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Graphing Functions of Two Variables in Spreadsheets

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1. Introduction

A widespread way of using spreadsheets is dynamic graphing. 2D modelling is used mainly (see [1], and many articles in ejSiE, e.g. [2], [3], [4], [5]), but spreadsheets can also be used as an effective 3D modelling tool. An implementation of the orthographic parallel projection is shown in [6] where 3D figures with solved visibility and increasing intricacy (up to irregular, non-convex) are projected on Excel charts. A method of creating (not true) $z = f(x, y)$ graphs using Excel surface charts is in [7].

This article presents a method of graphing functions of two variables in spreadsheets. The method uses parallel orthogonal projection. Projection $\mathrm{A}''(x'', y'')$ of point $\mathrm{A}(x, y, z)$ onto the projection plane is governed by the equations [6]:

$$x'' = -x \sin\phi + y \cos\phi, \tag{1}$$

$$y'' = -x \sin\theta \cos\phi - y \sin\theta \sin\phi + z \cos\theta, \tag{2}$$

where angles ϕ and θ give the direction of the normal vector of the plane (Fig. 1). Changing the angles enables the user to view the graph from any direction.

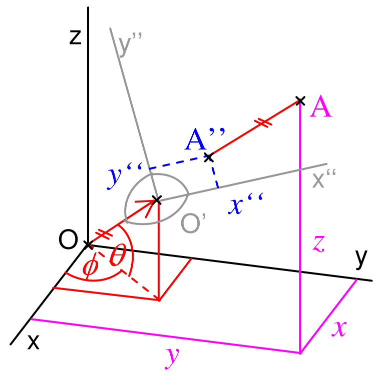

Figure 1: Orthographic projection $\mathrm{A}''(x'', y'')$ of point $\mathrm{A}(x, y, z)$

The graph is constructed as a mesh made over rectangle $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$ in the x and y directions. The method is demonstrated in two applications. The first method graphs functions that are continuous in the rectangle, the second one graphs functions that are not continuous in the rectangle. The visibility problem, that is, the problem of overlapping, is not solved. Creating the applications requires standard spreadsheet skills. Programming is used just for filling down and right, and for zooming.

The applications can be used in two ways. The first way is graphing. To get a graph of a $z = f(x, y)$ function requires just inputting the formula twice with some absolute references. The great advantage is that the data can be made visible, which helps to understand the principles of graphing. At pre-college level, despite the topic being out of the curriculum, the applications can be used to show students who are familiar with graphing $y = f(x)$ the appearance of graphs of functions $z = f(x, y)$. They are seen to be surfaces, and some of them are interesting and nice (the case of finding beauty in mathematics). Students can download the applications and experiment with the function formula to get even more interesting graphs. Hardly any school in author's country has software for graphing $z = f(x, y)$ functions, and if does, the students do not have it at home. The same holds for graphical calculators (as for graphical calculators vs. spreadsheets, see [8]). The second way is to use the applications as an example of applying constructivism with spreadsheets in informatics classes both at college and pre-college level, especially with students interested in mathematics and spreadsheets.

2. Graphing continuous functions

The entire application is in Figs. 2 and 3. First, intervals $[x_{\min}, x_{\max}]$ and $[y_{\min}, y_{\max}]$ are defined in H3:I3, K3:L3. Steps $\Delta x = (x_{\max} - x_{\min})/100$ and $\Delta y = (y_{\max} - y_{\min})/10$ are calculated in AA4 and AB4. Values y_j, $j = 0, \cdots, 10$ are calculated in Y6:AI6 so that $y_0 = y_{\min}$, $y_j = y_{j-1} + \Delta y$, $y_{10} = y_{\max}$. Values x_i, $i = 0, \cdots, 100$ are calculated in X7:X107 so that $x_0 = x_{\min}$, $x_i = x_{i-1} + \Delta x$, $x_{100} = x_{\max}$. Values $z_{ij} = f(x_i, y_j)$ are calculated in Y7:AI107. The user inputs the function formula in cell Y7. As he/she enters the cell, a comment appears that instructs him to put the address for x with fixed columns and the address for y with fixed rows. Then he/she clicks button "fill" to fill right and down by the macro

```
Range("Y7:AI7").FillRight : Range("Y7:AI107").FillDown
```

