is on the list of text functions. A dialog will appear as shown in Figure 16.
36. For the contents of the first box “Text1”, click in the top left cell of our “ones” grid. This should be M3. Tab to the next textbox of the dialog. Then click on the top left cell of the “twos” grid. This should be Y3. Notice that an extra box has appeared in the CONCATENATE dialog. This is good, because we need nine altogether. When you have strung together the top left cells of all our nine grids in this way, click OK. The cell in our POSSIBLES now has a list (with no commas) of the only numbers that can go in that cell.
37. Since the list of possibles can be long, we need a smaller font. Select the whole POSSIBLES grid and make it a very small font (say 8 pt or even 6pt).
38. We’re almost done! But we must first fix the little problem mentioned earlier. Did you spot it? It is simply this: in each of our grids from one to nine, we still report a “possible” even if the corresponding GAME cell is occupied! We ignored that, as it is much better to deal with it now. We need to modify our prototype CONCATENATE formula in cell B15 to filter out such cases. Instead, we would like to copy the already correct value from our GAME grid to the POSSIBLES grid. This gives us *context information* for the POSSIBLES. Finally, for technical reasons which emerge later, we convert our string of possible to a number by using the VALUE function. The final formula is shown here. You might have to adjust references.
`=IF(ISBLANK(B3),VALUE(CONCATENATE(M3,Y3,AK3,M15,Y15,AK15,M27,Y27,AK27)),B3)`
39. Now copy this formula (CTRL-C), then select the entire POSSIBLES grid (B15:J23), and go Paste Special, Formulas to get a final, correct list of possibles.

GAME											
	col1	col2	col3	col4	col5	col6	col7	col8	col9		
										1	1
row 1	1		7		5			9	6	1	
row 2	8				3					2	
row 3	4					9	5		8	3	
row 4	7	6						2	3	4	
row 5	3		4	8			1		9	5	
row 6			1	9	6		4			6	
row 7	5				8	2		7	4	7	
row 8	6						3			8	
row 9			2	3	4				5	9	
POSSIBLES										4	1
	2	2		24	2	48	2	4	2	1	
	29	259	569	2146	1217	11467	27	114	1217	2	
	2	2	6	12167	1217	1167	27	11	1217	3	
	9	589	589	1145	11	1145	8	58		4	
	2	25	5	257	27	57	67	56	7	5	
	2	258	58	257	27	57	78	58	7	6	
	9	119	9	116	119	116	69	116	11	7	
	9	1478	89	1157	1179	1157	289	118	121	8	
	9	11789	89	1167	1179	1167	689	1168	11	9	

