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Teaching Statistical Principles with a Roulette Simulation

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Teaching Statistical Principles with a Roulette Simulation

Abstract
This paper uses the game of roulette in a simulation setting to teach students in an introductory Stats course some basic issues in theoretical and empirical probability. Using an Excel spreadsheet with embedded VBA (Visual Basic for Applications), one can simulate the empirical return and empirical standard deviation for a range of bets in Roulette over some predetermined number of plays. In particular, the paper illustrates the difference between different playing strategies by contrasting a low payout bet (say a bet on "red") and a high payout bet (say a bet on a particular number) by considering the expected return and volatility associated with the bets. The paper includes an Excel VBA based simulation of the Roulette wheel where students can make bets and monitor the return on the bets for one play or multiple plays. In addition it includes a simulation of the casino house advantage for repeated multiple plays; that is, it allows students to see how casinos may derive a new certain return equal to the house advantage by entertaining large numbers of bets which will systematically drive the volatility of the house advantage down to zero. This simulation has been shown to be especially effective at theUniversityofCape Townfor teaching first year Statistics students the subtler points of probability, as well as encouraging discussions around the risk-return trade-off facing gamblers. The program has also been shown to be useful for teaching students the principles of theoretical and empirical probabilities as well as an understanding of volatility.

Keywords
Statistical Education, Probability, Risk, Roulette, Simulation

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1. Introduction

Spreadsheets have been used effectively to demonstrate the computational side of statistical analysis and to demonstrate results graphically (see, for example, Mills (2002) and Savage (2001) and the excellent history of the use of spreadsheets by Baker and Sugden (2003)). Spreadsheets also offer an excellent platform for empirical simulation, a notable case being their use to demonstrate the concept of sampling distributions with reference to statistical inference, using statistical bootstrapping procedures, see Rochowicz (2011). This paper demonstrates a parallel application of statistical simulation in the use of spreadsheets to demonstrate fundamental notions of probability, chance and the Central Limit theorem. An excellent vehicle for this which captures the imagination of students in introductory courses in Statistics is the game of Roulette. By combining the power of spreadsheet computation and graphical exposition with the flexibility of the inbuilt VBA facility we show here how we can effectively teach some basic fundamentals of probability in an environment which is stimulating for students and provides an excellent platform to further extend their understanding of statistical principles.

The use of Roulette as a tool for teaching is not new but has been used almost exclusively to consider the effect of simple play-to-win strategies; see Croucher (2005) and the often discussed "doubling-up" betting strategy; see Seal and Przasnyski (2005) and Marshall (2007). In this piece we focus on the house advantage of the casino to demonstrate that whatever the player's bet, the player's expected return is the same and that the longer the player plays, the closer their actual return will converge on this expected value. This leads to an interesting discussion and evaluation of different playing strategies. In fact is it at all possible to play a casino games such as Roulette in a rational attempt to make a positive return on one's capital?

1.1 Setting up the game of (European/Australian/South African) Roulette

The first teaching step is simply to set up a game that students can play. This is programmed in VBA as a spinning wheel on which one can place various bets. No split bets are included and the program accommodates red:black, even:odd, $1^{st}12$; $2^{nd}12$: ; $3^{rd}12$ and pure number bets as shown in the layout of Figure 1.

Students are able to place bets, spin the wheel by clicking on the "play" button and observe the dynamic outcomes of their bets. The spreadsheet which is programmed in VBA has a couple of interesting features:

1.2 Some interesting programming features of the Roulette Simulation

We first set up the Roulette wheel by defining the physical cells which will constitute the close-to-circular wheel. The spin-of-the-wheel is programmed as a conventional FOR NEXT programming loop which loops through the cells of the wheel to its final destination (the winning number) by changing the format of each cell it loops through to give the impression that the" ball" is moving. The program works as follows.