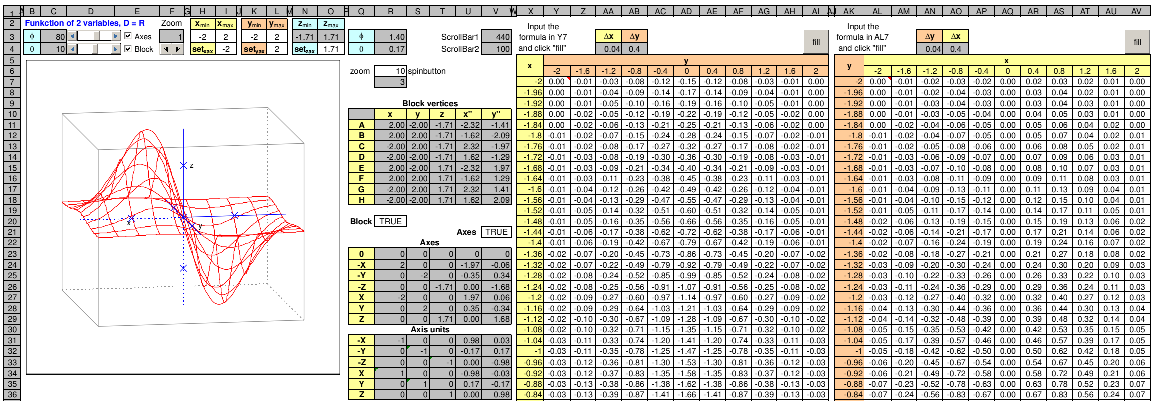

Figure 2: Function $z = 4x\mathrm{e}^{-(x^2+y^2)}$ and the first part of the application

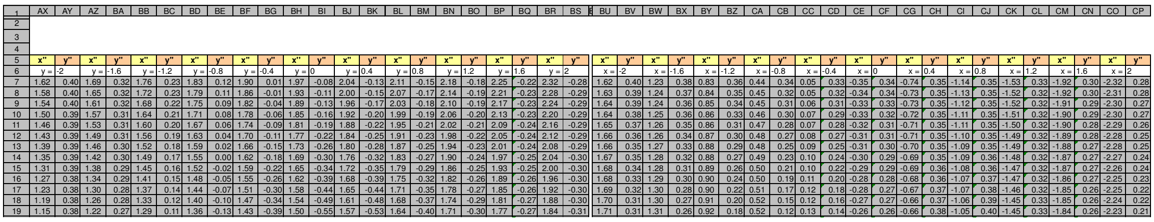

Figure 3: Function $z = 4x\mathrm{e}^{-(x^2+y^2)}$ and the second part of the application

Points (x_i, y_j, z_{ij}), $i = 0,\cdots,100$, $j = 0,\cdots,10$ in Y7:AI107 define 11 curves that are the intersections of the $z = f(x,y)$ graph and planes $y = y_j$. The curves comprise the part of the mesh in the x-direction. The coordinates are transformed to (x'', y'') in range AX7:BS107 using Eqs. (1), (2). The curves are drawn as xy line graphs. The chart axes are made invisible (they must not be switched off due to zooming). Points (x_i, y_j, z_{ij}), $i = 0,\cdots,10$, $j = 0,\cdots,100$ are calculated in AL7:AV107 and transformed to (x'', y'') in BU7:CP107 to give 11 curves in the y-direction to complete the mesh.

Function maximum $z_{\max}$ and minimum $z_{\min}$ are displayed in N3 and O3. Block $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}] \times [z_{\min}, z_{\max}]$ gives the bounds of the graph. The block edges are drawn as two-point xy line graphs. The vertices are calculated in range R11:V18 provided checkbox "Block" is ticked (linked with cell R20) otherwise they are set to zero to let the block collapse into point $(0,0,0)$. The length of the half-axes is defined in I4, L4, and O4. The beginning and ending points are calculated in range R23:V29. The axes are only shown if checkbox "Axes" is ticked (linked with cell V21) otherwise the cells are set to zero. The units are calculated in R31:V36 (marked by crosses "x" on the axes).