Figure 14 Possibles before checking for occupied cells

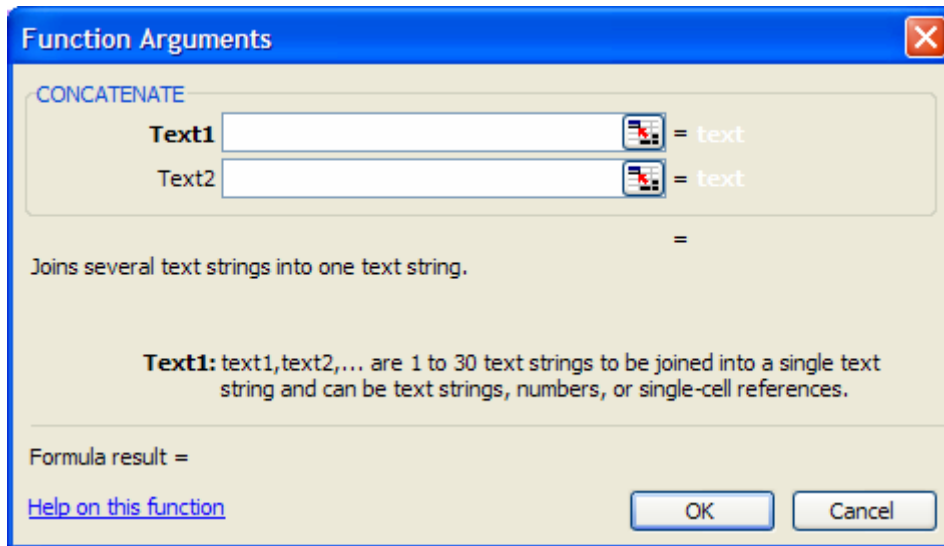


Figure 15

7 Conditional Formatting

40. The idea behind conditional formatting is to make certain data more visible, depending on its nature or usefulness. We'll be formatting the POSSIBLES grid so that it will be easy to see which numbers can be entered and how many possible numbers there are for any given square in the puzzle. Select cell B15. Go Format, Conditional Formatting and the dialog box (Figure 17) will appear. We are going to ask Excel to highlight the cells of our POSSIBLES grid in three different ways. We specify three conditions in a certain order. Excel checks the cell contents to see if it matches each condition, in order. If a match is found, the required format is applied and the remaining conditions ignored. If no match is found, no special format is applied. Here are our conditions in order.
 41. If the cell corresponds to one already occupied in the GAME grid, then use a light grey font. This gives us *context information* for the POSSIBLES.
 42. If the value is single digit (less than 10), then this is a *definite*. We indicate that by a bright green background colour in the cell.
 43. Finally, if the value is between 10 and 99 (just two POSSIBLES), we use an amber or light orange background.

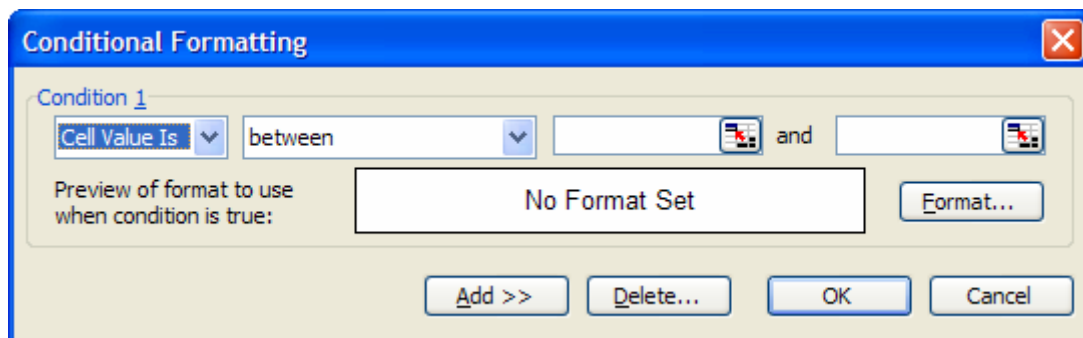


Figure 16 The first CF condition

44. Under “Condition 1” select “Formula Is” from the dropdown list. Enter =B3=B15. Then click on the “Format...” button and click on the Font tab. Change the colour of the font to a light grey. This is our first condition, designed to identify cells already completed in the GAME grid. *Do not close the dialog.*
45. Click on the Add >> button. Set “Condition 2” to the settings of Figure 18. This time we make the background green, not the font. You’ll find this under the Patterns tab after pressing the Format button of the conditional formatting dialog box. Because it will be highlighting all cells less than 10, any green cell numbers can be entered into the “Game” grid. This is our second condition.

