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Affordances of Spreadsheets In Mathematical Investigation: Potentialities For Learning

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This article, is concerned with the ways learning is shaped when mathematics problems are investigated in spreadsheet environments. It considers how the opportunities and constraints the digital media affords influenced the decisions the students made, and the direction of their enquiry pathway. How might the learning trajectory unfold, and the learning process and mathematical understanding emerge? Will the spreadsheet, as the pedagogical medium, evoke learning in a distinctive manner? The article reports on an aspect of an ongoing study involving students as they engage mathematical investigative tasks through digital media, the spreadsheet in particular. It considers the affordances of this learning environment for primary-aged students.

Keywords

school mathematics, investigations with spreadsheets, affordances, learning trajectories

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Abstract

This article, is concerned with the ways learning is shaped when mathematics problems are investigated in spreadsheet environments. It considers how the opportunities and constraints the digital media affords influenced the decisions the students made, and the direction of their enquiry pathway. How might these affordances shape the way the learning trajectory unfolds, the learning process and how mathematical understanding emerges? Will the spreadsheet, as the pedagogical medium, evoke learning in a distinctive manner? The article reports on the affordances of the spreadsheet environment for students investigating number tasks, an aspect of an ongoing study involving students as they engage mathematical investigative activities through digital media. It considers the affordances of this learning environment for primary-aged students.

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Introduction

How might using spreadsheets shape mathematical learning? When considering how learning and learning trajectories might differ when mathematical tasks are engaged through the digital pedagogical media of the spreadsheet, characteristics of that engagement need to be examined. In what ways might the learning experience be different from engaging mathematical phenomena with other media such as paper-and-pencil? This article identifies and discusses the common affordances that spreadsheets offered 10-year-old students when they investigated mathematical tasks involving number patterns. Each affordance is accompanied by a brief illustrative excerpt, frequently drawn from fuller episodes. The function of these brief excerpts is to illustrate the particular affordance rather than for analysis of the mathematical thinking exclusively. The data were drawn from an ongoing study concerned with the ways investigating mathematics with spreadsheets might influence the learning process. Firstly, research involving spreadsheet environments in mathematical education is considered. This prefaces an examination of data in relation to those particular affordances.

Affordances in a digital environment are the opportunities that the environment offers the learning process. They may facilitate or impede learning. Affordances are a potential for action, the capacity of an environment or object to enable the intentions of the student within a particular problem situation [1]. We might consider them as perceived opportunities offered through the pedagogical medium, in relationship with the propensities and intentions of the user. Affordance implies the complementarities of the learner and the environment. They are not just abstract physical properties [2], but the potential relationships between the user and the 'artifact' [3]. Important in this discussion is the symbiotic relationship between the digital media and the user. While the digital medium exerts influences on the student's approach, and hence the understanding that evolves, it is his/her existing knowledge that guides the way the technology is used, and in a sense shapes the technology. The student's engagement is influenced by the medium, but also influences the medium [4].

Spreadsheets in mathematics education: research and practice

Spreadsheets have given mathematicians and mathematics students a tool to extend the capacity and speed of computation. This has enabled students to better focus on the underlying mathematical ideas rather than on routine mathematical manipulation. Ploger, Klinger and Rooney [5] investigated the use of spreadsheets in developing algebraic thinking in a fifth grade class. They found that children learnt to pose problems and to create their own explanations while using spreadsheets to explore powerful mathematical ideas. Unencumbered by numerical computation involving large or decimal numbers, and using formulae in meaningful ways, the young children gained access to the predictive quality of algebraic thinking. This allowed them to pose rich 'What if...?' questions.

Another aspect that has frequently been associated with spreadsheet environments is the notion of multiple representations. The ability to link and explore visual, symbolic, and numerical representations simultaneously in a dynamic way with digital technologies has been recognised extensively in research (e.g., Tall, [6]). Ainsworth, Bibby, and Wood [7] suggested that multiple representations promote learning. Giving the learner the scope to visualise both in tabular and graphical form clearly gives the spreadsheet a major advantage as a learning tool. Baker and Biesel [8] found some advantage to a visual instructional style, modelled by spreadsheet usage, in their investigation of how children best understand averages. Lemke [9] maintained that visual-graphical representations available in software such as spreadsheets have the potential to allow students to develop mathematical concepts and relationships. Seeing an immediate change to a graph, when a table value is altered, is certainly a powerful method of imaging the relationship between the two.

Associated with this affordance is the notion of visualisation. While the debate is inconclusive as to the positioning of visualisation in mathematics [10], there is greater consensus regarding the positive role of visualisation or graphic approaches in the facilitation of understanding in mathematics education [11, 12]. Spreadsheets allow the learner flexibility to quickly rearrange information and re-engage with activities from fresh perspectives. In a study involving primary school-aged students solving problems using spreadsheets, Calder [13] has described how the particular nature of the spreadsheet environment framed the emergence of subgoals in the investigative path. They can also manage large amounts of realistic data more easily than pencil-and-paper technology, allowing students to more easily explore social and political debates through a mathematical lens.