The scene can be zoomed up to 300 % and down to 10 % by clicking spinbutton "Zoom". Properties Max and Min are set to 30 and 1. The spinbutton is linked with cell R6. Cell R7 contains the formulas =30/R6. Cell N3 contains the formula =R6/10. Zooming changes the scale of the chart axes by the macro

```
Private Sub SpinButton1_Change()
 With ActiveSheet.ChartObjects(1).Chart
 .Axes(xlCategory).MinimumScale = -1 * Range("R7")
 .Axes(xlCategory).MaximumScale = 1 * Range("R7")
 .Axes(xlValue).MinimumScale = -1 * Range("R7")
 .Axes(xlValue).MaximumScale = 1 * Range("R7")
 End With End With
End Sub
```

Zoom 1 corresponds to scale $[-3,3]\times[-3,3]$. Scrollbars 1 and 2 that govern angles ϕ, θ are linked to cells V3 and V4. Properties Max and Min are set to 720 and 0, and 180 and 0. Small Change is 1 and Large Change is 5. Angles ϕ and θ are calculated in cells C3 and C4 by the formulas =V3-360 and =V4-90. Thus $-360° \le \phi \le 360°$ and $-90° \le \theta \le 90°$. That allows rotating the graph and viewing it from any side. The angles are transformed to radians in cells R3, R4. Columns Q:W, Z:AH, AM:AU, AX:CP, and rows 8:106 are to be hidden. The result is in Fig. 4 where function $z = 2x^2 + y^2$ is graphed. The functions $z = x^2 - y^2$, $z = \mathrm{e}^{-(x^2+4y^2)}$, and $z = 2xy/(x^2 + y^2 + 1)$ are graphed in Fig. 5.

