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P-Value Approximations for T-Tests of Hypothesis

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Keywords

Mathematical Statistics, T-Tests of Hypothesis, P-Values, Numerical Integration, and Simulation

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Mathematics can be analyzed in different ways and each method supports the other with the same results. This paper describes a number of approaches for finding the p-values necessary for making decisions about statistical t-tests of hypothesis. The concepts of areas under the Student's t-curve and the mathematical connections between tests of hypothesis, probabilities and areas under a curve are presented. The EXCEL function TDIST and various approximation techniques from numerical analysis are discussed. Numerical analysis techniques include Simpson's Rule for integration and Monte Carlo integration. Also an approximation from the National Bureau of Standards is provided. Comparisons of results from each method are presented. Numerical approximations are shown to be as important and as accurate as exact solutions.

Keywords: Mathematical Statistics, t-tests of hypothesis, p-values, Numerical Integration, and Simulation.

1. Introduction

Analyzing concepts in various ways helps in understanding them. Comparisons of results and connections of ideas are made possible with technology. The applications of EXCEL to doing mathematics allow the user to apply several approaches to the study of numerous concepts. Spreadsheets enable the user to proceed through mathematical concepts in algorithmic, step-by-step ways. The actual calculations that are applied to achieve successful outcomes are illustrated. EXCEL can conduct various statistical techniques by performing calculations in the traditional ways, or by using numerical analysis techniques and the Data Analysis Toolpak. Data Analysis Toolpak is an add-in for EXCEL. Many additional statistical techniques not found in the EXCEL spreadsheet alone can be done using Toolpak. Although the Data Analysis Toolpak does not have the capabilities to do certain techniques that software such as SPSS, Minitab or SAS can, EXCEL can be used to demonstrate mathematical concepts that cannot be accomplished in other ways. And although a statistician would probably not use EXCEL to do quantitative research, EXCEL has its place in displaying the mathematics behind actual calculations.

This paper exhibits the mathematics behind the determination of p-values for t-tests of hypothesis. Also this paper illustrates that EXCEL with its Built-in function TDIST and the add-in Data Analysis Toolpak provides the same results correctly and without the mathematical details.

Through this discussion the user witnesses how approximations are made and learns a variety of different mathematical methods for finding the p-values for t-tests of hypothesis. Describing the t-distribution, the value of it in quantitative research, the importance of p-values in decision analysis and statistical tests of hypothesis are emphasized.

2. Tests of hypothesis and p-values

A test of hypothesis is a common approach to statistical inference. An assertion is made regarding a population parameter. The parameter is a numerical value representing the population. The objective of statistical research is to make an inference or a prediction about the parameter from information collected in a sample. The most popular inferences include those about means, differences between and among means, proportions, correlations, and slopes in linear regression. There are many more.

The following steps are applied when conducting a test of hypothesis [5]: a) State the null hypothesis and alternative hypothesis; b) Set the alpha or significance

level (usually set at 0.05); c) Determine the test statistic; d) Determine the p-value; e) Decide to reject or accept the null hypothesis (reserve judgment) based on the p-value.

In research, a null hypothesis and an alternative hypothesis are usually stated, as part of the research question. Essentially the null hypothesis tests the theory that there are no differences between distribution parameters. The alternative or research hypothesis claims there is a difference between the distribution parameters. If there is a difference in the distributions and this difference would be statistically significant, the null hypothesis is rejected. Statistically, the p-value is the smallest observed significance level [5] for which a researcher rejects the null hypothesis. That is the smaller a p-value, the more evidence exists to support the alternative hypothesis. This p-value determination enables a researcher to make decisions, regarding whether the data collected indicates significant differences or not in the population. Usually for alpha, the significance level is 0.05, a p-values less than or equal to this number indicates there are significant differences regarding the specific parameters being analyzed.

Mathematically, the p-value is the area under a curve defined for a specific probability distribution and requires integral calculus or some approximation technique to achieve correct results. There are numerous ways of evaluating areas under curves using integration techniques. In determining the area under the t-probability distribution only numerical integration and approximation techniques can be applied. These areas cannot be found other ways because the integrals representing the areas under the t-curve cannot be evaluated in the closed analytic form found in standard calculus textbooks [4]. On the interval (a, b), the area under the t-probability distribution requires determining the value of the following integral

$$\int_a^b f(t)dt$$

where $f(t)$ is defined [3] by:

$$f(t) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\Gamma(\frac{v}{2})} \left(1 + \frac{t^2}{v}\right)^{-\frac{v+1}{2}} \quad (1)$$

where v is the number of degrees of freedom and Γ represents the gamma function defined by the integral

$$\Gamma(x) = \int_0^{\infty} e^{-u} u^{x-1} du \quad (2)$$

is the calculated test statistic. There also is no analytical closed way to evaluate the integral representing the gamma function [4]. But the gamma function is built-in to EXCEL.

A number of ways to integrate this probability density function $f(t)$ is discussed in the following sections.