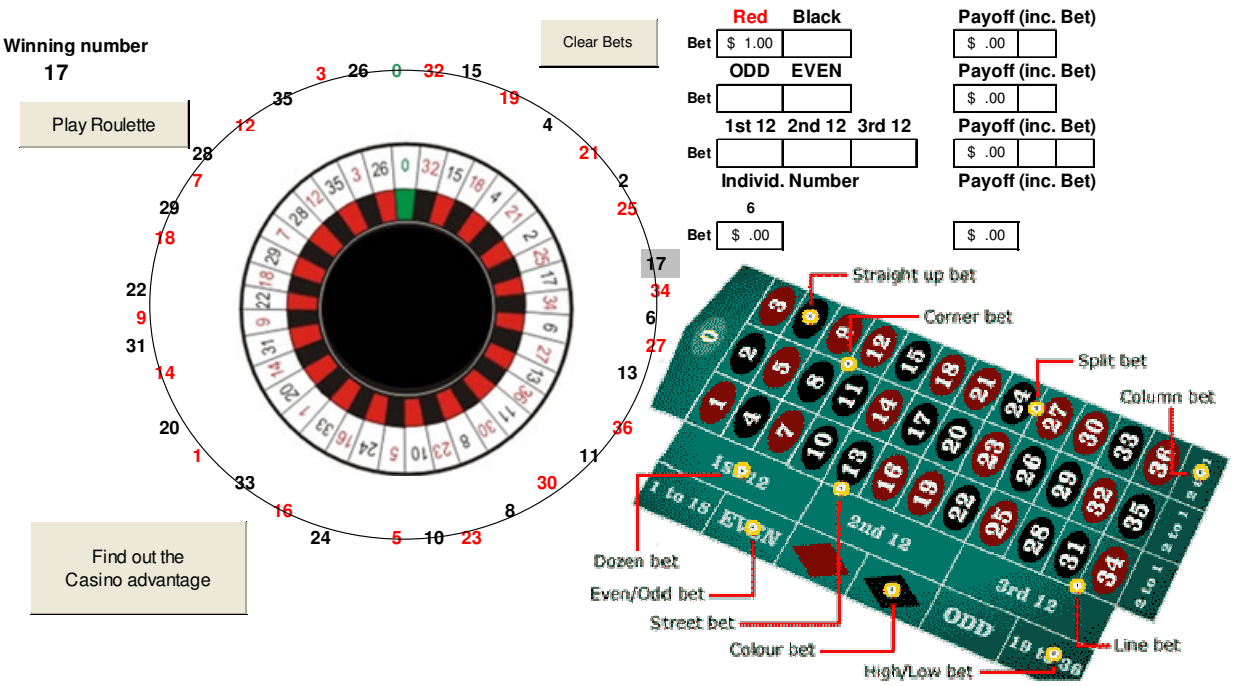

Figure 1. The Basic Spreadsheet layout of the Roulette Simulation

First the cell references of each of the Roulette wheel numbers are given range names in the sequence they occur on the Roulette wheel; the range names are then stacked into a vector ***rnl*** and the associated numerical values are stacked into the vector ***rn***. Thus, for example, the cell with reference "k3" contains the Roulette number "0" , is given the range name "r_0" and assigned the numerical value 0. The VBA code for this case is as follows:

```
Range("k3").Name = "r_0"
rnl(1) = "r_0"
rn(1) = 0
```

This code is repeated for all the numbers in the vector ***rn*** (corresponding to the sequence of numbers on the wheel, namely 0, 32, 15, …, 26.
Secondly, the number of spins is randomly determined between certain natural limits. *nspins* is the total number of spins (including the last incomplete spin) with a minimum of 5 and maximum of 15. *finalnums* is the (randomly determined) point in the sequence of numbers at which the final (incomplete) spin terminates.

```
nspins = 5 + Round(10 * Rnd, 0)
finalnums = Round(37 * Rnd, 0)
```