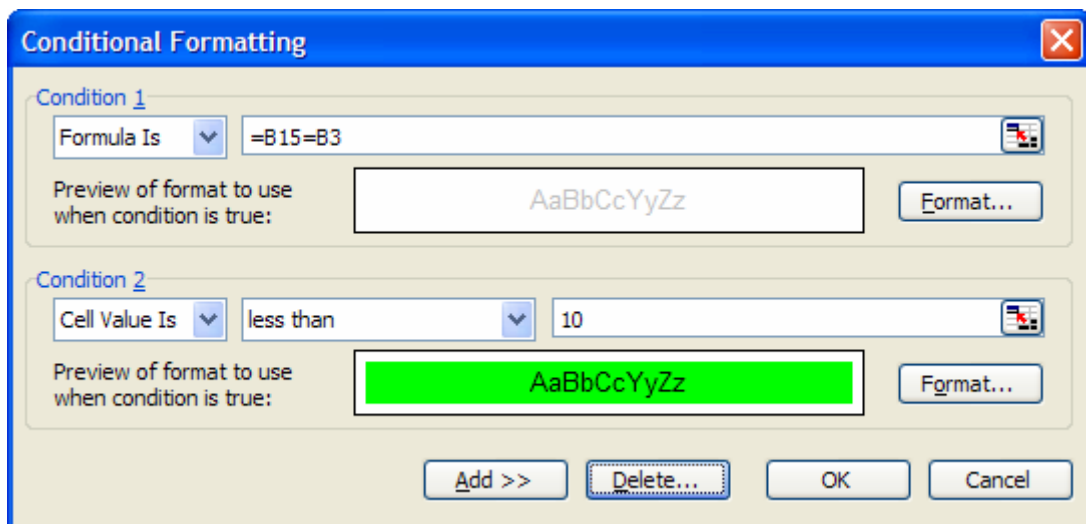


Figure 17 The first two CF conditions

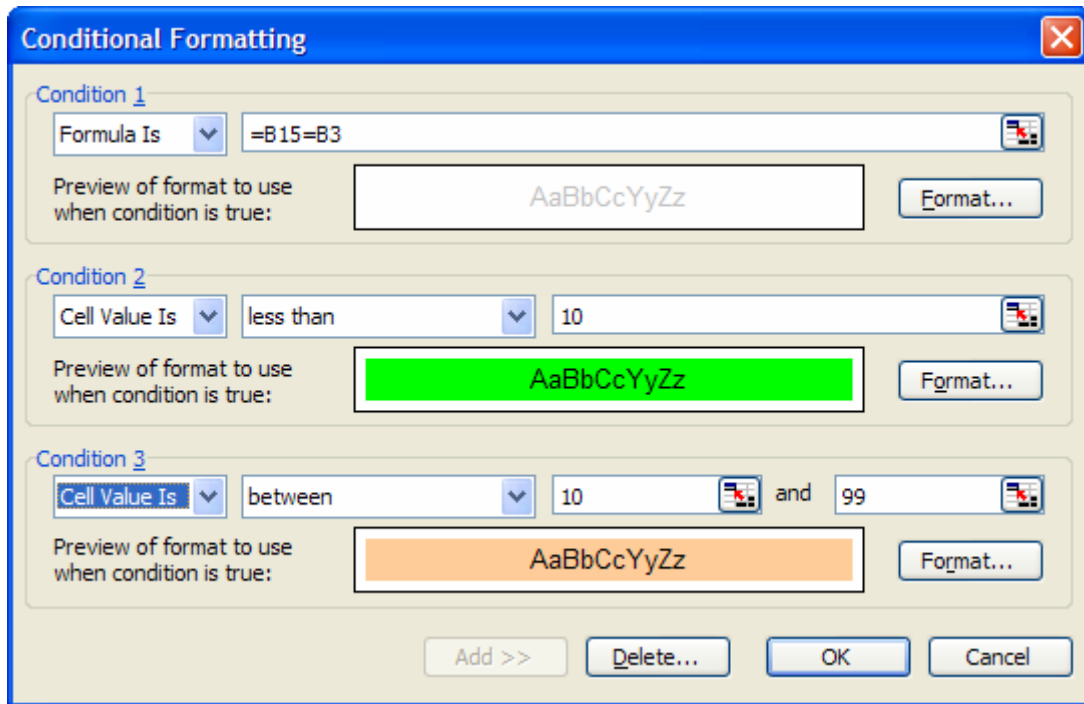


Figure 18 All three CF conditions

46. For the third condition, click again on the Add >> button. Use Cell Value Is between 10 and 99. Your final dialog should look like Figure 19. When you are satisfied with it, click OK.
47. We have specified the condition only for the first cell. The simplest way to spread this all over the POSSIBLES grid is by using the Format Painter. With the first cell of POSSIBLES still selected, click on the Format Painter button. Then select the entire POSSIBLES grid. The conditional formats will now be applied. You should see a range of colours. Simple Sudoku puzzles may now be solved by filling in the GAME cells highlighted green. Try it out.
48. An unwanted side-effect of the previous step is to mess up the border of the POSSIBLES grid. Unfortunately, we cannot restore this by copying the games cell, and then going Paste special, formats; this will destroy our conditional formatting. It seems that we must do step-by-step. Please let us know if you think of a better way.

8 Advanced

49. With just a little extra effort, we can make our model much more useful. We now have the information about possible numbers that can go in each vacant cell. The following observation is fundamental, and will allow us to solve most Sudokus. *Consider the POSSIBLES grid. Suppose that, in any row, column, or block, a number appears in just one of the 9 POSSIBLES. Then it must go in the corresponding GAME cell.* We can spot some of these ourselves. But if we can get Excel to highlight these, the whole thing becomes much easier.
50. Select cell M12 and click on the Insert Function button. If it does not appear automatically, select the "COUNTIF" function (from under the "Statistical"

heading in the dropdown menu). We will then add the cell range (M3:M11) that we plan to search in. Type in the Criteria box “one” (this is what we are looking for – the number “1” - and that is what we want to have counted). Check that your dialog box looks like Figure 20.