The facility of spreadsheets to immediately test and reflect on output influenced the learning process. The almost instantaneous nature of the feedback, coupled with the interactive nature of the engagement, allows for the ease of exploration of ideas. Discussion is stimulated, as the results of prediction or conjecture are viewed rapidly and are more easily compared. This enhances the emergence of logic and reasoning as students investigate deviations from expected output, or the application of procedures. Students also required greater accuracy when applying procedural structures, to be more explicit with entering mathematical manipulations [14]. Others have indicated that these affordances, when facilitated appropriately by the teacher, may lead to students exploring powerful ideas in mathematics, learning to pose problems, and create explanations of their own [15, 16]. They reported improved high-level reasoning and problem solving linked to learners' investigations in digital environments. In a study of grade three children using spreadsheets to explore fractional number problems, Drier [17] reported that the students reinforced and extended their rational number knowledge, while exploring many mathematical concepts in an integrated manner.

While acknowledging that spreadsheets were designed for accountancy or financial purposes rather than mathematics education, S. Johnston-Wilder and Pimm [18] nevertheless contend that spreadsheets offer features that enhance mathematical teaching. The visual and interactive elements of working in a spreadsheet environment as well as the ability to explore number patterns, solve equations both numerically and graphically, operate on and transform vast amounts of data, and then represent them graphically for analysis, are particular affordances of the spreadsheet environment. Monaghan [19] identified the use of iterative refinement as an element of thinking algebraically that the spreadsheet is particularly suited to. Meanwhile, P. Johnston-Wilder [20] while discussing spreadsheets use in statistics, acknowledged its usefulness, but warned of the potential to mislead novice learners in this area due to structural aspects of the graphing process, and the requirement that the student aggregate the data within frequency tables before graphing.

Advantages have been found in the development of algebraic thinking [5]. The use of a spreadsheet allowed children to explore number patterns algebraically. Researchers have identified other benefits that spreadsheets offer within investigative approaches. These include its interactive nature, its suitability for linking concepts, and its capacity to give immediate feedback. They have found, significantly, that young students learn to pose problems and to create explanations of their own. Wilson, Ainley, and Bills [21] reported that spreadsheets give opportunities for the conceptualisation of algebraic variables, while Battista and Van Auken Borrow [14] also found that spreadsheets facilitated the development of algebraic thinking. Other researchers have identified how the use of spreadsheets in the preliminary stages of algebra courses enhanced conceptual understanding of equations and their solutions [22]. They advocated that spreadsheets be utilised in mathematics programmes beyond the investigation of variation and patterns, but also in the areas of relations and transformations. While those who support the use of spreadsheets to develop algebraic thinking describe the generalisation of numerical patterns as a key aspect of that development, there are aspects of numeracy and number investigations that are also suitable for exploration using spreadsheets. Several mathematics education researchers (e.g., Baker & Biesel [8]), have utilised spreadsheets to help children develop a better understanding of various numerical concepts such as equivalent fractions and exponential numbers, and in doing so have gained some insights into the way children's understanding develops.

Attributes, such as the interactive nature of the engagement and the multi-representation of data, coupled with appropriate teacher intervention, enable the learner to not only explore problems but to make links between different content areas that might otherwise have developed discretely. They allow students to model in a dynamic, reflective way, and enhance students' ability to model mathematically [23]. They also foster risk taking and experimentation [13] allowing space for students to explore. This exploration requires some scaffolding, however as it may not occur spontaneously. The visual image may provide the stimulus, but it is the subsequent thinking that is key to the learning process. Imagining consequential possibilities are part of that response. The effect on student engagement and motivation when using digital technologies in school mathematics programmes has also been noted. Higgins and Muijs [24] found much work pertaining to the positive effects on motivation and attitude, and while this enthusiasm might relate to the novelty factor initially, it can't be ignored, given the correlation between students' attitudes to learning in mathematics, and their understanding. Other researchers have likewise found positive motivational effects through using spreadsheets in classroom programmes (e.g., Drier, [17]).

The particular affordances of spreadsheets have the potential to facilitate learning in mathematics. They allow for relative ease when exploring ideas in problem solving (either numerically or visually), and stimulate discussion as the results of prediction or conjecture are viewed rapidly, allowing them to be more easily compared. This aspect facilitates the development of logic and reasoning, with students promptly seeing the effects of gaps or errors in their logic or application of procedures. It gives students the opportunity to immediately test and reflect on their conjectures in an interactive environment, allowing them to concentrate more on conceptual understanding. Shifting the computational responsibility to the computer also enables the learner to explore and focus more on conceptual understanding. This process, and the associated dialogue, enhances mathematical thinking, and allows learners to explore mathematical concepts beyond that which they would usually encounter [25]. In the following sections, the ways that spreadsheets might facilitate mathematical thinking is considered, after a brief description of the research situation.