3. The t-probability distribution (The Student's t-distribution)

The t-distribution or Student's distribution discovered by Gosset [5] is a continuous probability distribution that is used to study relationships of many kinds. The distribution approaches the normal bell-shaped distribution as the sample size gets large. For each sample of size n , the degrees of freedom $n-1$ based on the sample size is used to display the shape of the t-distribution. This distribution is defined by a function $f(t)$ called the probability distribution or probability density function. For a distribution to be continuous the area under the curve defined by that function must be 1 and the probability that a value is in a certain interval (a, b) is given by the integral

$$\int_a^b f(t) dt \quad (3)$$

Since the probability density function defined by the t distribution cannot be evaluated by direct application of integration, approximations are necessary [4].

4. T-tests of hypothesis

A number of steps are usually implemented for conducting tests of hypothesis. They include: Determine the null hypothesis and alternative hypothesis. Calculate the t-statistic. The t-statistic is based on the t-probability distribution. Determine the rejection region using the p-value. If $p \leq .05$ reject null hypothesis and state that the findings did not occur by chance. That is the null hypothesis is rejected and the researcher can claim there is a significant difference between hypothesized mean and the sample mean for a one sample independent t-test or

claim there is a difference in 2 means for an independent samples t-test for 2 groups.

The t-test is a specific parametric technique applied to making decisions and inferences about population means and the difference in 2 population means among many others. T-tests of hypothesis are invaluable in research involving quantitative methods.

5. Assumptions for parametric t-tests of hypothesis

When conducting parametric tests of hypothesis, a number of assumptions are necessary in order to conduct valid t-tests of hypothesis. 1) When two populations are compared, the samples collected must be independent. 2) Data must come from a normal population or two normal populations. 3) When analyzing two samples from two populations, the variances of each group are equal and 4) Data collected must be random. Once these assumptions are justified then the researcher can proceed with the t-test of hypothesis [5]. Since the goal of this paper is to show various approximations of p-values for t-tests of hypothesis, these assumptions are not verified.

6. Usefulness of the t-test of hypothesis

The t-test of hypothesis is a valuable inferential technique for research experiments where the sample size is small or where the population variance or standard deviation is unknown. The study of statistics enables a researcher to analyze sample data and to infer or predict the population parameters. The disappointing fact about learning statistics is that many ideas are studied with too much reliance on technology. Much of the mathematics behind these calculations is conducted by a computer. Any student would benefit from calculating and applying different techniques for the sequential process used to find correct and approximate p-values.

Examples where t-tests of hypothesis are used include:

- a) Suppose you wish to study whether there is difference in mean weights for men to women at a certain age.
- b) Suppose you wish to study whether the manufacture of 17 ounces of a can of peas is on average 17 ounces.
- c) Also you could study whether there is a difference in means of a particular concept where a test is given before a treatment and then a test is given on the same material after some practice.

A variety of t-tests of hypothesis exist. This paper describes the following t-tests of hypothesis: a) One sample independent t-test; b) Two samples independent t-test; and c) Paired samples t-tests.

7. Multiple ways for finding p-values for t-tests of hypothesis

EXCEL is a useful software tool for conducting tests of hypothesis and determining the p-values for t-tests of hypothesis. Numerous ways to determine p-values include:

- a) A p-value can be found by using the EXCEL command TDIST. The TDIST syntax incorporates the calculated t, the number of degrees of freedom and whether a 1-tailed or 2-tailed test is conducted. That is “=TDIST(t, df, 1 or 2)” will calculate a specific p-value.
- b) When using the Data Analysis Toolpak, a t-test of hypothesis is available for two independent samples where variances are equal or unequal and for paired dependent samples t-tests. Add-in Data Analysis Toolpak is found by going into EXCEL. For installing Toolpak in MS Office 2010 see <http://office.microsoft.com/en-us/excel-help/load-the-analysis-toolpak-HP001127724.aspx>. The Data Analysis Toolpak cannot conduct 1 sample t-tests of hypothesis.
- c) Since the determination of p-values involves finding areas under the t curve, numerical integration is also possible. And since the area under the t-curve cannot be performed analytically, the application of the numerical integration technique called Simpson’s Rule is useful. The use of Data Analysis Toolpak, TDIST and Simpson’s rule all illustrate the same p-values. And these three methods yield the same answers as usually found in tables’ books and textbooks [5].

Simpson’s Rule [2] is a numerical analysis technique that is often applied where there is no analytic integral form that can be evaluated. The technique is based on an even number of subintervals n and is given by

$$S_n = \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(x_n)] \quad (4)$$

where the approximation is made over the interval (a, b) and n is the number of subintervals.

- d) Monte Carlo integration is another technique used to approximate the value of an integral. Select a large number of points over an interval

randomly. Estimate the value of the function by taking the interval width (b-a) and multiply this value by the average value of the function given by:

$$\frac{1}{n} \sum_{i=1}^n f(x_i).$$

The examples illustrate applications of Monte Carlo integration. That is the Integral from a to b is approximately equal to

$$(b - a) \frac{1}{n} \sum_{i=1}^n f(x_i) \tag{5}$$

- e) The National Bureau of Standards (NBS) provides another approximation [1] for finding the right tail area under the t probability distribution. This rational approximation to p-values was implemented before computing technologies became widely available.

