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Solving J. W. Kittinger's Excelsior III Jump in Excel

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Abstract
The article brings a spreadsheet model of J. W. Kittinger's legendary jump from the stratosphere performed in 1960. The most likely solution to the fall is presented. Some data published about the fall turned out to be contradictory. The discrepancies are pointed out. The main results are that (1) it is impossible that Kittinger reached supersonic speed and (2) it is possible that he reached the published maximum speed of 274 m/s but not at the published altitude and not if he used his parachute system in the published way.

Keywords
Kittinger, stratosphere, free fall, drag

1. Introduction

On August 16, 1960, USAF Captain (later Colonel) Joseph W. Kittinger carried out his legendary jump from the helium balloon Excelsior III at the altitude of 31,300 m. It has been the highest, fastest and longest sky-dive ever made [1].

Taking advantage of the 50th anniversary, the author of this paper decided to model the fall with his students. However, discrepancies were found in the published data. According to [1], the top speed was 614 mph (274 m/s), reached at 27,400 m with fully deployed 6-ft drogue parachute. According to [2], it was 714 mph (319 m/s), reached at the same altitude. That is a supersonic fall at the altitude. However, acceleration due to gravity at 31,300 m is 9.71 m/s^2. The distance travelled was 3,900 m. The formula for free fall in vacuum gives time 28 s and speed 275 m/s. Reaching 319 m/s is impossible. Reaching 274 m/s in the thin air might be possible, but hardly with an open 6-ft chute.

A model of Kittinger's fall was presented by Mohazzabi and Shea [3]. They found that the initial altitude to reach top speed of 274 m/s at 27,430 m should be 34,670 m and not 31,330 m. The difference is an intolerable 3,340 m.

This paper brings the results of the author's investigation into the problem. An Excel model of the fall is created and the data published about the fall are analyzed. The data are presented in section 2. Unfortunately, Kittinger's book "The long, lonely leap" (Dutton, NY, 1961) has been out of print for long time and it is impossible to obtain. The theory of high altitude free fall in the US Standard Atmosphere 1976 [4] is presented in section 3. A detailed description of the spreadsheet model is given in section 4. The analysis of the fall is made in section 5. The most likely solution to Kittinger's fall is found. Some facts about the fall turned out to be contradictory. The discrepancies are pointed out. The findings are summarized in section 6.

A numeric model of low altitude free fall when the properties of the air and body are constant is in [5]. The model presented in this paper enables the user to analyse free fall from the stratosphere when the properties of the air, but also of the body, are changing. From the aspect of spreadsheet skills, a method is shown how to find the nearest value to a given value by the VLOOKUP function (the VLOOKUP function finds the equal or the nearest less value). The IF function is applied largely to implement the correct values and formulae. The application uses the Euler's method of solving differential equations, which is simple and transparent even at secondary level. No programming is used. The method of solving the problem is an example of reasoning in investigation.

2. Facts published about Kittinger's jump

Conversion to SI: 1 mile = 1 609.344 m, I foot = 0.3048 m, 1 pound = 0.45359237 kg; results rounded to 3 valid figures; time rounded to integers.

Ref. [1]: The parachute system consisted of a spring-loaded 3-ft (0.941 m) pilot chute to provide the pull to open a 6-ft (1.83 m) stabilization drogue chute to prevent deadly flat spin, and a conventional 28-ft (8.53 m) chute. Kittinger had 155 lb of gear and he totalled 313 lb (142 kg). The ground radar altimeter showed 102,800 ft (31,300 m) above sea level when he jumped. The 3-ft pilot chute opened at 15 s. The 6-ft parachute opened at 16 s at 96,000 ft (29,300 m) and stabilized Kittinger in a feet-to-earth position. Kittinger continued accelerating until 90,000 ft (27,400 m) where he reached the top speed of 614 mph (274 m/s),

0.9 of the speed of sound at the altitude. The 28-ft chute opened at 17,500 ft (5,330 m) at 4 min and 38 s (278 s). Kittinger landed at 13 min and 45 s (825 s).
Ref. [3]: The speed at 50,000 ft (15,200 m) was 250 mph (112 m/s). Kittinger reached: 40,000 ft (12,200 m) at 2.5 min (150 s); 30,000 ft (9,140 m) at 3.5 min (210 s); 20,000 ft (6,100 m) at 4.4 min (264 s).
Ref. [6]: Kittinger had breathing difficulties from 90,000 to 70,000 ft (27,400 to 21,300 m).