Research methods

The article draws on data from an ongoing study that considers how processing mathematical tasks through digital pedagogical media might influence the learning process. For this particular discussion, the research took place in a setting involving spreadsheet environments, where the researcher was able to gain ethical access without compromising students' ongoing programs. The research situation involved ten-year-old students, attending five primary schools, drawn from a wide range of socio-economic areas. There were students from each school, who had been identified as being mathematically talented through a combination of problem-solving assessments and teacher reference; forty-five students in total. The students participated in four two-hour sessions, once a week, over four weeks, using spreadsheets to investigate mathematical problems. They received some instruction on using spreadsheets as well as ways to use them as a tool to explore the problems. Their discussions were audio recorded and transcribed, their onscreen output was printed out, observations were recorded, and any written recordings collected. Each group was interviewed after they had completed their investigation. These data, together with informal observation and discussions, formed the initial basis for the research. The transcripts were then systematically analyzed for patterns in the dialogue, within and between the groupings. Initially, the various research methods were considered individually, but for this analysis, the dialogue and output was meshed and considered from an interpretive frame.

Discussion of results

In what ways is mathematical understanding reorganised when mathematical phenomena are engaged through digital pedagogical media, the spreadsheet, in particular? The following sections address key affordances of the spreadsheet pedagogical environment on the learning process and how the students understanding evolved in distinctive ways due to these influences. A discussion of the differences that typify the mathematical learning encounters when engaged through spreadsheets prefaces an analysis of the ways in which the students mathematical thinking might be reorganised.

How the learning experience differed

Central to this research study was the nature of the learning experience when mathematical phenomena were engaged through the pedagogical medium of the spreadsheet. Coupled with this, was the consideration of ways learning trajectories might differ in a spreadsheet environment from the investigation of mathematical phenomena through other pedagogical media. This section considers what the characteristics of the learning encounters were, and what opportunities the medium afforded that were particular to the spreadsheet. The data in the research were illustrative of various affordances offered by the spreadsheet medium. They are accompanying excerpts, lifted from more comprehensive analysis, to illustrate these alternative features of the spreadsheet environment.These are discussed first, with their ramifications for the shaping of the learning trajectories addressed in the following section.

One characteristic of the spreadsheet environment that the data indicated was influential in the learning process was the visual, tabular structure of the output produced. It allowed for clearer comparisons to be made between adjacent cells or columns, and more direct links to be drawn between input and output. Other researchers have likewise discussed how visualization, enabled through using spreadsheets, enhanced conceptual development [8, 26]. The students were able to easily transform a column or table of values, a process that facilitated the perception and confirmation of relationships and emerging informal conjectures. This, coupled with other affordances such as the immediate feedback, enhanced their opportunities to interpret and make decisions more readily. The facility to compare output more easily left space in the investigative process for other influences such as personal value judgments and experimentation. These tables were typically generated by formulas; they were a function of the formulas engendered by the students' interpretations and intentions to model the situations. The data were also indicative of how this characteristic shaped the students' subsequent interpretation and explanations while also influencing the evolution of their learning trajectory in particular ways. The students used visual referents when forming and explaining their emerging generalizations and theories.

Viewing the visual representation simultaneously with a symbolic form enabled the students to alter the symbolic and observe the effect on the table structure, and the numerical data within it. This helps them to see the connections between those forms. The visual representation of output, in either tabular or graphical form, was an affordance of the spreadsheet environment, as was the facility to view and interact with multi-representations of the data. For example, students produced the following workbook when investigating a task that involved exponential growth.