Finally, the main loops are performed, looping through complete spins and then the final (partial) spin of the wheel. For each spin the program then loops through the sequence of numbers (stored in the vector **rn**) as they occur on the Roulette wheel

```
For i = 1 To nspins
If (i < nspins) Then
numnums = 37
Else
numnums = finalnums
End If

For j = 1 To numnums
Number = rn(j)
```

At each loop we colour the number as a red or black or a green according to Roulette convention.

```
Select Case Number
Case 32, 18, 21, 25, 34, 27, 36, 30, 23, 5, 16, 1, 14, 9, 19, 7, 12, 3
colour = 3
Case 15, 4, 2, 17, 6, 13, 11, 8, 10, 24, 33, 20, 31, 22, 29, 28, 35, 26
colour = 1
Case 0
colour = 50
End Select
```

We then colour the background of the cell and then switch the colour back to the original background. In the actual program this is done several times to create the impression of a flashing ball and then by looping across the cells in the *j* loop to create the impression of a moving ball.

```
Range(rnl(j)).Font.ColorIndex = colour
Range(rnl(j)).Interior.ColorIndex = 16
Range(rnl(j)).Interior.ColorIndex = xlNone
Range(rnl(j)).Font.Bold = True

Next j
Next i
```

The loop continues for *nspins*. For each complete spin the program moves round the complete wheel and then in the final spin stops at the randomly selected number. It is important to mention here that the computer generated game of Roulette described here is not a true representation of the physical game of Roulette. This is because the computer game uses pseudo random numbers derived from some random number seed rather than physically generated random numbers. However the representation of reality is assumed to be close enough to fulfil all our intended demonstrative purposes.

When the students have become familiar with how Roulette works and are clear on the bets and possible outcomes of the bets according to casino rules, we are in a position to discuss betting strategy for the player and how it is that the casino can expect to win at Roulette.

2. The House Advantage of the Game of (European) Roulette

Viewed objectively and from a purely statistical perspective, it might not seem sensible for someone to spend a lot of time playing Roulette in a casino. We illustrate this with reference to so-called European roulette, the game with one "zero". The game in this form is played in casinos in the UK, Europe and Australia. For the purposes of this exposition we have put the statistical results required in an Appendix; this lends rigour to the discussion if required but since this paper is targeted at first year students the focus is on learning principles through the effective use of the spreadsheets. The results in the Appendix show, for example, that in the game of European Roulette (also played in Australia and South Africa), whatever the bet the house (casino) can expect a constant expected return of 2.7% per spin of the wheel. Moreover, the standard deviation or so-called volatility, of this return decreases steadily, the greater the number of plays. So the casino can expect that over multiple plays their actual return will steadily converge to this number of 2.7%. How does this affect the gambler? Well it means that the gambler can play any array of bets and often be up in the short-term but it becomes increasingly unlikely that he or she is ahead, the longer the period of play. Thus it appears that this really is a case of "when up, head for the door"! The problem with this strategy of course is that it is not clear if the gambler should ever return (if they are attempting to maximise return!). If a gambler wins on the first play, the strategy which maximises return would be to leave the casino for ever. However that's not much fun if the gambler enjoys gambling. (If the gambler doesn't find gambling fun and is there to attempt to win money, the statistics of the exercise are against him!)

If the gambler just likes to participate and wants to be certain to constrain his losses he could place a bet on each number (including zero). This way he always wins exactly $36 but it has cost him $37 so he has a certain loss of $1 per play. The volatility of this strategy is zero and the casino is happy to collect the cash each time. Of course this "riskless", but sure-fire-way-of-making-a-loss strategy isn't followed very often. A more exciting option might be to place a bet on a number. This way it is quite possible the gambler could have a big win in the short term. In fact such a strategy wouldn't be very popular for the casino because of the high volatility of the return. A worst case scenario for a casino would be if one and only one player at a Roulette table bet the maximum bet on a single number. Then it could take many plays for the decreasing volatility to push the return back to 2.7% in favour of the casino. In such a case either the gambler or the casino would win big on a particular night. The casino, of course, is much more interested in just winning 2.7% of the handle (casino jargon for the total money bet on Roulette over some period). In fact to limit the problems associated with patrons winning big on low-odds bets the casino always places maximum bet limits on the Roulette tables.