The right tail area is approximated by the following:

$$\frac{1}{4(1 + a_1x + a_2x^2 + a_3x^3 + a_4x^4)^4} + error \tag{6}$$

where

$$x = \left(t^{3/3} \left(1 - \frac{2}{9d} \right) - \frac{7}{9} \right) \left(\frac{2}{9} + t^{4/3} \frac{t}{9d} \right)^{-1/2} \tag{7}$$

and

$$\begin{aligned} a_1 &= 0.196854 & a_2 &= 0.115194 \\ a_3 &= 0.000344 & a_4 &= 0.019527 \end{aligned}$$

and d is the number of degrees of freedom and t is the calculated test statistic.

The independent samples t-test, two independent samples t-test and the paired samples t-test are discussed and illustrated with Simpson’s Rule, Monte Carlo Integration and the NBS Approximation [1] in the examples that follow.

8. Examples

Examples below demonstrate Simpson’s Rule, Monte Carlo Integration

and the NBS Approximation. Examples 8.1 and 8.2 show the area under the t probability distribution from 0 to 1.5 with 98 degrees of freedom. Example 8.3 shows the approximation of the right tail area (probability) above 1.5 with 98 degrees of freedom

Example 8.1: Implementation of Simpson’s Rule on a spreadsheet

In this example, Simpson’s Rule is displayed. Below the figure are the details on the implementation of EXCEL.

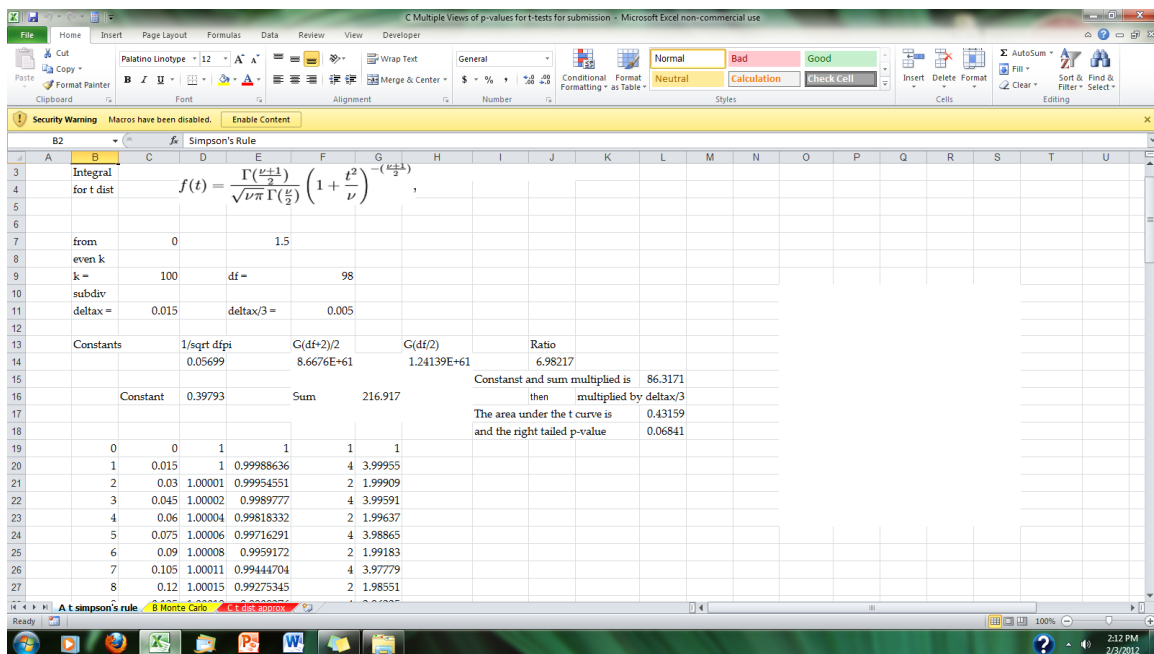


Figure 1: Simpson’s Rule

The following are the EXCEL formulas entered in each column along with what is calculated.

- Column B: =B19+1; counts iterations
- Column C: =\$C\$7 =C19+\$C\$11; breaks interval into even number of segments
- Column D: =(1+C19^2/\$F\$9); calculates 1+t²/ν
- Column E: =D19^(-1*(\$F\$9+1)/2); calculates (1+t²/ν)^{((-ν+1)/2)}. This is the function without the constant part.
- Column F: Start with 1 and end with 1 and alternate 4 2 4 2 4
- Column G: =E19*F19

Notice that the TDIST command presents the same value.

The constant, $\frac{1}{\sqrt{\pi v}} \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})}$ is independent of integration and is pulled out of the integral and calculated separately.

The product of 1/3h and deltax completes the calculation.

In Figure 1, n = 100 iterations are displayed. Column G shows the product of Columns E and F. Summing these values and multiplying by the constant term provide the result in first section. This constant must be multiplied by the sum of the 100 iterations and the constant 1/3h from Simpson's Rule. The answer is .06841. The results from Simpson's rule and the EXCEL command TDIST are the same.