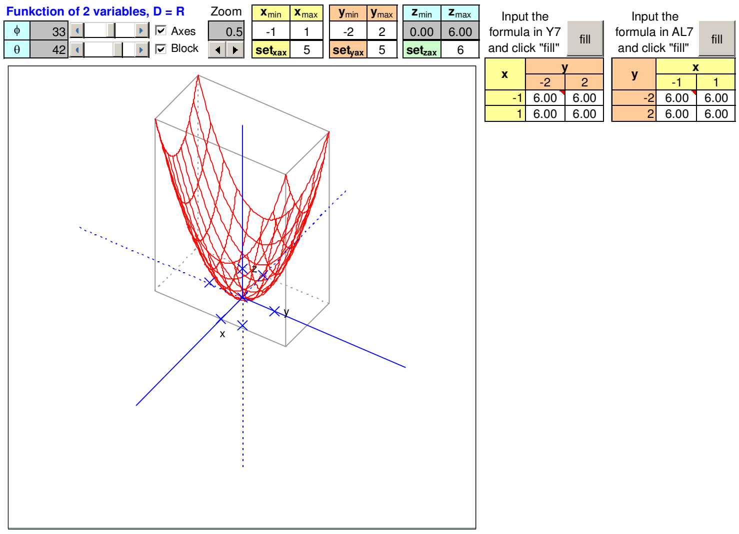

Figure 4: Function $z = 2x^2 + y^2$

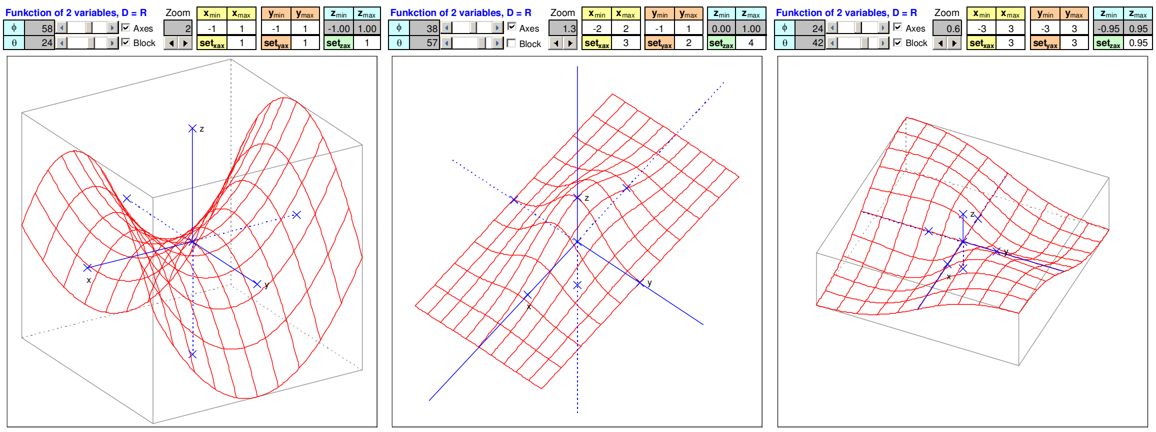

Figure 5: Function $z = x^2 - y^2$ (left), b) $z = \mathrm{e}^{-(x^2+4y^2)}$ (middle), c) $z = 2xy/(x^2 + y^2 + 1)$ (right)

3. Graphing discontinuous functions

The case occurs if the interval $[x_{\min}, x_{\max}]\times[y_{\min}, y_{\max}]$ contains (x, y) pairs that are not from the definition domain of the function $z = f(x, y)$. Then, the Excel formula of the function returns an error, which ISERROR function can detect. On that condition, the contents of the cell should be "nothing", which function NA() can provide.

The curves that comprise the mesh are calculated in the same way as in the previous section, however, they are drawn not from columns AX:BS and BU:CP but CR:DM and DO:EJ (Fig. 6) where the formula =IF(ISERROR(address),NA(),address) is applied, e.g. it is =IF(ISERROR(AY7),NA(),AY7) in cell CS7. If the formula in cell AY7 gives an error, which happens if the (x,y) pair is out of the definition domain, then ISERROR gives true, and IF sets CS7 to "nothing" thus no point appears in the chart. If the formula in AY7 gives a value, i.e. (x,y) is from the definition domain, then ISERROR yields false, IF sets CS7 to AY7, and the point appears in the chart. The points are drawn separately (point xy graph is used). 600 points are drawn for each curve to visualize it sufficiently. Leaving the NA() function out causes that the points that should be excluded appear on axis x''. The application with various graphs is in Fig. 7.