3. Free fall in the US Standard Atmosphere 1976

Acceleration due to gravity at altitude z above sea level is given by the equation

$$g(z) = g_0\left(\frac{r_0}{r_0 + z}\right)^2, \qquad (1)$$

where $g_0 = 9.80665$ m/s^2 is the acceleration at sea level, and $r_0 = 6{,}356{,}766$ m is the effective radius of the Earth [4].
In the air, resistance force (drag) acts, which is given by the formula [7]

$$\mathbf{F_D} = -0.5AC(v)\rho(z)v\mathbf{v}, \qquad (2)$$

where A is the projected area, which is the maximum cross-section area of the body perpendicular to the velocity vector $\mathbf{v}$, $\rho(z)$ is the density of the air dependent on altitude z, and $C(v)$ is the drag coefficient dependent on the shape of the body and speed v. If the speed is subsonic, that is, smaller than approximately 0.8 of the sonic speed (Mach 0.8 = 0.8 M), then coefficient C is virtually constant relative to the speed. If the speed is between 0.8 M and 1.2 M, then coefficient C becomes a rapidly increasing function of the speed.
The equation of motion of a body of mass m falling free in the air from altitude h is (note that $\mathbf{v} = -v\hat{\mathbf{k}}$, where $\hat{\mathbf{k}}$ is the z-axis unit vector)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g(z) - 0.5\frac{A}{m}C(v)\rho(z)v^2,\ v(0) = 0,\ z(0) = h. \qquad (3)$$

The layers and properties of the US Standard Atmosphere up to 32 km are in Table 1 [4].

Layer b	**Height** z_b **(m)**	**Density** ρ_b **(kg/m3)**	**Temperature** T_b **(K)**	**Temperature lapse rate** L_b **(K/m)**	**Speed of sound** c_b **(m/s)**
0	0	1.225	288.15	–0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

Table 1. Layers of the US Standard Atmosphere 1976

Temperature lapse rate L_b is constant within layer b. Density ρ_b, temperature T_b, and speed of sound c_b hold at bottom z_b of layer b.
Density $\rho(z)$, $z_b \le z \le z_{b+1}$, is given alternatively by the equations

$$\rho(z) = \rho_b \left(1 + L_b (z - z_b)/T_b\right)^{-(\beta/L_b + 1)}, \tag{4}$$

$$\rho(z) = \rho_b \exp(-\beta(z - z_b)/T_b), \tag{5}$$

where $\beta = g_0 M/R$, $R = 8.31432$ J/mol.K is the gas constant, and $M = 28.96461$ kg/kmol is the mass per kilo mole of the air. Eq. (4) holds for layers $b = 0, 2$ where $L_b \neq 0$, while Eq. (5) holds for layers $b = 1$ where $L_b = 0$.

Temperature $T(z)$, $z_b \leq z \leq z_{b+1}$, is given by the equation

$$T = T_b + L_b (z - z_b). \tag{6}$$

Speed of sound is given by the equation $c = \sqrt{\gamma RT / M}$, where $\gamma = 1.4$ is the ratio of the molar heat capacity at constant pressure to that at constant volume. Substituting for T from Eq. (6) and using $c_b = \sqrt{\gamma R T_b / M}$ gives

$$c = c_b \sqrt{1 + L_b (z - z_b)/T_b}. \tag{7}$$

4. The spreadsheet model

Suppose Kittinger did not exceed Mach 0.8, or if, then for a very short time. Then, coefficient C can be taken constant relative to speed v. Replacing the differentials by differences Eq. (3) takes the form

$$\Delta v = \left(g(z) - K\rho(z)v^2\right)\Delta t, \ v(0) = 0, \ z(0) = h, \tag{8}$$

where coefficient

$$K = 0.5AC/m \ \left(m^2/kg\right) \tag{9}$$

only depends on the shape and orientation of Kittinger's body and its mass. The recursive formula

$$v_{n+1} = v_n + \left(g(z_n) - K\rho(z_n)v_n^2\right)\Delta t, \ v_0 = 0, \ z_0 = h, \ \Delta t = \mathrm{t}_{\max}/n \tag{10}$$

results from Eq. (8), where $t_{\max}$ is the duration of the fall, and n is the number of dividing points of the interval $[0, t_{\max}]$.

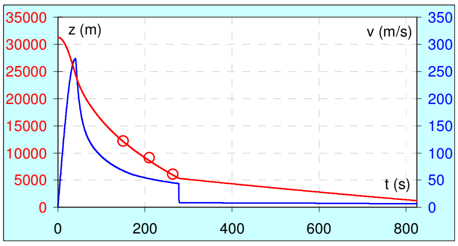
Kittinger jumped and reached the maximum speed in layer $b = 2$ of the Standard US Atmosphere 1976. Solving Eq. (8) in Excel by the Euler's method through layers $b = 0, 1, 2$ is transparent and requires no programming. The application is in Fig. 1.