Example 8.2: Monte Carlo integration on spreadsheet

In this example Monte Carlo integration is displayed. Below the figure are the details on the implementation of EXCEL

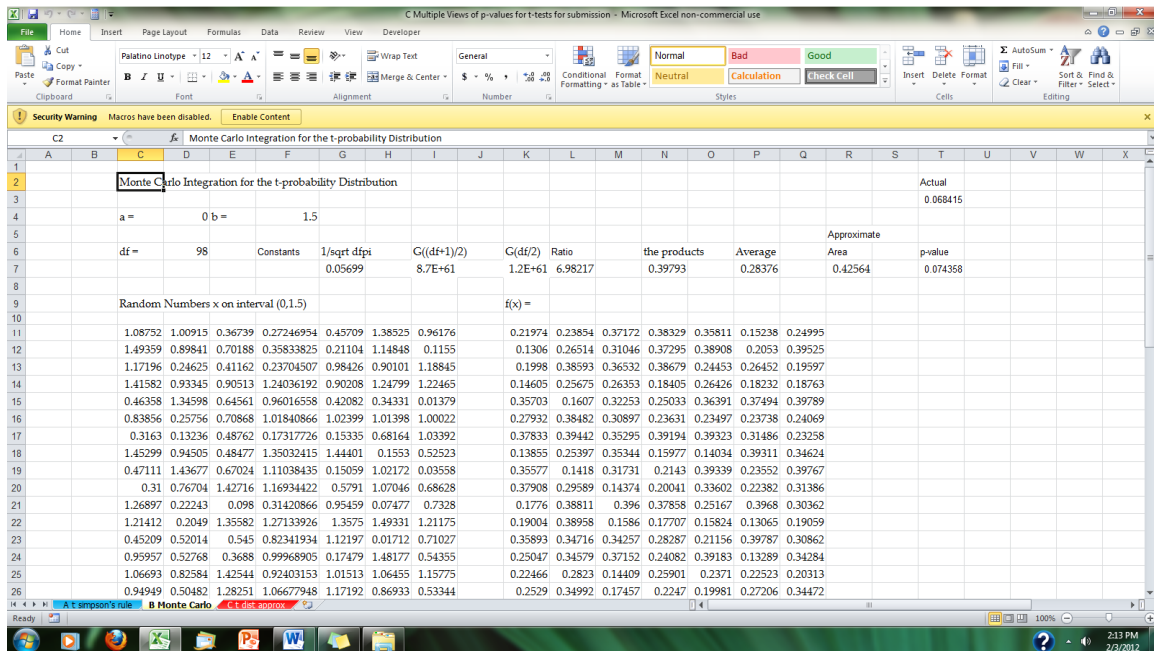


Figure 2: Monte Carlo integration

Cell formulas are as follows:

Column C: $=\$D\$4+(\$F\$4-\$D\$4)*RAND();$ generates random numbers on the interval (0, 1.5)

Column K: $=\$N\$7*(1+C11^2/\$D\$6)^{-((\$D\$6+1)/2)}$ uses

$(1 + \frac{t^2}{v})^{-\frac{(v+1)}{2}}$ to evaluate this expression at the generated random numbers.

Next the syntax for 1/square root (v times pi): =1/SQRT(\$D\$6*PI())

And for Gamma(v+1)/2: =EXP(GAMMALN((\$D\$6+1)/2))

And for Gamma(v+1): =EXP(GAMMALN((\$D\$6)/2))

Ratio of the gammas is calculated and then the product: =\$L\$7*\$G\$7

The constant [1] is saved until the end. Then in

Cell K49: =AVERAGE(K11:Q46) calculates the average of all values of the function evaluated at the generated random numbers.

Cell K51: =(\$F\$4-\$D\$4)*\$K\$49; approximates the area.

Cell K53: =0.5-K51; approximates the p-value

Note that the GAMMALN function in EXCEL is equal to LN($\Gamma(x)$) and is the natural log of the gamma function. That is =EXP(GAMMALN(df+1)/2) calculates $\Gamma(\frac{v+1}{2})$ and similarly =EXP(GAMMALN(df/2)) calculates $\Gamma(\frac{v}{2})$

As before the constant term $\frac{1}{\sqrt{\pi v}} \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})}$ is kept out of the calculations until the end.

The first section (first seven columns) of the table is a set of random numbers generated on the interval (0, 1.5). The right side of the table shown includes the t distribution function formula (1) evaluated at these random numbers. The average of all these values is shown. This value is then multiplied by 1.5 minus 0. The approximation of the integral [1] on the interval (0, 1.5) is 0.060403. This result is only 1 calculation. Each time Monte Carlo integration is used a different p-value approximation is found. Only 1 cell with a random number and only 1 function evaluation are shown. The rest of values are based on the same format and are copy and pasted down and over to get all the numbers shown.

Example 8.3: The NBS approximation

In Example 8.3 the approximation from the National Bureau of Standards [1] is displayed.