1	CR	CS	CT	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK	DL	DM
2																						
3																						
4																						
5	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"
6	y =	-3	y =	-2.4	y =	-1.8	y =	-1.2	y =	-0.6	y =	0	y =	0.6	y =	1.2	y =	1.8	y =	2.4	y =	3
7	-1.38	1.15	-0.84	1.10	-0.30	1.08	0.24	1.10	0.78	1.30	1.32	#####	1.85	0.09	2.39	0.29	2.93	0.31	3.47	0.29	4.01	0.25
8	-1.39	1.14	-0.85	1.10	-0.31	1.08	0.23	1.10	0.77	1.30	1.31	#####	1.85	0.09	2.39	0.29	2.93	0.31	3.47	0.29	4.01	0.25
9	-1.39	1.14	-0.85	1.10	-0.31	1.08	0.23	1.10	0.77	1.30	1.31	#####	1.85	0.08	2.38	0.29	2.92	0.31	3.46	0.29	4.00	0.24
10	-1.39	1.14	-0.86	1.10	-0.32	1.08	0.22	1.10	0.76	1.30	1.30	#####	1.84	0.08	2.38	0.28	2.92	0.31	3.46	0.28	4.00	0.24
11	-1.40	1.14	-0.86	1.10	-0.32	1.07	0.22	1.10	0.76	1.30	1.30	#####	1.84	0.08	2.38	0.28	2.92	0.30	3.45	0.28	3.99	0.24
12	-1.40	1.14	-0.86	1.09	-0.32	1.07	0.21	1.10	0.75	1.30	1.29	#####	1.83	0.07	2.37	0.28	2.91	0.30	3.45	0.28	3.99	0.24
13	-1.41	1.13	-0.87	1.09	-0.33	1.07	0.21	1.09	0.75	1.30	1.29	#####	1.83	0.07	2.37	0.27	2.91	0.30	3.45	0.27	3.99	0.23
14	-1.41	1.13	-0.87	1.09	-0.33	1.07	0.21	1.09	0.75	1.30	1.28	#####	1.82	0.06	2.36	0.27	2.90	0.29	3.44	0.27	3.98	0.23
15	-1.42	1.13	-0.88	1.09	-0.34	1.07	0.20	1.09	0.74	1.30	1.28	#####	1.82	0.06	2.36	0.27	2.90	0.29	3.44	0.27	3.98	0.23
16	-1.42	1.13	-0.88	1.09	-0.34	1.07	0.20	1.09	0.74	1.30	1.28	#####	1.81	0.06	2.35	0.26	2.89	0.29	3.43	0.27	3.97	0.23
17	-1.43	1.13	-0.89	1.09	-0.35	1.06	0.19	1.09	0.73	1.30	1.27	#####	1.81	0.05	2.35	0.26	2.89	0.29	3.43	0.26	3.97	0.22
18	-1.43	1.12	-0.89	1.08	-0.35	1.06	0.19	1.09	0.73	1.30	1.27	#####	1.81	0.05	2.35	0.26	2.88	0.28	3.42	0.26	3.96	0.22
19	-1.43	1.12	-0.89	1.08	-0.36	1.06	0.18	1.09	0.72	1.30	1.26	#####	1.80	0.04	2.34	0.25	2.88	0.28	3.42	0.26	3.96	0.22
20	-1.44	1.12	-0.90	1.08	-0.36	1.06	0.18	1.08	0.72	1.30	1.26	#####	1.80	0.04	2.34	0.25	2.88	0.28	3.42	0.26	3.95	0.22