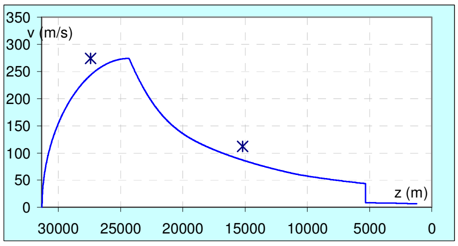
Acceleration at sea level g_0 and effective radius r_0 of the Earth are in cells C4:C5. Gas constant R and molar mass M of the air are in cells C6:C7. Parameter β is calculated in cell C8. The starting altitude h and Kittinger's mass m are in cells C9:C10. Table 1 is in cells O18:T21. The data published for the fall are in cells O25:P27 and R25:S26. They are shown by circles in the chart on the left and by crosses in the chart on the right.

The fall has three parts: 1) no chute deployed, 2) 6-ft chute deployed, 3) 28-ft chute deployed. The following parameters correspond to them: coefficient K_1 for the first part (cell C12); time t_{12} of deploying the 6-ft chute (cell C13), coefficient K_2 for the second part (cell C14); time t_{23} of deploying the 28-ft chute (cell C15), coefficient K_3 for the third part (cell C16), and time t_{max} of landing (cell C17).

J. W. Kittinger's jump

	B	C	D
4	g_0	9.80665	m/s^2
5	r_0	6356776	m
6	M	0.028965	m
7	R	8.314	m^2
8	β	0.03416	kg
9	h	31300	m
10	m	142	kg
12	K_1	0.00292	1/m
13	t_{12}	41	s
14	K_2	0.00732	1/m
15	t_{23}	278	s
16	K_3	0.196	1/m
17	t_{max}	825	s
19	n	15000	
20	Δt	0.055	





	E	F	G	H	I	J
14	41	0.00292	0.044	9.73	24331	274
15	278	0.19600	0.710	9.79	5332	43.6
16	825	0.19600	1.088	9.80	1217	6.8

v_{max}		v_{max}	
274	m/s	0.92	M

	t (s)	K (kg/m^2)	ρ (kg/m^3)	g (m/s^2)	z (m)	v (m/s)	Δv (m/s)	c (m/s)	v/c
18	0	0.00292	0.015	9.71	31300	0	9.71	303	0.00
19	0.06	0.00292	0.015	9.71	31300	0.53	9.71	303	0.00
20	0.11	0.00292	0.015	9.71	31300	1.07	9.71	303	0.00
21	0.17	0.00292	0.015	9.71	31300	1.60	9.71	303	0.01
22	0.22	0.00292	0.015	9.71	31300	2.14	9.71	303	0.01
23	0.28	0.00292	0.015	9.71	31300	2.67	9.71	303	0.01
24	0.33	0.00292	0.015	9.71	31299	3.20	9.71	303	0.01
25	0.39	0.00292	0.015	9.71	31299	3.74	9.71	303	0.01
26	0.44	0.00292	0.015	9.71	31299	4.27	9.71	303	0.01
27	0.50	0.00292	0.015	9.71	31299	4.81	9.71	303	0.02
763	40.97	0.00292	0.044	9.73	24331	274.215	0.10	298	0.92
764	41.03	0.00732	0.044	9.73	24316	274.221	-14.48	298	0.920
765	41.08	0.00732	0.044	9.73	24301	273.42	-14.40	298	0.92

b	z_b (m)	ρ_b (kg/m^3)	T_b (K)	L_b (K/m)	c_b (m/s)
0	0	1.225	288.15	-0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

given data

t (s)	z (m)
150	12200
210	9140
264	6100

given data

z (m)	v (m/s)
27400	274
15200	112

Figure 1: The most likely solution to the fall

Number n of subintervals is in cell C19 (15,000 is enough; an author's MATLAB® model using RK4 method gives the same results). Time increment Δt is calculated in cell C20.

The time points are calculated in column E by the formula (cell E19)
=E18+$C$20, starting with 0 in cell E18.

Coefficient K is calculated in column F by the formula (cell F18)
=IF(E18<=$C$13,$C$12,IF(E18<=$C$15,$C$14,$C$16)).

Air density ρ is calculated in column G by the formula (cell G18)
=IF(I18<=P$19,Q$18*(1+S$18/R$18*(I18-P$18))^(-C$8/S$18-1),
IF(I18<=P$20,Q$19*EXP(-C$8/R$19*(I18-P$19)),Q$20*(1+S$20/R$20*(I18-P$20))^(-C$8/S$20-1))).