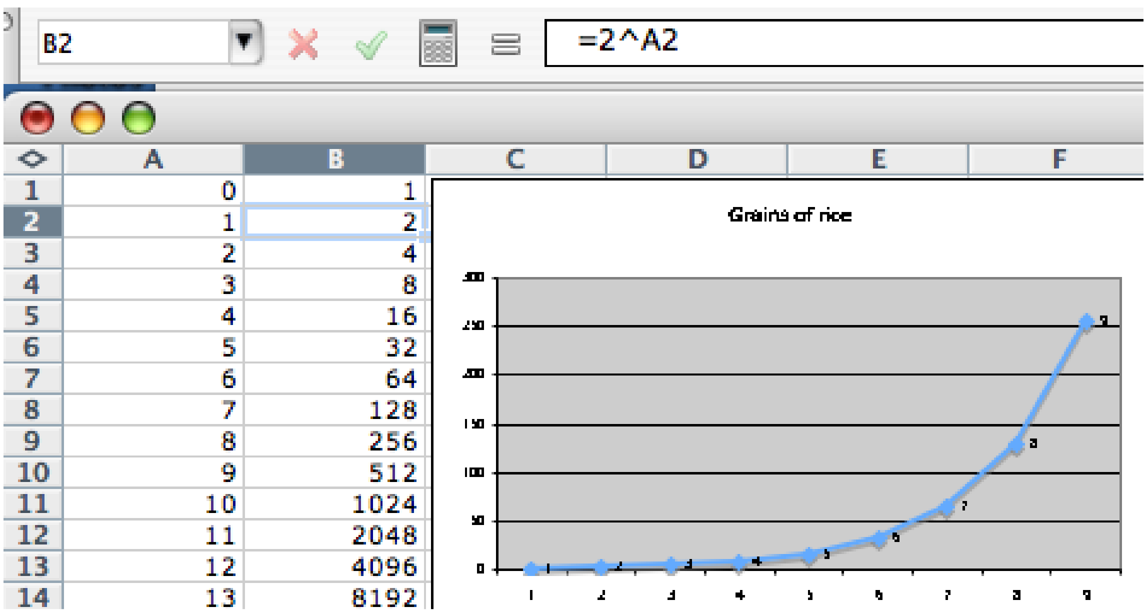

Figure 1: Multi-representations of data from Rice Mate investigation involving powers of two.

Associated with this was the affordance of the spreadsheet and ICT in general to give immediate feedback. The students were able to change formulas or numerical values and get a relatively instant response to their input. This appeared to foster a more experimental, exploratory approach, as the students were willing to pose informal conjectures, immediately test them and reflect on the output.

The episode below was illustrative of this characteristic and its influence on the learning pathway. The pupils were investigating the 101 times table task (see figure 2), and having explored it with a table, have been through several iterations of their conjecturing approach.

101 times table

Investigate the pattern formed by the 101 times table by:

- **Predicting what the answer will be when you multiply numbers by 101**
- **What if you try some 2 and 3 digit numbers? Are you still able to predict?**
- **Make some rules that help you predict when you have a 1, 2, or 3-digit number. Do they work?**
- **What if we used decimals?**

Figure 2: 101 times table task.

They explored how the product changes when 3-digit numbers are multiplied by one hundred and one. The following output was produced:

A	B	C
101	101	10201
101	102	10302
101	103	10403
101	104	10504

Adam: *101, 102, 103, 104, so there's a pattern- you've got your 101 and in the middle you've got 20, 30, 40, 50.*
Beth: *Quite right.*

Although, on the surface their observation was not quite right (the final digit is not 1 in each case), there was an apparent consensus of interpretation. There are several viewpoints that might be occupied in the discussion of this, one being that the pupils were indicating the B column in the observation rather than the A, that is "you've got your 101" implied 101, 102, 103, 104 as opposed to 101 in each instance. This seemed to be reasonable, given the following dialogue, and was later confirmed. The clarity of the output in its visual structured form may have contributed to Adam and Beth's mutual interpretive accord. Nevertheless, fresh impetus was given to their investigative path from the evolving perspectives. They reset their sub-goals by adjusting the type of 3-digit number to enable further insights into the pattern.

Carl: *So what would 126 be?*
Adam: *Would it be 10706, 120706, 12706?*

These predictions emerged from their evolving informal conjecture borne of the previous engagements. The pupils' confidence and willingness to attempt variations and refine their generalisation was evident. They then tried 126:

101	126	12726

Adam: *12726.*

Carl: *So it's the same thing.*
Beth: *The first two and the last stay the same and then the outside numbers are added together.*
Adam: *Let's predict 135, it's going to be 13635.*
Beth: *Let's check.*

101	135	13635

Their generalisation while articulated with visual referents (i.e. the nature and position of the digits) was consistent with the output. Their ongoing informal conjecture emerged through cycles of setting of investigative sub-goals based on their engagement with the task, the affordances of the environment, and the interpretation and reflection on the output. They considered what the pattern might be if decimals were used:

Beth: *Okay do a few with decimals, 4.35.*

They entered 4.35 into their workbook producing the following output:

101	4.35	439.35

Adam: *Try a higher one 43.5.*

101	43.5	4393.50

Adam: *4393.50, a whole new can of worms here.*
Beth: *Although the numbers look the same.*

They considered the output as it appeared on the screen:

101	435	43935
101	4.35	439.35
101	43.5	4393.50

They inputted another:

101	0.435	43.935

Beth: *They are the same numbers but just with the decimal.*

The pupils were able to test values and obtain an immediate response. This allowed their predictions and evolving conjecture to take shape, as output was able to be considered quickly and either discarded or folded within their interpretation. Discussion was stimulated, as the results of prediction or conjecture were viewed rapidly and were more easily compared and reflected upon. This enhanced their use of logic and reasoning as the pupils investigated, then endeavoured to explain deviations from the expected output, or opportunities that the output evoked. An observed comment from another situation further emphasised this attribute:

Tama: Highlight the row…Bingo. Just highlight and do it. Its done.

The spreadsheet's facility of giving an almost instantaneous response when data was inputted into a formula enabled the students to be more interactive and responsive to the output. They were able to test their emerging formative theories quickly and model situations relatively easily.

The spreadsheet enabled large amounts of data to be easily transformed perhaps by computational operations. The data were indicative of this affordance, and allied with the accuracy complicit to this, it removed the computational fetters of doing many repetitive known computations, giving access to investigating situations that might otherwise not be possible in the school situation. Rich mathematical tasks such as *Dividing one by the counting numbers (see Figure 3)* would not have been as accessible without the spreadsheet or another digital technology facilitating the accurate management of the large number of computations required to generalize and test the patterns.