The maximum bet limit scuppers one apparently simple winning strategy, that of simply doubling up when one loses. So, for example, the minimum bet on red at most Australian casinos is $1 and the maximum a bet of $200. A doubling up strategy from the minimum bet of $1 would require more than $200 if there was a run of more than 7 losses, and a run of exactly 8 losses has a probability of (19/37)^8, about 0.5%, so a 1 in 200 probability event. It would then take more than 200 winning plays to recover from the loss; thus although the doubling-up strategy has received lots of attention, one sees that it is not particularly interesting as it quickly leads to a dead-end.

2.1 The *Casino House Advantage* Tutorial

It can be seen that the expected gain or return for the player in both of the two different betting strategies mentioned above, is -2.703% of the value of the bets. In fact this result is quite easily generalised as follows: the return for any conceivable bet on Roulette, is -2.703%, an amount which is known as the casino "house advantage". It can also be shown that the standard deviation of this negative return falls according to the square root of the number of plays; with lower risk bets on Roulette (like *red* or *odd*) the standard deviation is much less than higher risk bets like *number* bets.

A tutorial which has been devised for first year students who have just been introduced to the notion of probability, demonstrates these concepts empirically. The tutorial is structured around the spreadsheet simulation of Roulette, a snapshot of which was shown in Figure 1. We have run this tutorial for four different semester courses, involving a total of some 3000 students in total. In addition, a version of this tutorial has been used in workshops with regular Roulette gamblers to explore their understanding of fundamental probability concepts. We relate our experience with this tutorial below. Participating students are shepherded through the following phases:

(i) In the initial phase of the tutorial, the game of Roulette is explained to the participants and they are encouraged to work in pairs and given free range to use the simulated Roulette game given on the first worksheet (tab *Play_Roulette*) to examine the return they receive on (repeated) contrasting bets of their choice. (The best way to demonstrate the fact that their expected return will remain constant but the variability of that return will differ greatly is to suggest they try contrasting bets; for example, a pair of students might elect to play and compare the returns of a *red* or *odd* bet versus a *number* bet).

(ii) The tutorial leader then gathers the class together for a discussion on the views on different Roulette strategies after some time has been spent on (i) above. Those students with firm convictions about the efficacy of *particular* strategies typically remain unconvinced after laboriously playing the simulated Roulette wheel a number of times. It is then agreed that it would be preferable to observe a large number of plays before reaching any conclusions about the efficacy of these strategies. The students then proceed to the second worksheet (tab *MultiplePlays*) which records and tabulates the results of large numbers of simulated plays. At this stage the cogent point about constant expected value across different bets generally becomes clearer. If we continue to use the contrasting bets suggested above, although the observed %profit/loss, known as the house advantage, varies across n plays, it is much more stable for the low risk bet (*red* or *odd*) than the high risk (*number*) bet. A typical spreadsheet output is shown in Fig 2. The greater the student sets the number of plays, the more their loss will converges on 2.7% . This convergence is much faster in the case of the low risk bet (*red* or *odd*).