Gravitational acceleration g is calculated in column H by the formula (cell H18)
=C$4*(C$5/(C$5+I18))^2. Altitude z is calculated in column I by the formula (cell I19)
=I18-J19*C$20, starting with 31,300 in cell I18.

Speed v is calculated in column J by the formula (cell J19)
=J18+K18, starting with 0 in cell J18.

Speed increment Δv is calculated in column K by the formula (cell K18)
=(H18-F18*G18*J18^2)*$C$20.

Speed of sound c is calculated in column L by the formula (cell L18)
=IF(I18<=P$20,T$19*SQRT(1+S$19/R$19*(I18-P$19)),
IF(I18<=P$21,T$20*SQRT(1+S$20/R$20*(I18-P$20)),T$21*SQRT(1+S$21/R$21*(I18-P$21)))).

Relative speed v/c is calculated in column M by the formula (cell M18) =J18/L18.

The top speed is shown in cells K15, M15 by the formula (cell K15) =MAX(J18:J15018).

The values of the parameters calculated in columns E:K that hold at time points t_{12}, t_{23}, and t_{max} are shown in cells F14:J16 by the formula (cell F14)

```
=IF(E14-VLOOKUP(E14,$E$18:$K$15018,1)<VLOOKUP(E14+$C$20,$E$18:$K$15018,1)-E14,
VLOOKUP(E14,$E$18:$K$15018,2),VLOOKUP(E14+$C$20,$E$18:$K$15018,2)).
```

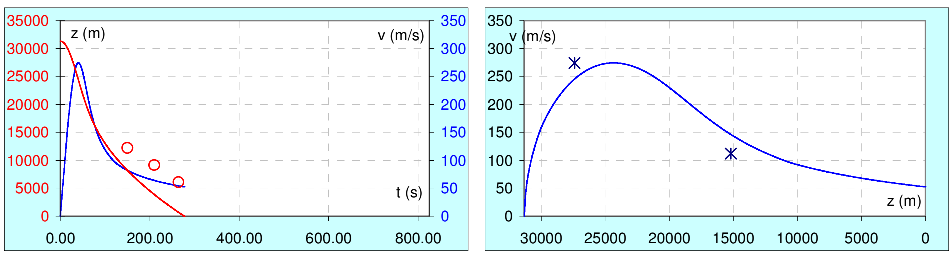
Function VLOOKUP(E14,..,2) looks for the value that is equal or nearest less to the value in E14. If there is no equal one but the next to the nearest less one is nearer to the value in E14, then VLOOKUP gives false result. Function IF finds which of the two values is nearer.

5. Analysis of the fall

Experimenting with the model started on the presumption that the 6-ft chute deployed at $t_{12} = 16$ s and the maximum speed 274 m/s was reached at the altitude of 27,400 m, as published. However, that turned out to be unfeasible. Even if $K_1 = 0$, which is impossible, then the model gives $K_2 = 0.00296$ (Fig. 2). As $K = 0.5AC/m$ (Eq. (9)) and $m = 142$ kg, then $AC \sim 0.84\ \mathrm{m}^2$. It holds that $C \sim 0.5$ for a sphere, $C \sim 0.8 - 1$ for a cube, $C \sim 1 - 1.3$ for a person in upright position, and $C \sim 1.4$ for a hollow semi-sphere opposite stream [8], [9]. If just $C \sim 1$, then $A \sim 0.84\ \mathrm{m}^2$, which definitely can not be the projected area of a 6-ft chute. It can also be seen in Fig. 2 that the altitude at $t_{23} = 278$ s is –31 m while the speed is 52.3 m/s, which means that Kittinger would have landed at deadly speed before opening the main chute. The result of this analysis is that if the maximum speed was 274 m/s, then the 6-ft chute must have deployed later than at 16 s.

J. W. Kittinger's jump

g_0	9.80665	m/s^2
r_0	6356776	m
M	0.028965	kg/kmol
R	8.314	J/mol.K
β	0.03416	kg
h	31300	m
m	142	kg
K_1	0.000000	m^2/kg
t_{12}	16	s
K_2	0.00296	m^2/kg
t_{23}	278	s
K_3	0.000000	m^2/kg
t_{max}	278	s
n	15000	
Δt	0.019	s