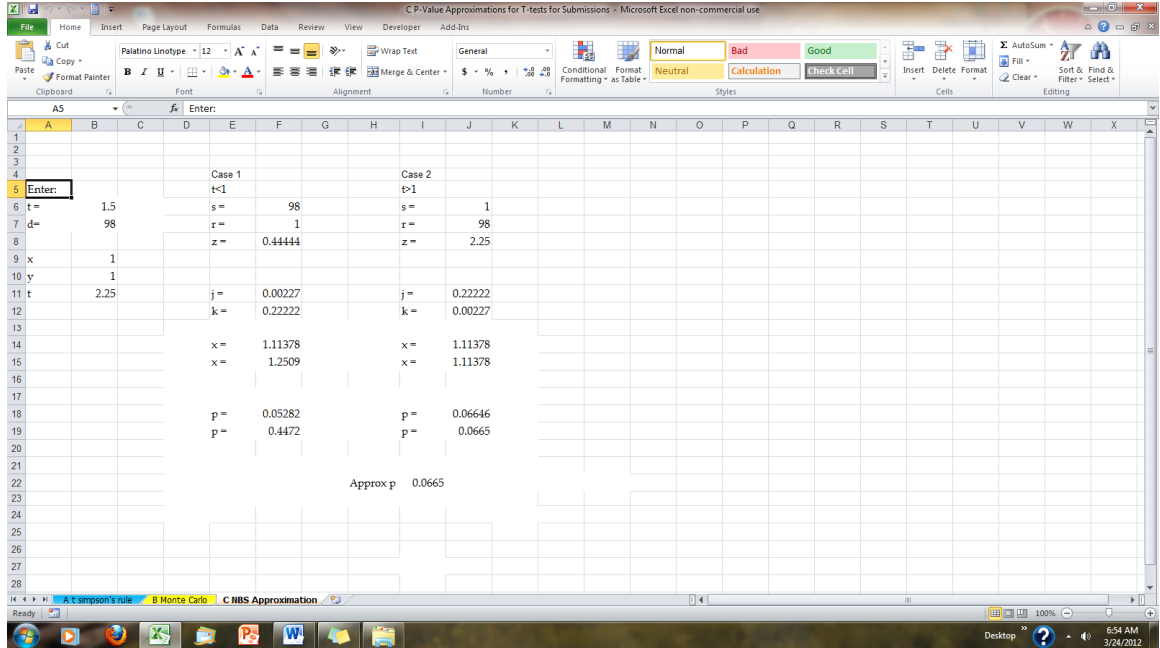


Figure 3: The NBS approximation

Details on this approximation [6] are described below:

In Cell B6 enter the calculated t

In Cell B7 enter degrees of freedom

In Cell B9 enter 1

In Cell B10 enter 1

In Cell B11 enter $B6^2$ (which is t^2)

Case 1 where t is smaller than 1,

Enter degrees of freedom (df): =B7 in Cell F6

Enter value for r: =B10 in Cell F7

Enter value for z: =1/B11 in Cell F8 which is 1/t

Enter $2/9/s$ that is: =2/9/F6 in F11

Enter $2/9/r$ that is: =2/9/F7 in F12

Next calculate the approximate x in parts

In Cell F14 enter: =ABS((1-F12)*F8^(1/3)-1+F11)/(SQRT(F12*F8^(2/3)+F11))

In Cell F15 enter: =F14*(1+0.08*F14^4/F7^3). This calculates the complete x value

Calculate the approximate p-value in two steps

In Cell F18 enter:

=0.25*(1+0.196854*F15+0.115194*F15^2+0.000344*F15^3+0.019527*F15^4)^(-4)

In Cell F19 enter: =0.5-ROUND(F18,4). This is the case where t is less than 1.

Case 2 where t is larger than 1,

In the section where s, r, and z are displayed the values had to be reversed compared to case 1 as shown:

Enter value for s: =B10

Enter value for r: =B7

Enter value for z: =B11

Enter 2/9/s that is: =2/9/J6

Enter 2/9/r that is: =2/9/J7

Next calculate the approximate x in parts

In Cell I14 enter : =ABS((1-J12)*J8^(1/3)-1+J11)/(SQRT(J12*J8^(2/3)+J11))

In Cell I15 enter: =J14*(1+0.08*J14^4/J7^3)

Again calculate the approximate p-value in two steps:

In Cell I18 enter:

=0.25*(1+0.196854*J15+0.115194*J15^2+0.000344*J15^3+0.019527*J15^4)^(-4)

In Cell I19 enter: =ROUND(J18,4). This step of rounding the result provides a very close result to those calculated by Simpson's Rule or Monte Carlo integration.

In Cell I22 enter: =IF(B6>1,J19,F19). This EXCEL command automates the approximate p-value based on the t calculated. This approximation and the p-value found by using =TDIST(t,df,1) are comparable. The p-value from the NBS approximation is 0.0665. The most accurate p-value is 0.06841 provided by Simpson's Rule and EXCEL's TDIST. The p-values from Simpson's Rule or EXCEL's TDIST are the most accurate because they agree with tables in any statistics textbook [5].

The following examples illustrate the applications of Simpson's Rule, Monte Carlo Integration and the NBS Approximation for the determination of p-values for t-tests of hypothesis

Example 8.4: One-sample t-test

Suppose a researcher is interested in determining if the ounces of pretzels filled in 16-ounce bags are actually on average 16 ounces. The following are the number of ounces on a production of randomly selected 16-ounce bags of pretzels:

15.965	15.939	15.503	15.789	15.840	15.345	15.541	15.973
15.743	15.822	16.427	16.050	15.782	16.571	15.821	15.820
16.095	16.127	15.948	15.895	15.871	16.178	15.928	15.799
15.827	15.999	15.728	15.690	15.687	15.715		

Do the values differ significantly from the advertised 16 ounces labeled on the bag? Test at the significance level of 0.05.