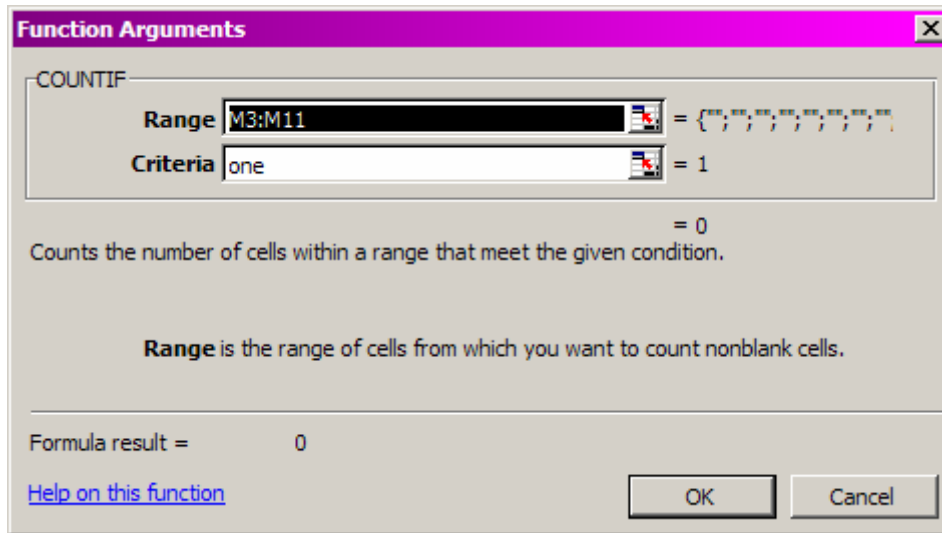


Figure 19

51. Copy the function across to cells N12 to U12 by using the fill handle..
52. Repeat the process in cell V3. This time, your range will be cells M3 to U3. Again, copy the formula down to V11 using the fill handle.
53. Repeat the above process on the remaining possible grids (i.e. numbers “one” to “nine”).