Dividing 1 by the Counting Numbers

When we divide 1 by 2, we get 0.5, a terminating decimal.

When we divide 1 by 3, we get 0.33333…, a recurring decimal.

Investigate which numbers, when we divide the number 1 by them, give terminating, and which give recurring decimals.

Figure 3. Dividing one by the counting numbers task.

The following excerpt illustrated how the students were able to better generalise number patterns by simultaneously transforming large amounts of data. They had already generated a table of values and formulated an emerging theory.

B1 =1/A1

	A	B	C	D
1	1	1		
2	2	0.5		
3	3	0.33333333		
4	4	0.25		
5	5	0.2		
6	6	0.16666667		
7	7	0.14285714		
8	8	0.125		
9	9	0.11111111		
10	10	0.1		
11	11	0.09090909		
12	12	0.08333333		
13	13	0.07692308		
14	14	0.07142857		
15	15	0.06666667		
16	16	0.0625		
17	17	0.05882353		
18	18	0.05555556		
19	19	0.05263158		
20	20	0.05		
21	21	0.04761905		

After several further interactions and refinements, Sara noticed something in the table of values:

Sara: *So that's the pattern. When the number doubles, it's terminating. Like 1, 2, 4, 8 gives 1, 0.5, 0.25, 0.125.*
Jay: *So the answer is terminating and is in half lots. Lets try that =0.125/2; gives 0.0625- which is there.*

She finds it on the generated output from above. They consider other values.

Sara: *If you take these numbers out they double and the answer halves.*
Jay: *That makes sense though, if you're doubling one, the other must be half.*
Like 125 0.008; 250 0.004.
Sarah: *What's next? Let's check 500.*
Jay: *Let's just go on forever.*

They generated a huge list of output, down to over 4260. The nature and structure of the spreadsheet enabled them seamlessly, yet intentionally, to generate large amounts of relevant data, thus fashioning the emerging theory.

Jay: *500 0.002; 1000 0.001.*

This indicated the relationship between the numbers that gave terminating decimals and the powers of ten. It led to a conjecture couched in visual terms:

Sara: *When you add a zero* [to the divisor], *a zero gets added after the point* [decimal point].

Sara was articulating an interpretation of the situation as envisaged through a particular school mathematics lens; for example, 5 gave an output of 0.2, 50 gave an output of 0.02, and 500 gave an output of 0.002. Their informal conjecture and conceptual understanding had evolved, filtered by the affordances of the spreadsheet environment and the particular interactions this medium evoked. The following was also recorded on a piece of working paper, as a list of the numbers that produced terminating decimals:

1, ~~2~~, ~~5~~, 10, ~~20~~, 100, 1000

After recording two and five, it appeared they noticed that these were factors of ten and subsequently crossed them out. This observation also occurred with the twenty and one hundred. This interpretation was later verified with the pupils. They had made sense of, explored, and generalised aspects of the investigation, culminating in the indication of a relatively complex notion of factors and the generalisation process.

While it would have been possible for the pupils to work out each of these computations individually and record them manually, this would have had limitations both in terms of motivation and interest for ten-year-olds. It probably would have disrupted the flow of their interpretive thinking and might also have incurred some computational errors. Yet this extended range of data enabled them to better explore and generalise the number patterns. The facility to manipulate large amounts of data influenced the manner of the students' engagement and learning in a particular way. An observed comment from one of the pupils also recognised this characteristic:

Sara: *You have unlimited room. You can go forever [seemingly]. You can fill down a whole lot more quickly than you can do with a calculator.*

It also meant that realistic data from more meaningful contexts that don't use 'tidy' numbers could be investigated at earlier levels, without the complexity of computation clogging the students' mathematical thinking processes. These affordances, in conjunction with the visual structure, opened opportunity for patterns to be recognized and explained more readily. The students were able to promptly assess their emerging formative conjectures and more easily model situations.

The particular ways actual learning trajectories might evolve

One of the key aspects of the engagement that was influenced by the spreadsheet as pedagogical medium was the initial engagement with the tasks. Across a range of activities the students, sometimes after a brief familiarisation of the problem, moved immediately to engagement within the spreadsheet environment. Usually this was to generate tables or columns of data, often through the use of formulas and the *Fill Down* function. For example, some of the students' comments regarding their initial approach to the tasks were:

Fran: Thought of a formula.

Ben: Because of spreadsheet, we went straight to formulas, looked for pattern, for a way to make the spreadsheet work.

For other groups there was a brief, preliminary phase of making sense of the intentions of the task. For instance:

Sara: Re-read to get into the maths thinking, then straight to a spreadsheet formula.

Beth: I looked at how it was written down and looked at all the patterns; then I sorted it out in my head then put it down [the formula] and if it wasn't right then try another one. Experiment.

