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An Application of an Optimization Tool to Solve Problems of Mechanics of Materials

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Abstract

This paper presents a study about using computational tools applied to a particular problem of Mechanics of Materials. Our purpose is, on one hand, to solve a structural problem in order to teach the application of an optimization tool, such as Excel Solver, by means of the calculation of the minimum weight in a shaft. On the other hand, we present an active learning methodology based on the creation of spreadsheets that contributes to enhancing the motivation of students. Since the evaluation of the subject takes into account the activity of creating the spreadsheets, the academic results have improved considerably.

Keywords

Interactive Design, Design Optimization, Interactive Learning Environments

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Abstract

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1. Introduction

This paper presents a study about using computational tools applied to a particular problem of Mechanics of Materials. This subject is studied by second year students in B.E. Mechanical Engineering Program. These students have previously learnt Statics, which is the basis for developing problems of Strength of Materials.

When learning a subject like Strength of Materials, students often become overwhelmed by a great amount of theoretical concepts. So, we are looking for a new way of teaching mechanics. Some authors, such as D. Elata [1] and P.S. Steiff [2], have tried to apply new methodologies. And sometimes it is very useful to employ an interactive learning environment, such as a computing environment. So, activities using computers are expected to enhance active learning [3-8].

From our point of view, the combination of experimental procedures with the use of the computer as a tool to solve problems has proved a good way to improve the motivation of students. Some previous work has been done in this sense. Figure 1 shows the schedule that has been designed to explain the methodology used in our laboratory [9].

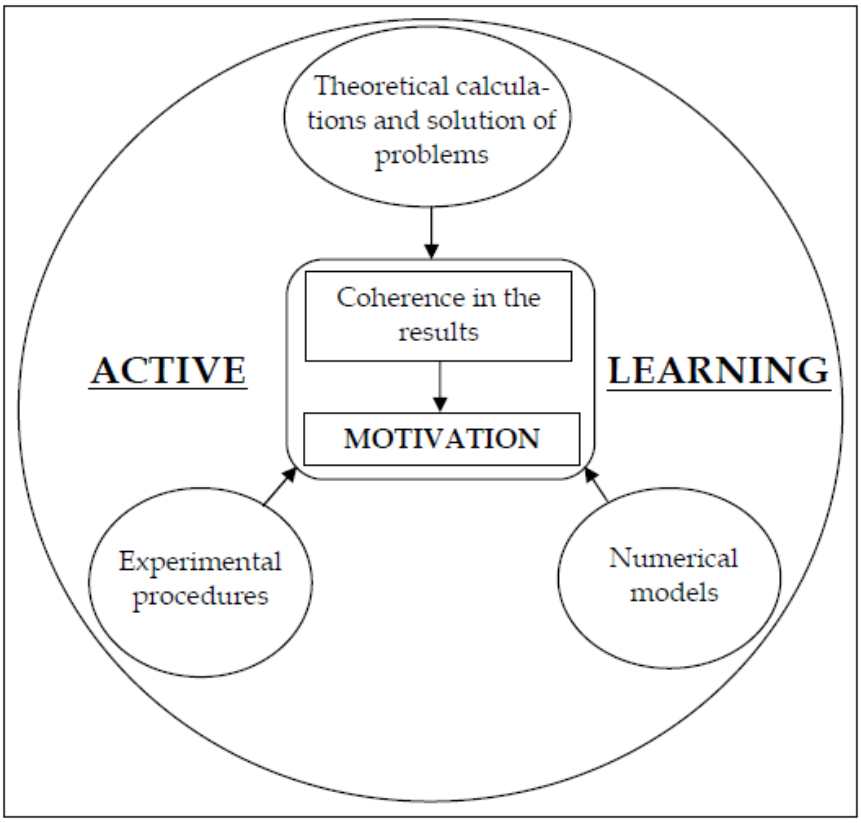

Figure 1. The active learning diagram applied to our laboratory [9].