16	0.00000	0.018	9.71	30056	155
278	0.00296	1.229	9.81	-31	52.3

v_{max}	
274.2	m/s

v_{max}	
0.92	M

t (s)	K (kg/m^2)	ρ (kg/m^3)	g (m/s^2)	z (m)	v (m/s)	Δv (m/s)	c (m/s)	v/c
0.00	0.00000	0.015	9.71	31300	0	9.71	303	0.00
0.02	0.00000	0.015	9.71	31300	0.18	9.71	303	0.00
0.04	0.00000	0.015	9.71	31300	0.36	9.71	303	0.00
0.06	0.00000	0.015	9.71	31300	0.54	9.71	303	0.00
40.51	0.00296	0.044	9.73	24356	274.1597	0.01	298	0.92
40.53	0.00296	0.044	9.73	24351	274.15984	0.00	298	0.92
40.55	0.00296	0.044	9.73	24346	274.15980	-0.01	298	0.92

b	z_b (m)	ρ_b (kg/m^3)	T_b (K)	L_b (K/m)	c_b (m/s)
0	0	1.225	288.15	-0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

Figure 2: Impossible solution when a chute deploys at 16 s and the top speed is 274 m/s

5.1 Analysis of the first part of the fall (no chute deployed)

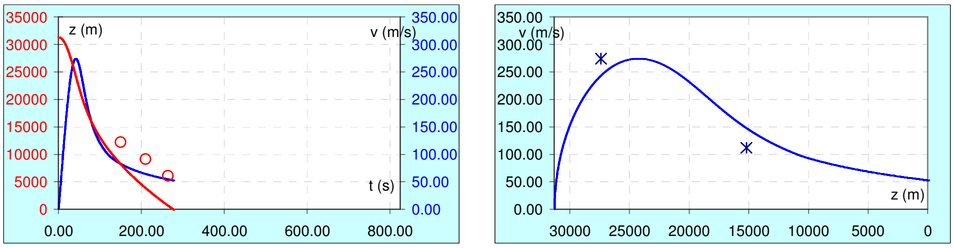
The analysis followed with the presumption that the 6-ft chute was opened later than at 16 s. Parameter t_{12} was set to 278 s (a big enough value) and parameter K_1 was iterated.

First: Parameter K_1 was iterated to reach maximum speed of 274 m/s. It was found that $K_1 = 0.00292$. The corresponding altitude and time are 24,300 m and 41 s (Fig. 3). Eq. (9) gives $A \sim 0.83\ \mathrm{m}^2$ of projected area if $C \sim 1$. Kittinger fell as sitting in an armchair due to his inflated pressure suit (see in [1]). Projected area of 0.83 m² is acceptable.

Second: K_1 was iterated so that the speed reached maximum at the altitude of 27,400 m. It was found that $K_1 = 0.0106$. However, the maximum speed is just 184 m/s (Fig. 4).

J. W. Kittinger's jump

g_0	9.80665	m/s^2
r_0	6356776	m
M	0.028965	kg/kmol
R	8.314	J/mol.K
β	0.03416	kg
h	31300	m
m	142	kg
K_1	0.00292	m^2/kg
t_{12}	278	s
K_2	0.000000	m^2/kg
t_{23}	278	s
K_3	0.000000	m^2/kg
t_{max}	278	s
n	15000	
Δt	0.019	s



278 0.00292 1.236 9.81 -97 52.5

v_{max}		v_{max}	
274.2	m/s	0.920	M

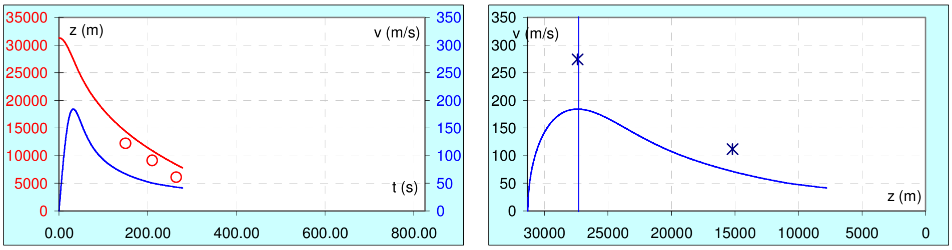
t (s)	K (kg/m^2)	ρ (kg/m^3)	g (m/s^2)	z (m)	v (m/s)	Δv (m/s)	c (m/s)	v/c
0.00	0.00292	0.015	9.71	31300	0.00	9.71	303	0.00
0.02	0.00292	0.015	9.71	31300	0.18	9.71	303	0.00
0.04	0.00292	0.015	9.71	31300	0.36	9.71	303	0.00
0.06	0.00292	0.015	9.71	31300	0.54	9.71	303	0.00
41.22	0.00292	0.044	9.73	24271	274.1953	0.00	298	0.92
41.24	0.00292	0.044	9.73	24265	274.1954	0.00	298	0.92
41.26	0.00292	0.044	9.73	24260	274.1953	-0.01	298	0.92

b	z_b (m)	ρ_b (kg/m^3)	T_b (K)	L_b (K/m)	c_b (m/s)
0	0	1.225	288.15	-0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