It was also clear from their dialogue and responses in the interviews, that the spreadsheets had provided not only a unique lens to view the investigation, but had possibly drawn a distinctive response in terms of investigative practice. Students experimented with various formulae within the spreadsheet environment. For example:

Cam: We put something and had a look and if it wasn't right, I'd just do another one and keep going.

Greg: I type what I think and try it.

This initial engagement allowed them to experiment with the intentions of the tasks and to familiarise themselves with the situation. They more readily moved from initial exploration, through prediction and verification, to the generalisation phase. The visual, tabular structure coupled with the speed of response facilitated their observation of patterns. Their language reflected this and frequently contained the language of generalisation. The data illustrated several versions of initial engagement. At times, the engagement involved familiarisation of the task (this could be ongoing), at other times, the purpose was the exploration of formula to produce an anticipated output, while in other instances it was to begin immediately the prediction or generalisation phases. The influence of this initial engagement permeated the subsequent ongoing interaction. It framed the ongoing interactions, interpretations and explanations as the students envisioned their investigation through that particular lens. The actual learning trajectories were shaped by that initial engagement of creating formulas or columns and tables of data to model the mathematical situation. The students' interaction with alternative representations promoted learning through the comparison or combination of representations, enabling broader perceptions than what might have been gained from a single representation. As well, when the students were required to relate different representations to each other, they had to engage in activity such as dialogue, interpretation, and explanation that enhanced understanding.

The spreadsheet environment was also influential in the generation of sub-goals as the students' learning trajectories unfolded. As they alternated between attending to the activities from the perspective of their underlying perceptions, and then reflecting on this engagement with consequential modification of their evolving perspectives, they set sub-goals that plotted their ongoing interaction. These were frequently reset in response to the output generated within the spreadsheet environment. The attributes that facilitated the modelling process and the facility to immediately test and reflect on emerging informal conjectures gave potential for the students' sub-goals to be shaped by the medium. For instance, when Awhi and Ben investigated the 101 times table, after trying multiplying by 2, they predict what the output will be when 101 is multiplied by 3:

Ben: *202.*
Awhi: *Now let's try this again with three. OK, what number do you think that will equal? 302?*
Ben: *No, 3003.*

They copied the formula down using the *Fill Down* function to produce the output below:

1	101
2	202
3	303
4	404
5	505 etc.

Ben: *Oh no, 303.*

The output was different from the predictions that their prevailing discourse had framed. The students appeared to use the table structure as a means to interpret the situation. It allowed them to more easily notice the relationship between the input and the output, and the ensuing pattern of the output values. Their perspective evolved and they re-engaged with the task from a fresh, modified stance.

Awhi: *If you go by 3, it goes 3 times 100, and zero, and 3 times 1, 303.*

The pattern that Awhi articulated was consistent with the output that the spreadsheet produced. Their informal proposal was confirmed and they reset their subgoal in the investigative trajectory accordingly.

Awhi: *OK, try 1919.*

The following output was produced:

193819

Interestingly, they seemed to disregard this output and formed a prediction based on their preconceptions.

Awhi: *Now make that 1818, and see if it's 1818 [the output].*

Ben: Oh look, eighteen 3, 6, eighteen.

There was a visual cognitive conflict, which made them re-engage in the activity, reflect on the output, and attempt to reconcile it with their current perspective. It caused them to reshape their emerging conjecture.

Awhi: Before it was 193619; write that number down somewhere (183618) and then we'll try 1919 again.
Ben: Yeah see nineteen, 3, 8, nineteen. Oh that's an eight.
Awhi: What's the pattern for two digits? It puts the number down first then doubles the number. This is four digits. It puts the number down first then doubles, then repeats the number.

The visual cognitive conflict made them reflect on their original, intuitive generalisation. It stimulated their mathematical thinking, as they reconciled the difference between what they expected and the actual output, and rationalised it as a new generalisation. This new generalisation was couched in visual terms. They used visual reasoning, referring to the type and position of the digits as they related to the input.

The data demonstrated how the students' interpretations of the situations they encountered were influenced by the visual, tabular structure. It allowed more direct comparison of adjacent columns and enabled them more easily to perceive relationships between numerical values on which to base their new sub-goal, often linked to an emerging informal conjecture. It enhanced their ability to perceive relationships and recognise patterns in the data. Seeing the pattern evoked questions. On occasion the students pondered why the pattern was there, and what was underpinning a particular visual sequence. While investigating in this environment, the students learnt to pose questions and sub-goals but also were encouraged to create personal explanations, explanations that were often visually referenced probably due to the pedagogical medium. It also gave opportunity through its various affordances for the students to explore powerful ideas and to explore concepts that they might not otherwise be exposed to. At times the learning trajectory evolved in unexpected ways. When the output varied, sometimes markedly, from what was expected, it caused tension that often led to the resetting of the sub-goal and substantial shifts in the way the student interpreted or engaged the situation.