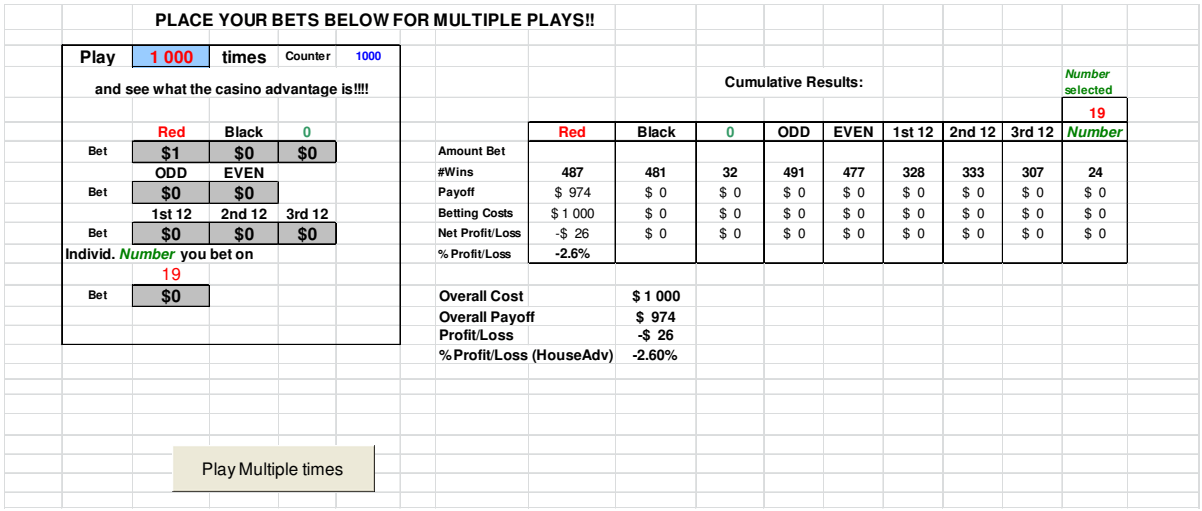

Figure 2: Spreadsheet showing % Profit/Loss for player who makes repeated bets of the same type

(iii) Brighter students are encouraged to explore the third worksheet (tab *CasinoAdvantage*) which empirically builds the distribution of the casino return or casino house advantage. This is explored in greater detail in 2.2 below. The students then observe the empirical results for their chosen bets and this is followed up by demonstrating that these findings in fact reflect the theoretical concepts of expected value and variance of a random variable.

Ensuing discussion on the findings of the tutorial has students attempting to find a strategy which might defy expectations (!), and also trying to find the least variable betting strategy! Our experience with this tutorial is that participants frequently arrive at the realisation that the casino's profitability hinges on the fact that it's house advantage is driven towards certainty as the total number of monetary bets made by all patrons, the so-called casino "handle", increases. Interesting points of discussion that have arisen include the realisation that the casino would always prefer large spreads of different bets to be placed in Roulette, as they would be at greatest risk when large bets are placed on high payout bets, for example a single player at a table playing a high bet on one number. This then leads the casinos to impose maximum bets (limits on the betting). More general statistical concepts are also explored in the discussion and through the use of an online quiz at the end of the tutorial, which asks students questions around the following themes:

a) Whatever the bet or combination of bets, the house advantage will, on average, be -2.7%. (We are here assuming no *en prison* or *la partage* rules in this representation of Roulette. These rules operate at some casinos and allow betters on *red/black* or *odd/even* or *high/low* to recover their stakes after a spin outcome of *zero*; such rules lower the house advantage).

b) The larger the number of plays the closer (on average) will the house advantage be to -2.7%

c) The chances of wins, long run expected returns and their variances can be calculated using simple probability rules.

d) The (empirical) outcomes (of the long run average returns) mirror the expected theoretical results provided sufficient plays (number of bets) are "observed".

Tutors are primed to prompt the students to think deeply about the role of the following:

- the amount bet,
- the number of plays (reflecting on the Law of Large numbers),
- the probability of a win (betting on say *red vs* a *number*),
- the number of simulations,

and the resulting effect on the overall profit/loss and the variability of this profit/loss both from an absolute monetary perspective and a percentage perspective (% profit/loss relative to the outlay). We then further extend these ideas (as outlined in the section below) by taking a look at the casino house advantage and how our simulation can show that this converges on a fixed value with diminishing returns.