Assuming assumptions are verified the necessary steps for analysis follow. In this example the null hypothesis and alternative hypothesis are: The null hypothesis is: The population mean is not significantly different from the sample mean. The alternative hypothesis is that the population mean is significantly different from the sample mean. The significance level is $\alpha = 0.05$. The rejection or acceptance of the null hypothesis is based on the approximate p-value.

In EXCEL, Data Analysis Toolpak cannot conduct a 1 sample t-test. But the following example 2 comparing 2 means can be analyzed using Toolpak.

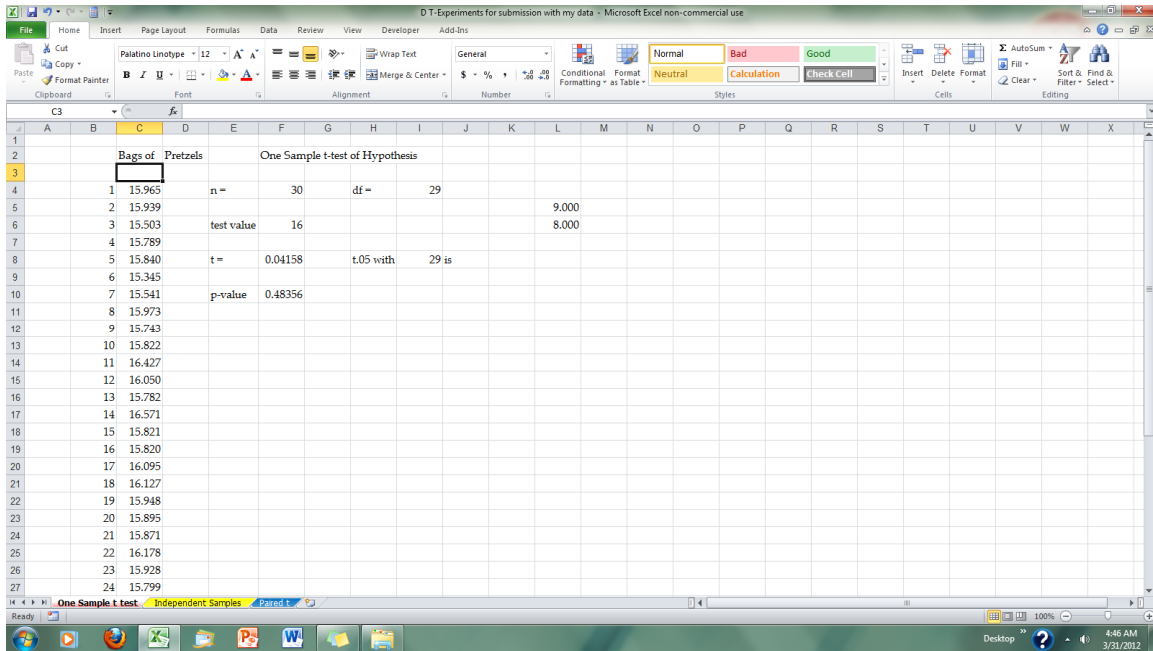


Figure 4: One sample t-test

In this example the researcher is trying to determine whether the sample mean is significantly different from the hypothesized mean of 16 ounces. In order to apply any of the techniques Simpson’s Rule, Monte Carlo Integration and the NBS Approximation the t statistic must be calculated. The t statistic is for a one-sample test of hypothesis calculated with the following formula:

$$t = \frac{\sqrt{n}(\bar{x} - \mu)}{s} \quad (8)$$

where n is the sample size, \bar{x} is the sample mean, μ is the mean of the population and s is the sample standard deviation. The calculated test statistic t is 0.04158. The approximate p-value with 29 degrees of freedom (df) for example 8.4 are found by using the following techniques:

- a) The use of the TDIST function with a t value of 0.04158 and df value of 29 and 1 tailed test provides the output displayed below
- b) Simpson’s Rule
- c) Monte Carlo Integration
- d) The NBS Approximation

The p-values from each of these methods are:

Using TDIST function p = 0.48356

Simpson's Rule: $p = 0.48356$
 Monte Carlo Integration $p = 0.483559$
 NBS Approximation $p = 0.4830$

In order to get the p-value in some of the techniques, 0.5 minus the value calculated by Simpson's Rule or Monte Carlo Integration must be performed. The techniques discussed in figure 1, figure 2 and figure 3 are applied to achieve the results of example 1. The answers are very close. The DATA Analysis Toolpak does not have capabilities for doing 1 sample t-test of hypothesis. For any p-values calculated, note that when they are less than or equal to 0.05, there are significant differences. But since the p is greater than 0.05 in all cases the hypothesis of no difference is not rejected. That is there is no significant difference between the hypothesized mean of 16 ounces and the sample mean for the 30 bags of pretzels.

Example 8.5: Two independent samples t-test

The researcher is studying whether there is a difference in the time it takes to implement a software component based on educational background of the person.