Figure 1 shows that the active learning that we propose aims to relate the theoretical concepts, the experimental procedures and the computational methods to improve the motivation of the students by means of the coherence in the obtained results.

Of course, it would be possible to add other learning methodologies to our proposal. That is the case of teaching through inquiry [10] or problem-based learning [11], that have proved good results in structural and construction engineering. But our study is focused on using computers.
It is worth saying that there are lots of studies related with the use of spreadsheets in the teaching-learning process as a way to encourage students in different disciplines, such as numerical

methods, computer aided design, mathematics, etc [12-14]. Also, there are some textbooks that combine theory explanations with modelling tools such as spreadsheets [15].
So, our purpose is to:

1) Present a structural problem to teach the application of an optimization tool in order to calculate the minimum weight in a shaft loaded with an axial force F and a torque T.
2) Develop a methodology that contributes to enhancing the motivation in the study, which is the main learning objective.

To achieve these objectives, an optimization tool is going to be used. Mathematical optimization seeks to find out what is best for problems in which the answer can be expressed as a numerical value. A mathematical optimization model consists of an objective function and a set of constraints expressed in the form of a system of equations or inequalities. Optimization models are used extensively in almost all areas of decision-making, such as mechanical engineering design [16-20], and can be applied in practically all fields of knowledge [21; 22]. Spreadsheets contribute to design decision problems [23; 24] and they are used in practically all fields of engineering [25, 26].

In the field of structural engineering design, the usual process to obtain a solution for a particular problem is described in Figure 2 [27].

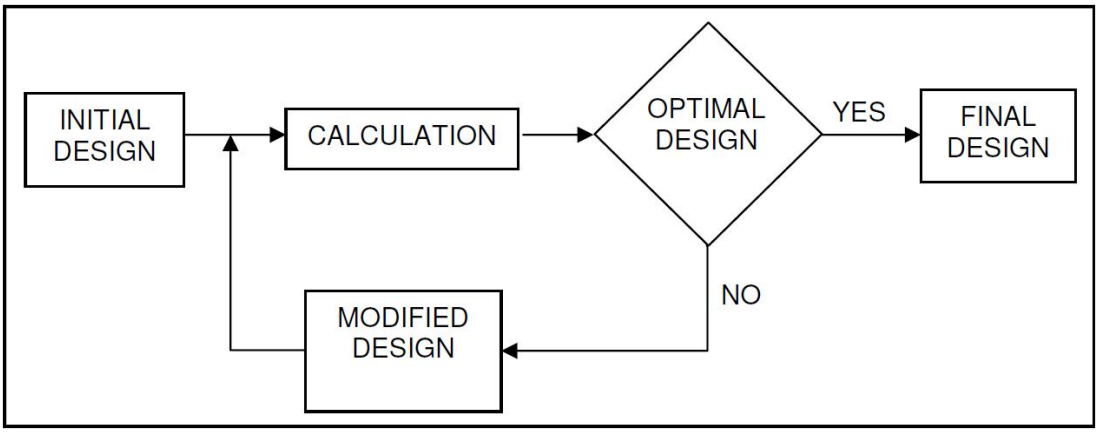

Figure 2. The process used to obtain a solution.

2. Description of the problem

In order to study some particular problems of mechanics, some applications have been developed, since we consider that spreadsheets take an important role in the motivation of students [28, 29].

First of all, some explanation must be done about failure theories. Failure theories (or yield criteria) are theories used in mechanics in order to determine a general state of stress.
In simple problems where there is only one principal stress to consider (such as simple axial loading), it is not necessary to apply any yield criteria. But when the problem contains a complex principal stress system, it would be very uneconomical to apply a trial and error method; so failure theories are a very useful tool.

So, in this work, a problem with combined loadings will be solved using the maximum shear stress theory (or Tresca criterion), that is applied for ductile materials such as steel or aluminium. This theory assumes that yielding is dependent on the maximum shear stress in the material reaching a critical value.