2.2 The Volatility (%Standard Deviation) of the House Advantage

As an extension of the above tutorial, it is useful to consider that if we repeat a number of fixed plays, we can estimate the (percentage) standard deviation or volatility of the house advantage. Thus, say we need to estimate the volatility of the house advantage when *n* people make a *red* bet at a casino (assume they are all on different tables!). We perform this experiment by repeating the above *n* person bet a number of times, say *numrep*. We allow *numrep* to be large enough (say 500 times) to obtain good volatility estimates.

We can now demonstrate two subtle phenomena.

As n increases, the volatility of the house advantage decreases. The Appendix shows us that the formula for the volatility for the above bet when $n = 100$ is

$$Std.dev\ (\%Gain\ over\ 100\ plays) = 100\sqrt{2^2\ \frac{18}{37}\frac{19}{37}/100} \quad \% = 9.9964\%$$

We can then demonstrate that if patrons change their strategy to a risky bet (say, a particular *number*) than the volatility increases markedly to

$$Std.dev\ (\%Gain\ over 100\ plays) = 100\sqrt{36^2\ \frac{1}{37}\frac{36}{37}/100} \quad \% = 58.378\% .$$

In both cases, of course, the expected house advantage is the 'magic figure' of -2.7%, but in the second case above we are likely to be quite far away from it.

The equally interesting question is how the number of replications *numrep* affects these figures. We can demonstrate to students that as *numrep* increases, the estimates for house advantage and volatility get closer and closer to their expected values.