Figure 3: Solution to the first part of the fall (no chute deployed, top speed 274 m/s)

J. W. Kittinger's jump

g_0	9.80665	m/s^2
r_0	6356776	m
M	0.028965	kg/kmol
R	8.314	J/mol.K
β	0.03416	kg
h	31300	m
m	142	kg
K_1	0.0106	m^2/kg
t_{12}	278	s
K_2	0.000000	m^2/kg
t_{23}	278	s
K_3	0.000000	m^2/kg
t_{max}	278	s
n	15000	
Δt	0.019	s



278 0.01060 0.537 9.78 7805 41.7

v_{max}		v_{max}	
184.2	m/s	0.61	M

t (s)	K (kg/m^2)	ρ (kg/m^3)	g (m/s^2)	z (m)	v (m/s)	Δv (m/s)	c (m/s)	v/c
0.00	0.01060	0.015	9.71	31300	0	9.71	303	0.00
0.02	0.01060	0.015	9.71	31300	0.18	9.71	303	0.00
0.04	0.01060	0.015	9.71	31300	0.36	9.71	303	0.00
0.06	0.01060	0.015	9.71	31300	0.54	9.71	303	0.00
32.17	0.01060	0.027	9.72	27402	184.223249	0.00	300	0.61
32.19	0.01060	0.027	9.72	27399	184.223330	0.00	300	0.61
32.21	0.01060	0.027	9.72	27395	184.223314	-0.01	300	0.61

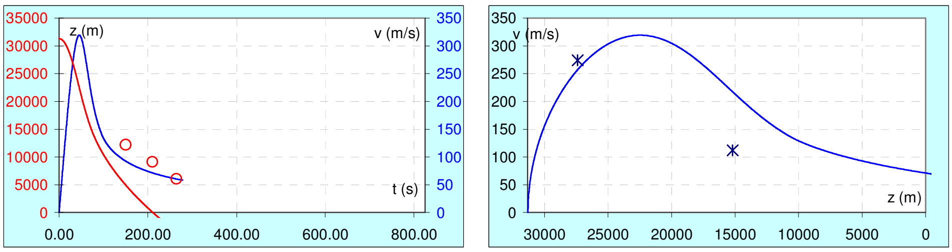
b	z_b (m)	ρ_b (kg/m^3)	T_b (K)	L_b (K/m)	c_b (m/s)
0	0	1.225	288.15	-0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

Figure 4: Solution to the first part of the fall (no chute deployed, top speed reached at 27,400 m)

Third: Parameter K_1 was iterated to reach maximum speed of 319 m/s. It was found that $K_1 = 0.00162$. The corresponding altitude and time are 22,500 m and 45 s (Fig. 5). The relative speed v/c and altitude z for $K_1 = 0.00292$ and $K_1 = 0.00162$ are graphed in Fig. 6.

J. W. Kittinger's jump

g_0	9.80665	m/s^2
r_0	6356776	m
M	0.028965	kg/kmol
R	8.314	J/mol.K
β	0.03416	kg
h	31300	m
m	142	kg
K_1	0.00162	m^2/kg
t_{12}	278	s
K_2	0.000000	m^2/kg
t_{23}	278	s
K_3	0.000000	m^2/kg
t_{max}	278	s
n	15000	
Δt	0.019	s



278 0.00162 1.813 9.82 -4277 58.3

v_{max}		v_{max}	
319.2	m/s	1.08	M

t (s)	K (kg/m^2)	ρ (kg/m^3)	g (m/s^2)	z (m)	v (m/s)	Δv (m/s)	c (m/s)	v/c
0.00	0.00162	0.015	9.71	31300	0	9.71	303	0.00
0.02	0.00162	0.015	9.71	31300	0.18	9.71	303	0.00
0.04	0.00162	0.015	9.71	31300	0.36	9.71	303	0.00
0.06	0.00162	0.015	9.71	31300	0.54	9.71	303	0.00
45.43	0.00162	0.059	9.74	22481	319.1832	0.00	297	1.08
45.44	0.00162	0.059	9.74	22476	319.1833	-0.01	297	1.08
45.46	0.00162	0.059	9.74	22470	319.1832	-0.02	297	1.08

b	z_b (m)	ρ_b (kg/m^3)	T_b (K)	L_b (K/m)	c_b (m/s)
0	0	1.225	288.15	-0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

Figure 5: Solution to the first part of the fall (no chute deployed, top speed 319 m/s)