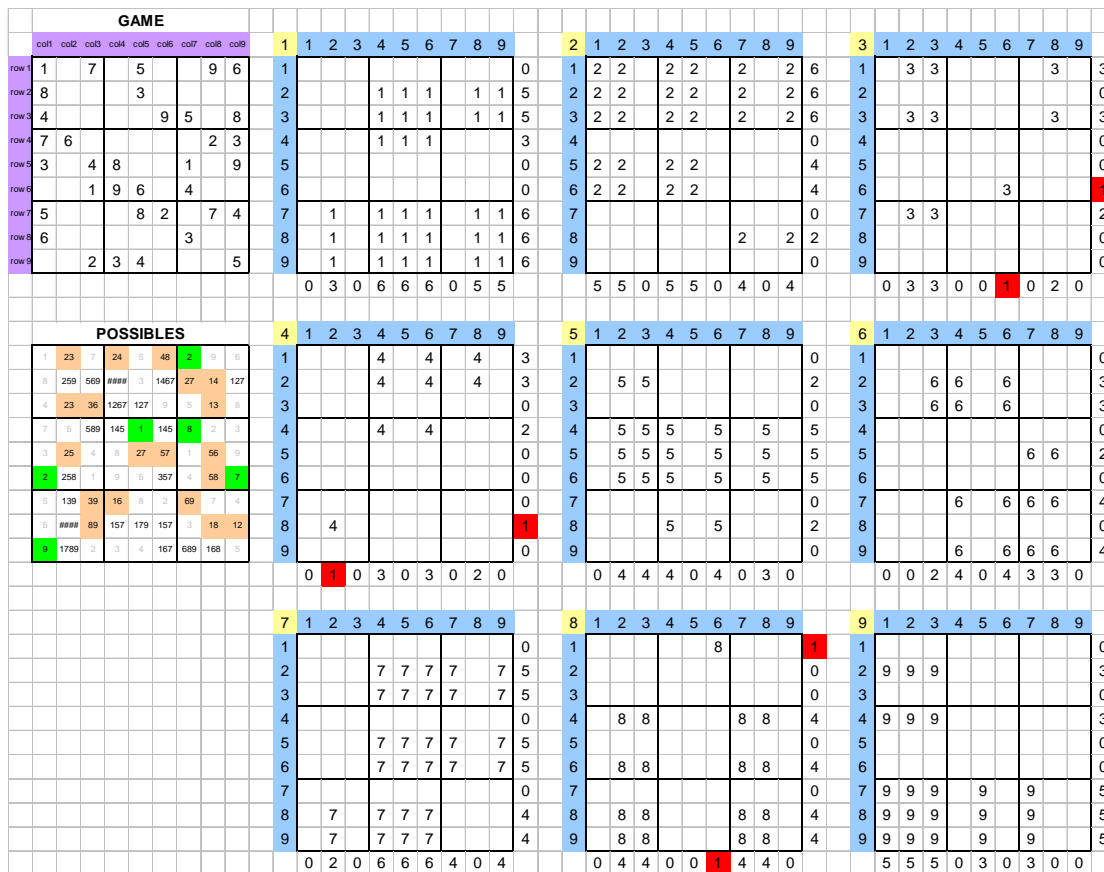


Figure 20 The final model

- 54. Once everything is completed, your spreadsheet should look like Figure 21. We are now going to include some Conditional Formatting for the calculations we have just included.
- 55. Select the various cells / columns / rows which add up the number of possibilities by performing multiple selections holding down the ctrl key. Your selections should look like those in Figure 21.
- 56. As explained earlier, open up the Conditional Formatting dialog box and change the setting to those shown in Figure 22. Click OK.

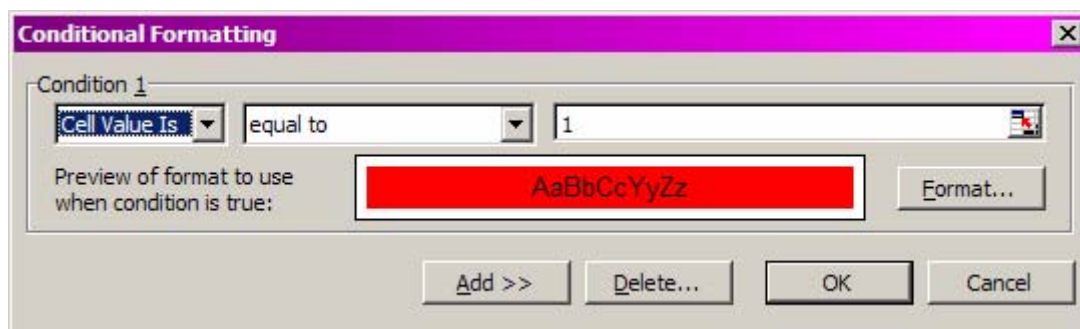


Figure 21 CF to identify definite placement locations

8.1 Exercises and extensions

1. Extend your model to compute the frequency of each value in the GAME grid. For example, there may currently be 3 ones, 5 twos, ... etc. All you need is a 9 by 2 table showing each number and its frequency. This information is sometimes useful. For example, if there are 8 sixes, then it becomes obvious where the ninth one must go. Use the COUNTIF function.
2. We concluded with a technique to highlight where values must go in certain rows or columns. Extend your model to highlight where a number must go in each of the 9 "numbered" areas. Hint: investigate the possibility of highlighting a block in one of these nine "numbered" areas with conditional formatting.
3. Use your model to help you to solve the following puzzles.

EASY								
1		7		5			9	6
8				3				
4					9	5		8
7	6						2	3
3		4	8			1		9
		1	9	6		4		
5				8	2		7	4
6						3		
		2	3	4				5

MEDIUM								
5	7			1			4	8
	2						6	
9			6		2			7
			4		9			
	4						2	
			1		5			
7			3		4			1
	3						5	
6	1			9			3	4

HARD								
	9	4				7	8	
	6						2	
2				8				4
9				1				7
		6	3		8	2		
3				2				9
7				6				3
	5						7	
	2	3				4	9	

Acknowledgement

I wish to thank Samantha Battle for her contribution to this article. Her cheerful nature and strong work ethic while working as my teaching assistant in 2006 were a constant encouragement to me. Her wonderful sense of humour made my daily work more bearable. Thank you, Sam. I wish you and your new husband Michael nothing but happiness and the very best in life.

Postscript for the teacher

If your class is learning basic set theory, students may be asked to express some the steps or strategies, concepts or operations for solving Sudoku in terms of set theoretic language. For example:

- a. The complement of the union of row, column and block values gives the set of POSSIBLES for a given cell.
- b. In a given row, column or block, the collection of POSSIBLES sets forms a cover or partition of the set of values yet to be placed. Sometimes, these little sets are interleaved, while others may be pairwise disjoint.
- c. The most difficult Sudokus require some form of trial and error such as *binary search with backtracking*. This approach may be based on POSSIBLES sets which are doubleton sets, i.e., just two possibilities exist for a certain cell. Students may be introduced to the very useful concept of a *binary tree* and its application here to solving the really fiendish Sudokus. A future article is planned to explore these ideas.