The following involves students investigating the Rice Mate problem. After some initial explorative skirmishing they entered:

	A
1	1
2	=A1*2

They *Fill Down* from cell A2 to produce the sequence of numbers they anticipated would give them the number of grains of rice for each square of the chessboard. They encountered something unexpected with the following output generated:

	A

1	1
2	2
3	4
4	8
…	…
26	33554432
27	67108864
28	1.34E+08
29	2.68E+08
…	…

Fran: *Ok that isn't supposed to happen.*
Tony: *1.34E +08 that makes a lot of sense.*

The output was unforeseen and in a form they weren't familiar with (scientific form). There was a tension between the expected and actual output causing them to reflect, adjust their position, and re-interpret.

This aspect and other affordances of the environment appeared to stimulate discussion. The students wanted to verbally articulate the rapidly generated output and discuss the connections they could see, not least when it was unexpected. Surprise provoked curiosity and intrigue, which allied with the interactive and visual nature of the experience, in the students' general view made the learning 'more fun and interesting'. This, in turn, enhanced the motivational aspects of working through the spreadsheet medium, a feature that emerged in the interview, survey, and observational data.

For some of the students, the pure novelty of the learning experience in a fresh context, seemed to allow them to break the fetters of their previous accumulation of mathematics learning, some or all of which may have been negative. For others, there was the intrinsic motivation that was fostered by the affordances the spreadsheet allowed the learner, that is, the potential to investigate complex problems in a reflective manner, to see visual representations of data simultaneously with symbolic forms, and the interactive nature of computer usage per se. Caution is needed where the data might have indicated the motivation was based superficially on novelty, as clearly the sustainability of this advantage would be limited if the spreadsheet was always available as a tool for problem solving.

Engaging the mathematical phenomena through a pedagogical medium that allowed the students to test informal conjectures, link the symbolic to the visual, and see the general through the specific, while being interactive and giving immediate feedback, enhanced the students' willingness and propensity to employ an investigative approach. They appeared to be more willing to take risks.

Risk taking

The learner's propensity and comfort to move beyond known procedures in recognisable situations, is indicative of their willingness to try fresh strategies in their approach to investigation and problem solving, By implication, problem solving contains an element of the unknown that requires unravelling and addressing through the application of strategies in new situations or in an unfamiliar manner. This requires a degree of creativity and a willingness to take conceptual or procedural risks of a mathematical nature. It is risk taking in a positive, creative sense as compared to risky behaviour. The data were indicative of the spreadsheet environment affording learning behaviours and responses that facilitated the learner's willingness to take risks while operating within an investigative cycle. For example:

> Fran: Using a spreadsheet made it more likely to have a go at something new because it does many things for you. You have unlimited room. You can delete, wipe stuff out.
>
> Tony: It was easy to try things – saved you rubbing it out, you press delete and it's gone. What else was good about it? – trying things out.
>
> Ben: We tried a couple of formulas and none of them were right but we could see what the formula might be, so we could change it around a bit.
>
> Ant: Yeah, like when we had to on the first activities when Dan had 8 then he had 11 we had to find what was different – we could try things out and see if that worked and change it.
>
> Sophie: I always find it good for me. I can put something in and if it's not quite right, I can change a couple of things and bang, it changes it automatically and I don't have to start from the beginning again.

These student comments reflect a certain comfort with trying things, knowing they can be easily modified, and with an awareness of the rapidity of that modification process. This seemed to allow the students to pose informal conjectures, to explore then reflect on them, before, perhaps after several investigative iterations, either validating or rejecting them. The offering and investigation of informal conjectures fostered mathematical thinking. While the data were illustrative of the spreadsheet shaping learning trajectories and offering a distinctive environment for the students to investigate mathematical tasks, they also indicated the spreadsheet's transformative qualities in mathematics education; of it behaving as a conduit for the reshaping or reorganization of their thinking.

The reorganization of mathematical thinking and understanding

The spreadsheet environment reshaped the students' approaches and their actual learning trajectories. It allowed them to engage in alternative processes and to envisage their interpretations and explanations from fresh perspectives. The mathematising facilitated by the medium was transformed by the visual, interactive nature of the investigative process. They used visual elements in their reasoning, while their explanations were punctuated with visual referents, such as the position and visual pattern of the digits. As such, the generalizations that emerged were couched in visual terms. They interpreted and explained their reasoning in alternative ways. There was a visual perspective to their mathematical thinking, while the visual tabular structure enhanced the possibility of seeing relationships in ways that might otherwise have been unattainable or inaccessible. Coupled with other affordances, such as the increased speed of the feedback, this visual dimension expanded the boundaries of what constituted mathematical knowledge, and gave students access to ideas earlier than teachers' usual expectation. It allowed a shift in focus from calculation techniques to a focus on mathematical thinking and understanding. Modelling the situations with various representations, and the capacity to think mathematically and generalize enhanced by the simultaneous viewing and translation between these alternative forms, also fostered the reorganization of the learners' thinking.