1	DO	DP	DQ	DR	DS	DT	DU	DV	DW	DX	DY	DZ	EA	EB	EC	EF	EG	EH	EI	EJ	EK	EL
2																						
3																						
4																						
5	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"	x"	y"
6	x =	-3	x =	-2.4	x =	-1.8	x =	-1.2	x =	-0.6	x =	0	x =	0.6	x =	1.2	x =	1.8	x =	2.4	x =	3
7	-1.38	1.15	-1.64	1.03	-1.91	0.94	-2.17	0.89	-2.43	1.02	-2.70	#####	-2.96	-0.34	-3.22	-0.21	-3.49	-0.26	-3.75	-0.35	-4.01	-0.46
8	-1.37	1.14	-1.64	1.03	-1.90	0.94	-2.16	0.89	-2.42	1.02	-2.69	#####	-2.95	-0.34	-3.21	-0.21	-3.48	-0.26	-3.74	-0.35	-4.00	-0.47
9	-1.36	1.14	-1.63	1.03	-1.89	0.94	-2.15	0.89	-2.42	1.02	-2.68	#####	-2.94	-0.34	-3.20	-0.21	-3.47	-0.26	-3.73	-0.36	-3.99	-0.47
10	-1.35	1.14	-1.62	1.03	-1.88	0.94	-2.14	0.89	-2.41	1.02	-2.67	#####	-2.93	-0.34	-3.20	-0.21	-3.46	-0.26	-3.72	-0.36	-3.98	-0.47
11	-1.35	1.14	-1.61	1.03	-1.87	0.94	-2.13	0.89	-2.40	1.02	-2.66	#####	-2.92	-0.35	-3.19	-0.22	-3.45	-0.26	-3.71	-0.36	-3.98	-0.47
12	-1.34	1.14	-1.60	1.03	-1.86	0.94	-2.13	0.89	-2.39	1.02	-2.65	#####	-2.91	-0.35	-3.18	-0.22	-3.44	-0.27	-3.70	-0.36	-3.97	-0.47
13	-1.33	1.14	-1.59	1.03	-1.85	0.93	-2.12	0.89	-2.38	1.02	-2.64	#####	-2.91	-0.35	-3.17	-0.22	-3.43	-0.27	-3.69	-0.36	-3.96	-0.47
14	-1.32	1.14	-1.58	1.03	-1.84	0.93	-2.11	0.89	-2.37	1.02	-2.63	#####	-2.90	-0.36	-3.16	-0.22	-3.42	-0.27	-3.69	-0.36	-3.95	-0.48
15	-1.31	1.14	-1.57	1.03	-1.84	0.93	-2.10	0.89	-2.36	1.02	-2.62	#####	-2.89	-0.36	-3.15	-0.22	-3.41	-0.27	-3.68	-0.36	-3.94	-0.48
16	-1.30	1.14	-1.56	1.03	-1.83	0.93	-2.09	0.89	-2.35	1.02	-2.62	#####	-2.88	-0.36	-3.14	-0.23	-3.40	-0.27	-3.67	-0.37	-3.93	-0.48
17	-1.29	1.14	-1.55	1.03	-1.82	0.93	-2.08	0.89	-2.34	1.02	-2.61	#####	-2.87	-0.37	-3.13	-0.23	-3.40	-0.27	-3.66	-0.37	-3.92	-0.48
18	-1.28	1.14	-1.55	1.03	-1.81	0.93	-2.07	0.89	-2.33	1.02	-2.60	#####	-2.86	-0.37	-3.12	-0.23	-3.39	-0.28	-3.65	-0.37	-3.91	-0.48
19	-1.27	1.14	-1.54	1.02	-1.80	0.93	-2.06	0.89	-2.33	1.03	-2.59	#####	-2.85	-0.37	-3.11	-0.23	-3.38	-0.28	-3.64	-0.37	-3.90	-0.48
20	-1.26	1.14	-1.53	1.02	-1.79	0.93	-2.05	0.89	-2.32	1.03	-2.58	#####	-2.84	-0.37	-3.11	-0.23	-3.37	-0.28	-3.63	-0.37	-3.89	-0.48

Figure 6: Function $z = 1/(xy)$ and the third part of the application

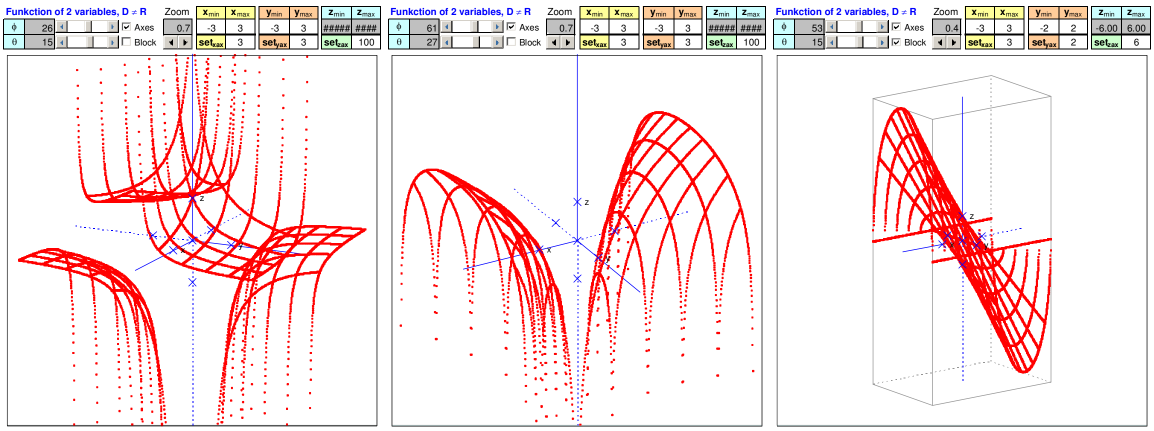

Figure 7: Functions a) $z = 1/(xy)$ (left), b) $z = \ln\left(x^2 - y^2\right)$ (middle), c) $z = x\sqrt{4 - y^2}$ (right)

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