In order to apply Simpson's Rule, Monte Carlo Integration and the NBS Approximation the test statistic t must be calculated using (9). The test statistic for two independent sample t-tests of hypothesis is -2.464 with 18 df.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \tag{9}$$

where $s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$; and n_1 and n_2 are the sizes of each sample and s_1^2 and s_2^2 are the variances of each sample.

The area under the curve in Simpson's Rule and Monte Carlo integration is determined by applying these techniques from 0 to the calculated t. The calculated test statistic t based on (9) is -2.464. And p is the right tailed area found by subtracting the approximated area found from 0.5. In order to apply any of the techniques described t must be positive. That is the use of =ABS(calculated t) is needed in arriving at correct results.

This example illustrates a two independent samples t-test. Compare the mean times in minutes to complete a setup of an in home electronic email account for college educated and high-school educated populations. Are there significant differences in the time in minutes it takes to complete the task based on education level?

The null hypothesis is that there is no difference in mean times for setting up an electronic mail system based on education level. The alternative hypothesis is that there is difference between mean times based on the education level. Alpha is set at 0.05. The data are:

College Times	High School Times
27	37
41	39
28	32
24	46
37	31
33	48
30	57
31	38
41	47
20	27

The figure displays the results from using Data Analysis Toolpak. In Toolpak select “t-test: Two Sample Assuming Equal Variances” or “t-test: Two Sample Assuming Unequal Variances.”

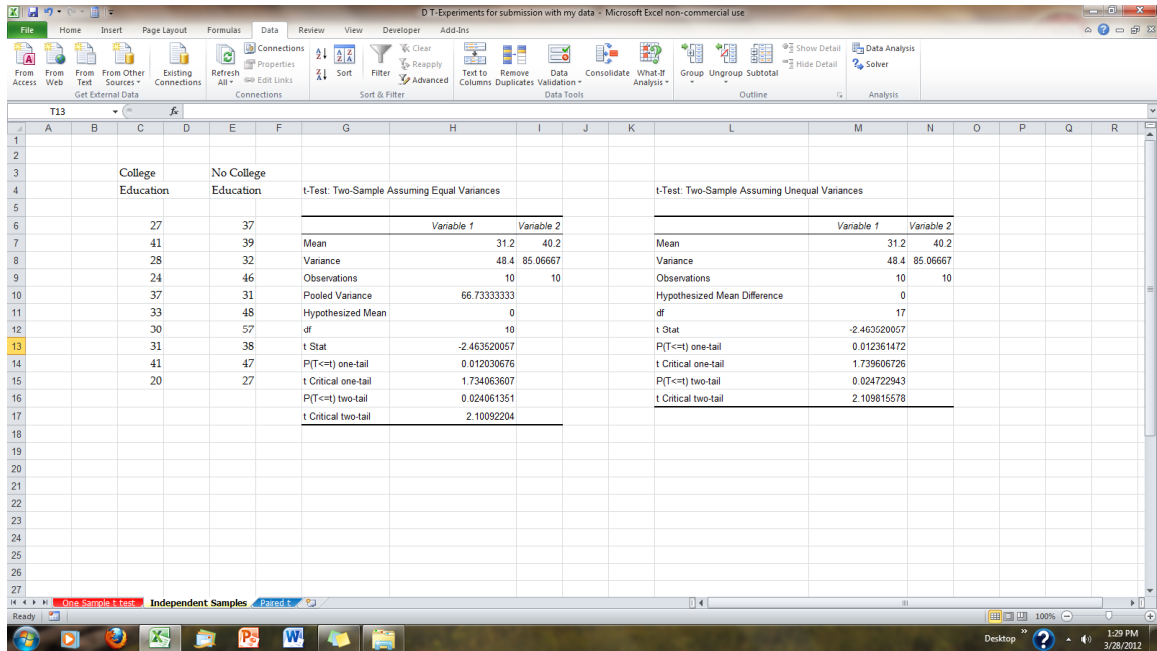


Figure 5: Two independent samples t-test

The p-values from each of the methods from figure 1, figure 2, and figure 3 with 18 df described above are:

- Data Analysis Toolpak: $p = 0.01202$ (Equal Variances)
- $p = 0.01240$ (Unequal Variances)
- Simpson’s Rule: $p = 0.01202$
- Monte Carlo Integration $p = 0.01233$
- NBS Approximation $p = 0.0114$

The results independent of approach are all less than or equal to 0.05. The conclusion is to reject the null hypothesis. There is a difference in mean times to setup an email account based on education level. If the researcher assumes equal variances the p-value is 0.01202 and if the researcher assumes unequal variances the p-value is 0.0124. The application of equal variances or unequal variances depends on the calculated variances of each sample. Notice the similarity and closeness of the approximate p-values.

Example 8.6: Paired samples t-test

This is an example of a paired or correlated samples t-test. Two professors scored 10 students on the same course material. Do they differ in their opinion about

scoring 10 students on the same exam? Is there sufficient evidence to suggest that the professors' scores are significantly different?

The null hypothesis is that there is no difference in the grades given by either professor. The alternative hypothesis is that there is a difference in the grades given by either professor. The significance level is 0.05.