Another aspect the data highlighted regarding the reorganization of thinking, was the nature of the students' initial engagement. Their approach was distinctive from the students in the classroom situation in that they immediately explored symbolic and tabular models of the situation - frequently with multiple, structured output, rather than a single numerical example. This framed the subsequent investigation of the mathematical activities, flavouring the investigative process and the explanations with this distinguishing perspective. Their dialogue also contained phrases and meanings particular to the medium. Investigating by processes such as *Fill Down* or using a spreadsheet formula, offered an alternative exploratory landscape with potential for the understanding to emerge in restructured ways. The speed and varying representations of feedback were also influential in the rearrangement of the students' methods and restructuring of the manner in which their learning trajectories and understandings evolved.

A particular element of this reorganization of thinking and understanding that the research revealed was concerned with the notion of visual perturbation. While cognitive conflicts have been discussed in previous research involving digital technologies (e.g., Kieren & Drijvers, [27]), the initiation of cognitive tension through the actual visual output differing from that which the students expected doesn't appear to have been documented. When the students anticipated an output suggested by their preconceptions, and the actual output produced differed, a tension arose. There was a gap between the expected output indicated by the learner's preconceptions, and the actual visual output produced by the pedagogical medium. The data were indicative of this visual perturbation evoking dialogue, and mathematical conjecture and reasoning of a distinctive nature, hence permitting a reshaping of the students' perspective, and the consequential potential for the reorganization of their thinking and understanding.

Conclusions

The speed of response to input, when using the spreadsheet, indicated their suitability for facilitating mathematical reasoning. When the students observed a pattern or graph rapidly, they developed the freedom to explore variations and, perhaps with teacher intervention, learned to make conjectures, and then pose questions themselves. This facility to immediately test predictions, reflect on outcomes, then make further conjectures, not only enhanced the students' ability to solve problems and communicate mathematically, but developed their logic and reasoning as the students investigated variations, or the application of procedures.

The data indicated that the spreadsheet environment gave an element of control to the learner that also seemed to enhance their willingness to take risks. As well, the students using the spreadsheets progressed more quickly into exploring larger numbers and decimals. This appeared to indicate a greater propensity for exploration and risk taking engendered by the spreadsheet environment. Aspects related to investigating in the spreadsheet environment such as the tabular format for output, the immediacy of the response to input, the facility to compute large amounts of data simultaneously, and to modify various elements quickly and easily, all appeared to engender confidence in the students to try things and take risks. The learning experience was also a relatively non-threatening, easily managed environment, conducive to making predictions, testing conjectures, and exploration without inhibition. In this regard, an advantage of working in an exploratory spreadsheet environment is that any cognitive conflict is predominantly non-judgemental [28]. Calder [28] also reported on the initiation of learners' informal conjectures in a spreadsheet environment, when the visual output produced unexpectedly differed from the output that was anticipated.

There is a need to balance the development of spreadsheet skills to enable entry into the spreadsheet environment, with the development of mathematical thinking. For the students to eventually work independently with the spreadsheet as a tool, they initially required an orchestrated sequence of skill development embedded in mathematical contexts (e.g., Burns-Wilson and Thomas, [29]). The aim should be for this approach to be replaced by appropriate mathematical problems that facilitate the use of spreadsheets, and for the skill development to only be driven by need. Initially, students should not be expected to invent and develop their own spreadsheet worksheets but as they gain experience, and their repertoire of skills develops, this would become possible. For the spreadsheet to be an influential pedagogical medium with investigative approaches to learning mathematics, this would certainly be desirable.

In rejoinder, aspects of the potential for spreadsheets in mathematics education are that they are interactive, give immediate feedback to changing data, enable multiple representations of data including visual, give students a large measure of control and ownership over their learning, and can transform large amounts of data. These attributes coupled with appropriate teacher intervention, enable the learner not only to explore problems, but to also make links between different content areas that might otherwise be developed discretely. They allow students to model in a dynamic, reflective way, facilitating a variety of learning styles that can be characterised by the terms: open-ended, problem orientated, investigative, active, and student centred [30].

They appear to give learners opportunities to develop as risk takers. Students made conjectures and immediately tested them in an informal, non-threatening, environment. This permitted the learners the opportunity to reshape their conceptual understanding in a fresh manner, to reorganise their mathematical thinking. This attribute enabled the participants to set, and then reset sub-goals, as they worked their way through an investigation [13]. The spreadsheet enabled different kinds of examples to be tested, compared and contrasted.

Spreadsheets, if used appropriately, enable mathematical phenomena to be presented and explored in ways which afford opportunities to initiate and enhance mathematical thinking, and make sense of mathematical situations. They allow the learner potential to look through the particular to the general [31]. When the learning experience differs with the use of spreadsheets, we can assume that learning trajectories and understanding will also differ. The spreadsheet doesn't operate in isolation, however. Its influence is inextricably linked to the pre-conceptions of the user, other societal and cultural discourses, and the nature of the learning process. The affordances of spreadsheet environment appear to influence the learning process in mathematics in distinctive ways, hence shaping the evolution of the mathematical understanding.

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