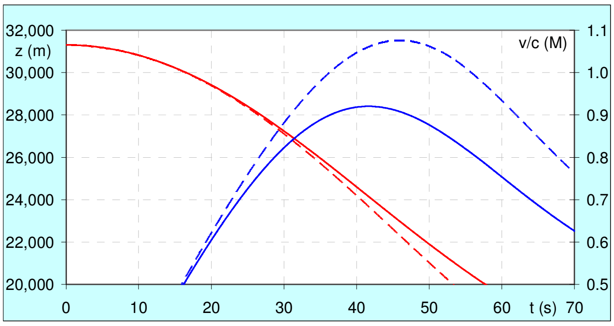


Figure 6: Relative speed (blue, right scale) and altitude (red, left scale) for $K_1 = 0.00292$ (full) and $K_1 = 0.00162$ (dashed), first 70 s

The maximum relative speed for $K_1 = 0.00292$ is 0.92 M, and it is reached at 24,200 m and 42 s. The maximum relative speed for $K_1 = 0.00162$ is 1.08 M, and it is reached at 22,300 m and 46 s. That is a supersonic fall. Eq. (9) gives $A \sim 0.46 \text{ m}^2$ if $C \sim 1$. Such a projected area only corresponds to head-to-earth or feet-to-earth position. Some time ago, an attempt was cancelled to break the sound barrier in free fall [10]. The jumper was to fall head-to-earth, stabilized just with his legs and hands straight back in a "V" shape. The analysis shows that it is possible to reach supersonic speed in this position; however, surviving is doubtful (see further). Kittinger's projected area was much bigger than 0.46 m². Thus, he could not reach supersonic speed.

There is another fact that indicates that $K_1 = 0.00292$ might be correct. As published, Kittinger had breathing difficulties between 90,000 ft (27,400 m) and 70,000 ft (21,300 m) [6] due to the helmet that was pressing against his throat [1]. It can be seen in Fig. 6 that if $K_1 = 0.00292$, then Kittinger was in transonic range, that is above 0.8 M, from about 27,600 m to 20,300 m, which meets well the range of breathing difficulties. In the transonic range, "Parts of your body may be going supersonic while others aren't, causing flutter waves pulling back and forth … that knocks him out of control" [10]. The turbulence might cause the problems with the helmet. The value $K = 0.00292$ is plausible for the first part of the fall.

5.2 Analysis of the second part of the fall (6-ft drogue chute deployed)

As shown above, the 6-ft stabilization must have deployed after reaching the maximum speed of 274 m/s at 41 s. Let it be deployed at 41 s. The main chute opened at 278 s at 5,330 m. Iterating K_3 under the condition that the 6-ft chute deploys at 41 s and the altitude at 278 s is 5,330 m yields $K_2 = 0.00732$. Then, the model gives speed 43.6 m/s at 278 s, which is acceptable to open the main chute (see link "T-10D" in [11]). The published data for the altitude against time correspond to the points (150 s; 12,200 m), (210 s; 9,100 m), and (264 s; 6,100 m). They are shown by circles in Figs. 1 – 5 and 7 in the chart on the left. The model meets well the points. However, the speed at 15,240 m should be 112 m/s, which cannot be fulfilled. The points (27,400 m; 274 m/s) and (15,200 m; 112 m/s) are shown by stars in Figs. 1 – 5 and 7 in the chart on the right. It can be seen that the $v(z)$ graph is not going through the points. If we take the point (15,200 m; 112 m/s) correct, then the 6-ft chute must have deployed later than at 41 s. Iterating both K_2 and the time of deploying the 6-ft chute shows that all the graphs go through the point just if the 6-ft chute deployed at 82 s and $K_2 = 0.00981$ (Fig. 7). Then, the model gives speed 37.6 m/s at 278 s when the main chute opened, which is acceptable. However, time 82 s seems to be too late to open the 6-ft chute to prevent Kittinger against the flat spin. Thus, $K_2 = 0.00981$ is less likely than $K_2 = 0.00732$.

Remark: If one wants to investigate the jump from the aspect of the chute characteristics, then the main question is whether the diameter of the chute was 6-ft when spread on the ground or when inflated. There may be differences of 50 % in the projected area (see section 5.3). Another question is the shape. We know nothing about that. Some drogue chutes have a circular ring slot canopy (see the photo in link "F-5" in [12]), some of them have an even more complicated shape [13]. Nevertheless, the area of a 6-ft diameter circle is 2.63 m². Then, Eq. (9) gives $C \sim 0.8$ for $K_2 = 0.00732$ and $C \sim 1$ for $K_2 = 0.00981$. The values are acceptable.