The data follow:

Professor 1	Professor 2
83.1	80.1
92.3	86.2
70.1	71.8
66.7	73.9
87.5	92.3
84.2	84.7
79.0	75.7
99.2	98.2
87.1	88.3
73.2	69.1

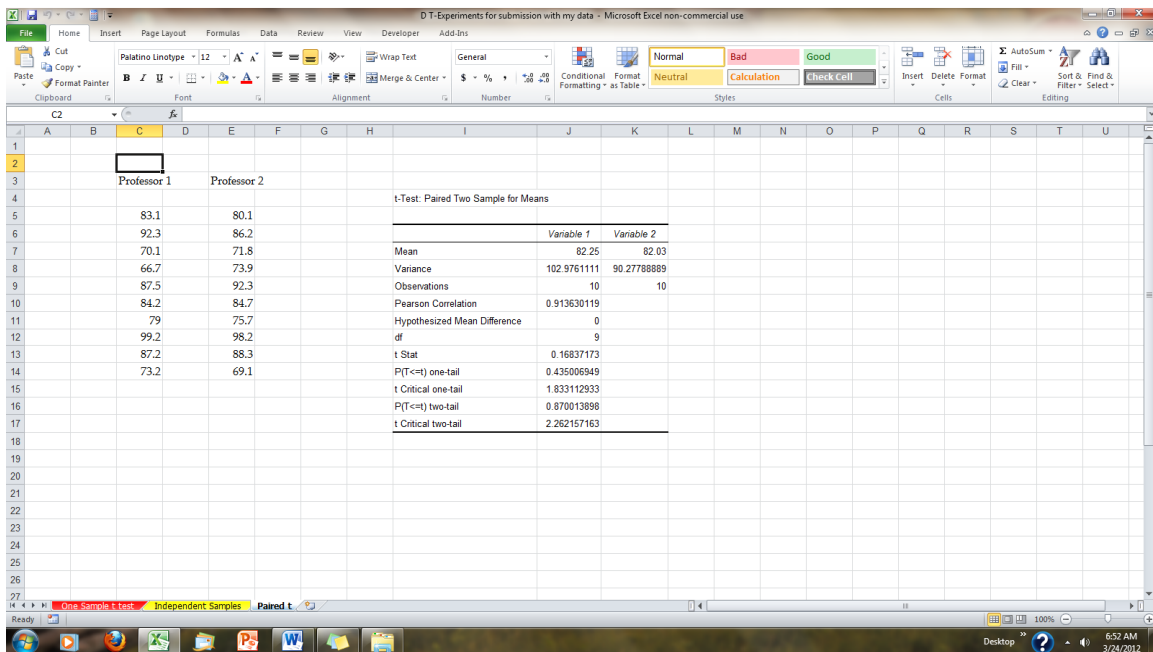


Figure 6: Paired samples t-Test

A researcher is studying whether the professors agree on their evaluation of a class of ten students.

Data Analysis Toolpak can be applied to conduct 2 independent samples and paired samples t-tests of hypothesis. Figure 6 displays the results of the paired t-test example on testing fairness in grading by professors. Note the closeness of the p-values.

The p-values from each of these methods are:

Data Analysis Toolpak:	p = 0.435
Simpson's Rule:	p = 0.435
Monte Carlo Integration	p = 0.4341
NBS Approximation	p = 0.432

Using TDIST with a calculated $t = 0.1684$ and 9 df (degrees of freedom) and 1 tailed the result is 0.435. The t test statistic is calculated by using:

$$t = \frac{\sqrt{n} (\bar{d} - 0)}{s_d} \quad (10)$$

where n is the number of paired differences, \bar{d} is the mean of the sample differences and s_d is the standard deviation of the sample differences.

The syntax “=TDIST(CALCULATED T, DF, 1)” calculates the p-value for two independent samples t-test and paired samples t-test when the t is calculated. In any case for the paired t-test of hypothesis, the p-values are greater than 0.05 and so the null hypothesis of no difference is not rejected.

Results using Toolpak, Simpson's Rule and Monte Carlo Integration are displayed underneath figure 6. Applying each technique shows similar results.

9. Conclusions

Mathematics is a field of study where multiple approaches to analysis can be applied. Methods may differ but results are always the same or very close to each other. Introducing various ways to find p-values enable the user to do more mathematics and develop an understanding of concepts impossible to be done without computers. Studying various approximations allows the learner to take part in the mathematical process. No black boxes are involved. Too frequently many researchers and students perform statistical analyses without any mathematical calculations and no understanding of the underlying process. As a

result the researchers and students are not exposed to the actual mathematical process involved in doing statistics. The techniques behind the statistical methods presented in this paper provide abundant details on what happens when calculating t values and p-values for t-tests of hypothesis.

Using the techniques described, approximations, their relevance in calculus, probability and statistics are demonstrated. In the process of applying different ways to analyze a problem the checking of one method against another is invaluable to learning.

Studying many different ways of solving or analyzing a mathematical problem also impacts professors. Professors must spend time developing and determining other ways to analyze and do mathematics. Also the professor should be able to develop an understanding of how students interact with a variety of techniques.

Approximations with a large dataset or large number of iterations usually illustrate more accurate p-values.

10. References

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