Figure 3 shows a circular solid shaft loaded with an axial force F and a torque T. So, it is a particular case of combined loadings.

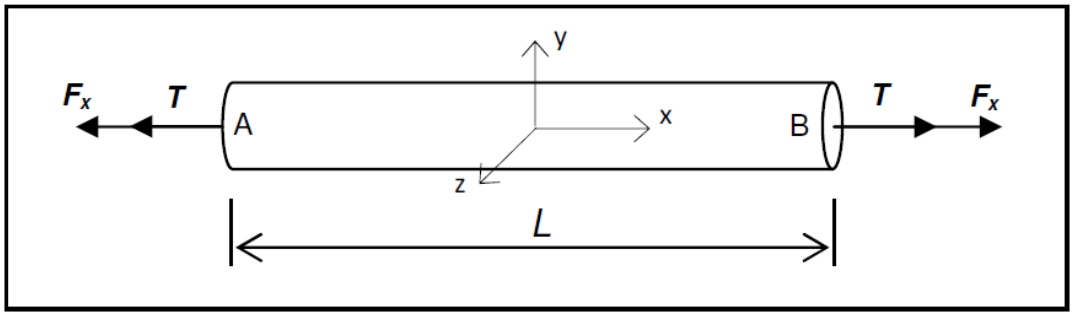

Figure 3. Circular solid shaft loaded with an axial force F and a torque T.

The purpose of this problem is to determine the minimum weight of the shaft. This can be done by means of the application of the maximum shear stress theory. Moreover, the maximum deflection due to the axial force and the maximum angle due to the torque, are going to be considered as a constraints for the problem.

3. Procedure for analysis

The proposed procedure for analysis is:

1) To calculate the normal and shear stresses that act on the shaft as a consequence of the applied forces and moments.
2) To determine the principal stresses.
3) To apply the corresponding failure theory to determine the equivalent stress in the structural component.
4) To calculate the deflection and the angle due to the combined loading (constraints).
5) To solve the equation using an optimization tool such as Excel Solver. In this tool, the four main elements are explained in Table 1:

Table 1. The four main elements of the Excel Solver optimization tool.

"Set Target Cell"	That contains the quantity we wish to optimize (the objective function value). In our case, the weight of the shaft.
"Equal To"	That specifies the direction of optimization. In this case, we try to find out the minimum weight of the shaft.
"By Changing Cells"	That is in order to optimize the value of the objective function by choosing an appropriate vector of decision variables. In our case, it is referred to the radius.
"Subject to the constraints"	That is in order to specify the corresponding constraints that define the problem. In our application the shaft will be subjected to the following three constraints: the yield stress of the material, the allowable deflection in tension and the allowable angle in torsion.

Now, the procedure for analysis is described in a more detailed way:

3.1. Normal and shear stresses

The stresses due to the axial force (normal stresses) and to the torque (shear stress) can be determined by equations 1 through 3:

$$\sigma_x = \frac{F_x}{A} = \frac{F_x}{\pi r^2} \qquad \text{(eq. 1)}$$

$$\sigma_y = 0 \qquad \text{(eq. 2)}$$

$$\tau_{xy} = \frac{2T}{\pi r^3} \qquad \text{(eq. 3)}$$

F_x, the applied axial force; T, the applied torque; A, the cross section area of the shaft, where r is the radius of the shaft.

3.2. Principal stresses

To apply the corresponding failure theory, the maximum and minimum stresses are calculated by means of the equations 4 and 5:

$$\sigma_{\max} = \frac{F_x}{2\pi r^2} + \sqrt{\left(\frac{F_x}{2\pi r^2}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2} \qquad \text{(eq. 4)}$$

$$\sigma_{\min} = \frac{F_x}{2\pi r^2} - \sqrt{\left(\frac{F_x}{2\pi r^2}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2} \qquad \text{(eq. 5)}$$

3.3. Maximum shear stress theory

It is necessary to determine the equivalent stress, that equals the yield stress by means of the application of the maximum shear stress theory (equation 6):

$$\sigma_{eq} = \sigma_{\max} - \sigma_{\min} \leq \sigma_{UY} \qquad \text{(eq. 6)}$$

σ_{UY} is the yield stress of the material.