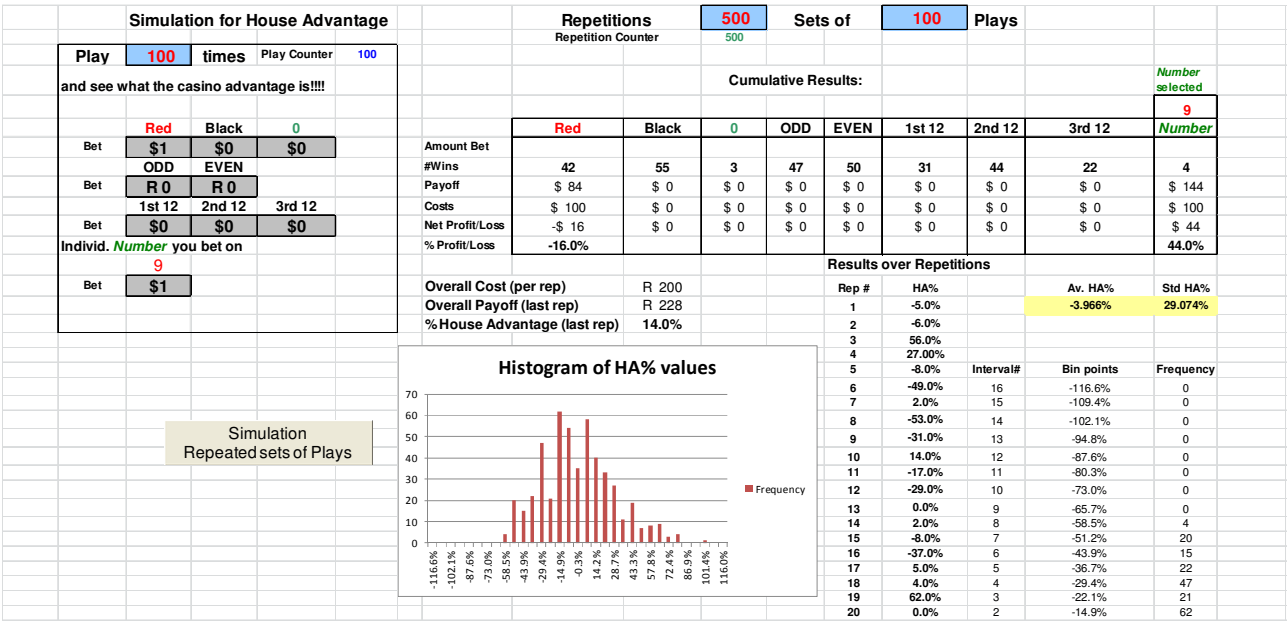

Figure 3. The Volatility of the House Advantage

2.3 Why Do People Play Roulette?

Our tutorial raises interesting issues around risk and return in gambling strategy. We have demonstrated that whatever betting strategy is employed, incorporating single, multiple or split bets, the returns are fixed. The volatility of the bets, however, can be seen to vary widely. One might argue that, as with commercial activities, the higher variability bets should attract higher returns. But the compensating factor for gamblers, where of course gamblers expect and accept negative returns, is the thrill of a large win. This volatility reflects the 'excitement factor', as the higher payout bets are associated, as expected, with a greater volatility of return. Does this 'excitement factor' make more volatile bets and their associated higher payoffs more attractive? Some gamblers, it would appear, seek out the 'excitement factor' and do not expect to be compensated for choosing it. Some gamblers in contrast might simply seek 'time at the wheel'. They might be quite happy to place Roulette bets which maximise their "duration" at the table. This would require a repeated low risk bet like *red* or *odd*. So it depends upon which type of "gambling personality" is most prevalent, rather than any commercial notion of being compensated for risk.

These points make for interesting discussions with students and, in our experience, elevate the task of mastering elementary statistics and probability above the 'rote application of recipes' to a more nuanced one that incorporates attitude to risk and return and the conundrums of our human condition!

3. Conclusion

We have described the use of a simulated game of Roulette as a tutorial exercise for introductory Statistics classes. As described in our "tutorial" section, our experience with this tool has been that it opens up a range of insights into standard statistical concepts such as risk and return, the distinction between empirical and expected (theoretical) results as well as random variation, in a fun real-world context. The use of the spreadsheet in the guise of MSExcel as a vehicle for repeated sampling allows students to 'observe' the effect of

random variation and to monitor long run trends and averages. The ability to compare these with the calculated theoretical expectations gives new credibility to these (often poorly conceptualised) statistical manipulations. The tutorial has a further spin off in that it captures the imagination of some students by creating an awareness of the power of Excel to model real life situations. The tutors and teachers of this course found that there were a small but nevertheless encouraging group of students who found the synergy between Excel skills and a growing understanding of the discipline of Statistics to be particularly stimulating and powerful. For most students the core statistical concepts remain challenging and difficult to master but the visual nature of Excel based tutorials and simulations gives teachers a new avenue in attempting to forge breakthroughs in understanding. These impressions have been formed on the basis of surveys conducted with students who have completed the relevant statistics courses.

The focus of this paper has been on the more qualitative experience of the authors with using this teaching tool. The introductory course referred to has also subjected the students to more traditional and formal forms of testing of the concepts taught in this tutorial. An online test in the Excel environment probes these students' understanding of the statistical concepts of statistical significance, expectation and distributions (of both individual measurements and average values) using exercises which involve repeated sampling from known distributions. Around 1100 students were exposed to this test and around 70% met the requirements for passing. Subsequent interviews with random sub sets of these students confirmed that these tests both consolidating the learning which occurs in the computer lab based tutorial sessions and improved understanding of subtle statistical concepts with which students traditionally struggle.

The authors are of the opinion that the approach outlined in this paper of using Excel as a 'statistical laboratory' in which fundamental statistical concepts can be visually and experientially demonstrated is beneficial. It is believed that this is especially true for teaching in contexts where many students have poor educational backgrounds, particularly in maths and science, and, in which many students are learning in a language that is not their mother tongue.