5.3 Analysis of the third part of the fall (28-ft main chute deployed)

As published, the 28-ft main chute deployed at 5,330 m at 278 s. The elevation of the landing area is 1,220 m (see the map [1] and Google Earth). Kittinger landed at 825 s. The distance of 4,110 m was travelled in 547 s, which gives 7.5 m/s of average descent rate.

In case the speed at 5,330 m and 278 s is 43.6 m/s (the first variant of the second part, $K_2 = 0.00732$), then the model gives $K_3 = 0.196$ and landing speed of 6.8 m/s, which is standard (see link "T-10D" in [11]). The solution is graphed in Fig. 1. In case the speed at 5,330 m and 278 s is 37.6 m/s (the second variant, $K_2 = 0.00981$), then the model gives the same values, that is $K_3 = 0.196$ and landing speed of 6.8 m/s. The solution is in Fig. 7.

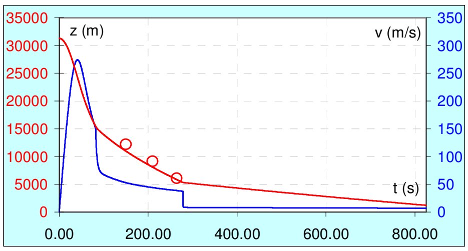
Remark: The published 28-ft was the nominal diameter of the chute (when the chute is spread on the ground). Comparison with the T-10D airborne troop parachute (nominal diameter 35 ft, inflated diameter 25.7 ft) gives inflated diameter 20.56 ft, which yields a projected area of 30.8 m². Eq. (9) gives $C \sim 1.8$. That seems to be too big if compared with $C \sim 1.4$ that holds for a hollow hemisphere. However, the photo on page 871 in [1] clearly shows that Kittinger did not descend vertically but he was gliding. Then, the air flowing around the canopy generates lift that has the same effect as if the drag coefficient increased. Thus, the value $K_3 = 0.196$ is acceptable for this part of the fall.

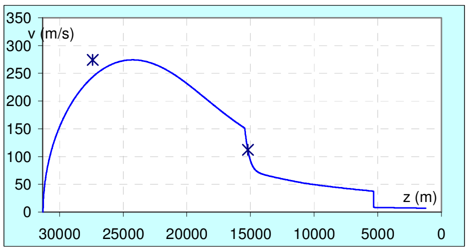
J. W. Kittinger's jump

g_0	9.80665	m/s²
r_0	6356776	m
M	0.028965	kg/kmol
R	8.314	J/mol.K
β	0.03416	kg
h	31300	m
m	142	kg

K_1	0.00292	m²/kg
t_{12}	82	s
K_2	0.00981	m²/kg
t_{23}	278	s
K_3	0.196	m²/kg
t_{max}	825	s

n	15000	
Δt	0.055	s





82	0.00981	0.180	9.76	15452	151.2
278	0.19600	0.710	9.79	5331	37.6
825	0.19600	1.088	9.80	1217	6.8

v_{max}	
274.2	m/s

v_{max}	
0.92	M

t (s)	K (kg/m²)	ρ (kg/m³)	g (m/s²)	z (m)	v (m/s)	Δv (m/s)	c (m/s)	v/c
0.00	0.00292	0.015	9.71	31300	0	9.71	303	0.00
0.06	0.00292	0.015	9.71	31300	0.53	9.71	303	0.00
0.11	0.00292	0.015	9.71	31300	1.07	9.71	303	0.00
0.17	0.00292	0.015	9.71	31300	1.60	9.71	303	0.01
41.19	0.00292	0.044	9.73	24271	274.2288	0.00	298	0.92
41.25	**0.00292**	**0.044**	**9.73**	**24256**	**274.2289**	**-0.02**	**298**	**0.92**
41.30	0.00292	0.045	9.73	24241	274.228	-0.04	298	0.92

b	z_b (m)	ρ_b (kg/m³)	T_b (K)	L_b (K/m)	c_b (m/s)
0	0	1.225	288.15	-0.0065	340.29
1	11,000	0.36391	216.65	0	295.07
2	20,000	0.08803	216.65	0.001	295.07
3	32,000	0.01322	228.65	0.0028	303.13

Figure 7: The less likely solution to the fall

6. Conclusion

J. W. Kittinger's jump from the altitude of 31,300 m was investigated in this paper. The findings are: (a) The 6-ft stabilization chute must have deployed at 41 s or later; (b) Supersonic speed was impossible to reach; (c) Top speed of 274 m/s was possible to reach but at the altitude of 24,300 m; (d) If the maximum speed was reached at 27,400 m, then it was 184 m/s. The most likely solution to the fall is graphed in Fig. 1.

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