3.4. Constraints of the problem

Now, substituting equations 4 and 5 into equation 6, the first constraint will be obtained. The process is detailed in equations 7 and 8:

$$\sigma_{UY} \leq \left[\frac{F_x}{2\cdot\pi\cdot r^2} + \sqrt{\left(\frac{F_x}{2\pi r^2}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2}\right] - \left[\frac{F_x}{2\pi r^2} - \sqrt{\left(\frac{F_x}{2\pi r^2}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2}\right] \qquad \text{(eq. 7)}$$

$$\sigma_{UY} \leq 2\sqrt{\left(\frac{F_x}{2\pi r^2}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2} \qquad \text{(eq. 8)}$$

Equation 8 is a constraint referring to the yield criteria, that must be taken into consideration for the calculation of the optimum radius, that will provide the minimum weight of the shaft.
So, the first constraint has been determined by means of the application of the maximum shear stress theory and it is referred to the yield stress of the material. Moreover, in a structural problem, such as the one presented in this paper, there must be some limitations given by standards or by the designer. In our case, students define the two following constraints: the allowable deflection due to the axial force (δ) and the allowable angle due to the torque (θ).
Equations 9 and 10 define these limitations:

$$\delta \leq \frac{F_x L}{EA} \qquad \text{(eq. 9)}$$

$$\theta \leq \frac{2TL}{G\pi r^4} \qquad \text{(eq. 10)}$$

F_x, the applied axial force; L, the length of the shaft; E, the modulus of elasticity; A, the cross section area of the shaft; T, the applied torque; G, the shear modulus.
So, equations 8, 9 and 10 define the three constraints of the problem.

3.5. Solving the problem

To solve this problem, an objective function must be defined. As we try to find out the minimum weight (W) of the shaft, this function will be the equation 11:

$$W = \rho L \pi r^2 \qquad \text{(eq. 11)}$$

ρ is the density of the material (kg/m^3). This equation is defined in cell G18 (Figure 4).
So, Figure 4 shows the developed spreadsheet to solve the problem. In this spreadsheet, equations 8, 9 and 10 are defined in cells C17, C20 and C23, respectively.