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Appendix

A Statistical Overview of (European) Roulette play

We consider a simple (one chip on one position) bet in the game of European Roulette, the game with 37 numbers including one zero, as distinct from the American variant which has 38 numbers including two zeros. We let P(win) = p and P(loss) = q with $p + q = 1$. In general if we let X be the number of successes (wins) over n independent plays, then X follows a binomial distribution ($X \sim B(n, p)$). Assume a bet of b; and let the gross payout per unit of bet be w, thus a "win" yields a gain of $b(w-1)$ and a "loss" yields a "gain" of $-b$. Therefore,

$$E(Gain\ per\ play) = b(w-1)p + (-b)*(1-p) = -b(1-pw)$$

In percentage terms, the expected percentage loss per play is:

$$E(\%Gain\ per\ play) = -100(1-pw)\%$$

To compute the variance of this outcome over some fixed number of plays, say n plays, we invoke the fact that since $X \sim B(n, p)$, $Var(X) = npq$. Thus for n trials where we bet b units per play, the variance of the betting outcome is:

$$Var(Gain\ over\ n\ plays) = Var\ (\#wins * b * (w-1) + (n - \#wins)(-b)) = w^2\ b^2\ npq\ ,$$

$$Std.dev\ (Gain\ over\ n\ plays) = wb\sqrt{npq}\ .$$ In percentage terms we have:

$$Var(\%Gain\ over\ n\ plays) = 100\frac{w^2 pq}{n}\ \%\ ;\ Stddev(\%Gain\ over\ n\ plays) = 100w\sqrt{\frac{pq}{n}}\ \ \%$$

For illustrative purposes, let us consider two particular cases

A.1 Case1: A Bet of 1 unit on *red* or *black* or *even* or *odd*.

In this case, $p = 18/37$, $q = 19/37$ and $w = 2$

$$E(Gain\ per\ play) = -\frac{1}{37} = -0.02703$$

$$E(\%Gain\ per\ play) = -2.703\%$$

For a single play ($n = 1$),

$$Var\ (Gain\ per\ play) = 2^2\ \frac{18}{37}\frac{19}{37} = 0.99927\ (99.927\%)$$

$$Std.dev.\ (Gain\ per\ play) = \sqrt{2^2\ \frac{18}{37}\frac{19}{37}} = 0.99964\ \ (99.964\%)$$

For 100 plays,

$$E(\%Gain\ over\ 100\ plays) = -2.703\%$$

$$Std.dev\ (\%Gain\ over\ 100\ \ plays) = 100\sqrt{2^2\ \frac{18}{37}\frac{19}{37}/100}\ \ \ \% = 9.9964\%$$

For 10 000 plays,

$$E(\%Gain\ over\ 10\,000\ plays) = -2.703\%$$

$$Std.dev\ (\%Gain\ over\ 10\,000\ plays) = 100\sqrt{2^2\ \frac{18}{37}\frac{19}{37}/10\,000}\ \ \ \% = 0.99964\%$$

A.2 Case2: A Bet of 1 unit on any one number of the 37 Roulette numbers.

In this case, $p = 1/37$, $q = 36/37$ and $w = 36$.

$$E(Gain\ per\ play) = -\frac{1}{37} = -0.02703$$

$$E(\%Gain\ per\ play) = -2.703\%$$

For a single play ($n = 1$),

$$Var\ (Gain\ per\ play) = 36^2\ \frac{1}{37}\ \frac{36}{37} = 34.080 \quad (3408.0\%)\ .$$

$$Std.dev.\ (Gain\ per\ play) = \sqrt{36^2\ \frac{1}{37}\ \frac{36}{37}} = 5.8378 \quad (583.78\%)$$

For 100 plays,

$$E(\%Gain\ over\ 100\ plays) = -2.703\%$$

$$Std.dev\ (\%Gain\ over 100\ \ plays) = 100\sqrt{36^2\ \frac{1}{37}\frac{36}{37}/100} \quad \% = 58.378\%\ .$$

For 10 000 plays,

$$E(\%Gain\ over\ 10\,000\ plays) = -2.703\%$$

$$Std.dev\ (\%Gain\ over\ 10\,000\ \ plays) = 100\sqrt{36^2\ \frac{1}{37}\frac{36}{37}/10\,000} \quad \% = 5.8378\%\ .$$