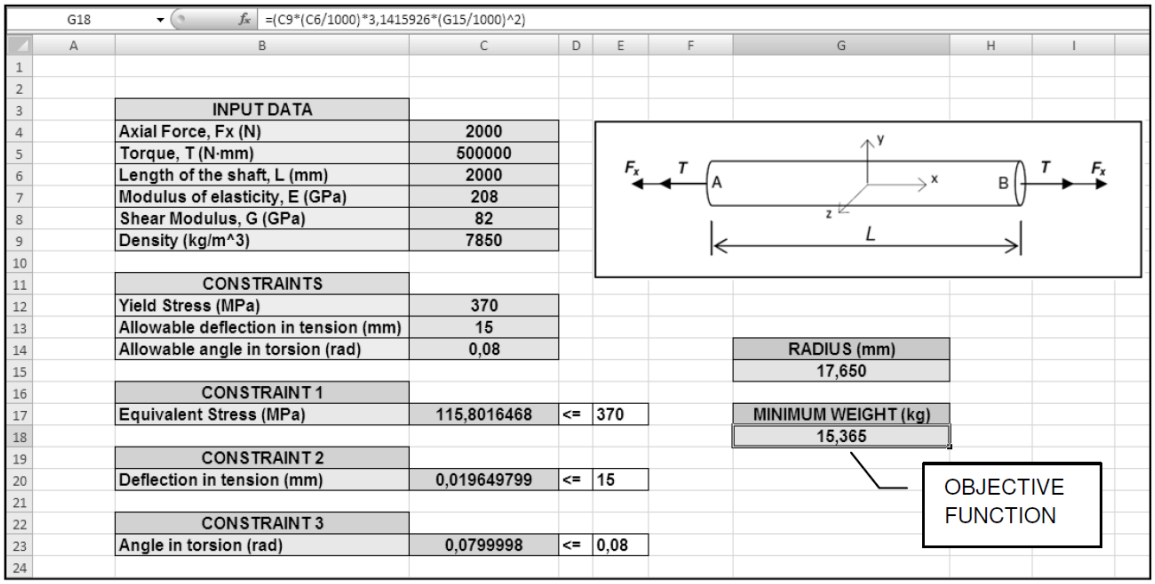

Figure 4. Spreadsheet used for the problem.

So, students are able to study the variation of the radius in the shaft when modifying any of the input data parameters. The radius relates with the weight of the shaft.

Since the objective function and the constraints are not linear functions of the decision variables, it is worth noting that when dealing with a nonlinear problem [30], it is convenient to start from initial values for the decision variables [31]. So, the Solver follows a path from the starting values (based on our knowledge of the problem) to the final solution values, depending on the direction and curvature of the objective function and constraints [32].

Excel Solver, by means of the GRG Method (Generalized Reduced Gradient algorithm), can find a locally optimal solution to a well-scaled model [33]. This locally optimal solution means that Excel Solver has found a "peak" (maximizing) or a "valley" (minimizing).

4. Teaching-Learning strategy

Once this problem has been understood, students are encouraged to solve some more problems related with the one that has been explained previously in detail. This is the strategy proposed to motivate students in their learning process.

As an example, Figure 5 shows the new problem, that is a modification of the first one.

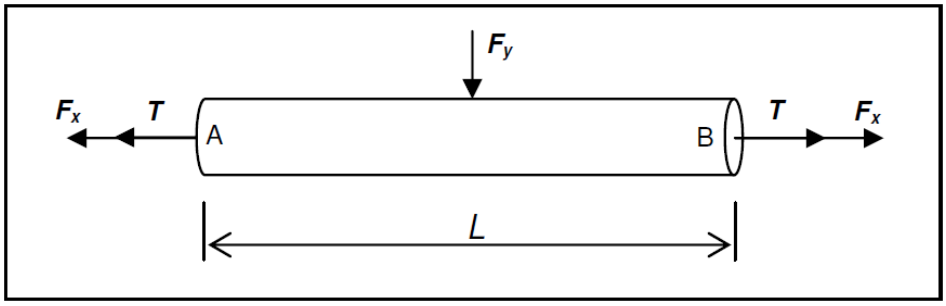

Figure 5. The new problem.

The new problem considers the shaft supported at points A and B and a shear force (F_y) acting in the middle of the shaft. The force F_y produces bending. The deflection due to bending (δ_y) will be taken into account as a new constraint of the problem (equation 12):

$$\delta_y = \frac{F_y L^3}{48 E I_{zz}} \qquad \text{(eq. 12)}$$

I_{zz} is the moment of inertia of the cross section of the shaft, that equals to $r^4/4$.
Of course, to calculate the equivalent stress there will be a new axial stress due to the bending moment (equation 13), that will be added to the equation 1:

$$\sigma_x = \frac{M_b r}{I_{zz}} \qquad \text{(eq. 13)}$$

M_b is the bending moment, that is $(F_y L)/4$. In this particular case, this new axial stress has to be added to the axial stress due to the axial force that is considered in equation 1. In the same way, students will consider the new axial stress in the determination of the equivalent stress; equations 6, 7 and 8. With this new data, students develop the new spreadsheet, that is shown in Figure 6.

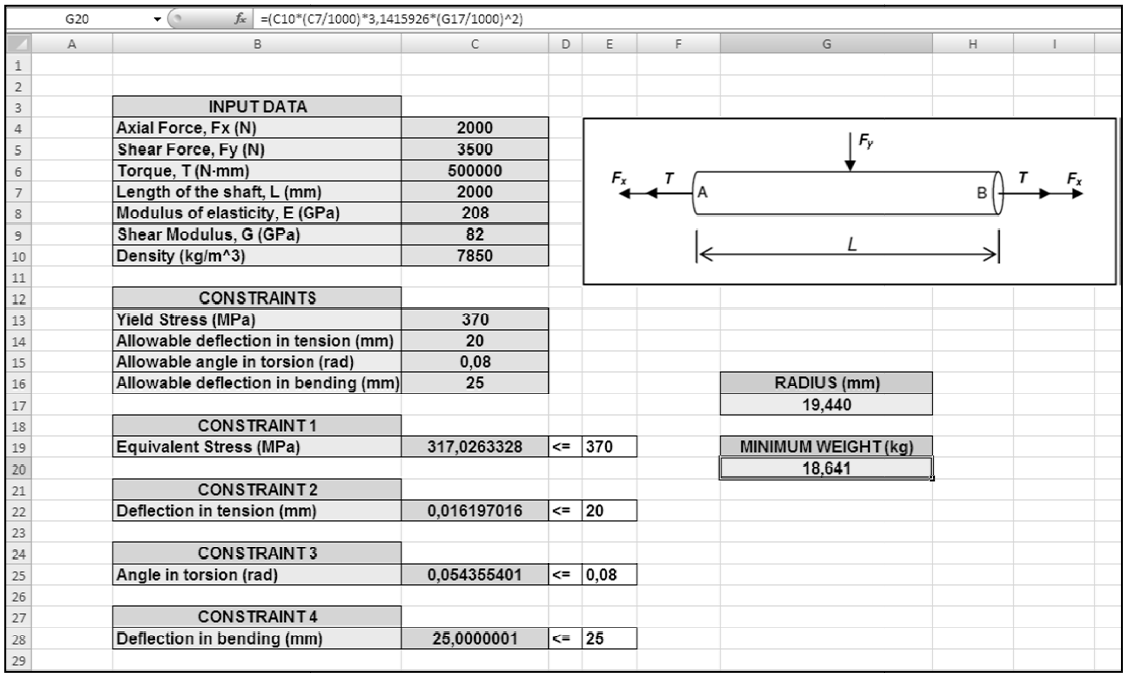

Figure 6. The new modified spreadsheet.

5. Conclusions

The aim of this teaching-learning strategy was to apply an alternative way to teach mechanics of materials. Using spreadsheets offers a computing environment in which students are able to set up the mathematical relationships for quite sophisticated systems in a similar manner in which they would do it by hand. So, one of the main advantages is that students can interfere with the program.

So, the tool created by students is easy to use and they enjoy learning the subject. It is observed that a reinforcement of students understanding of concepts and procedures of mechanics of materials take place.

When working in the laboratory, the use of spreadsheets is complemented with theory explanations and procedures developed in experiments. With regard to the work presented in this paper, it is worth saying that students have previously worked in this way, developing spreadsheets that allow the study of shear stresses in bolted connections, torsion in shafts, bending in beams and buckling in columns. So, the final step is to apply the failure theories when acting combined loadings by means of the application of the optimization tool Excel Solver. This type of nonlinear programming problems are solved with the GRG method (Generalized Reduced Gradient algorithm).

The aim is that, at the end of the course, students have a collection of such spreadsheets. This methodology contributes to enhancing the motivation in the study and since the evaluation takes into account the activity of creating the spreadsheets, the academic results have improved considerably.

The work presented in this paper is our first step in applying this methodology in class with computers. This course is concerned with basic concepts and procedures of strength of materials. There is an advanced course of mechanics of materials for third year students. In that course, it is pretended to continue using this type of tools; however, by now those students work with a software based on the Finite Element Method, such as ANSYS.

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