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Mass, Measurement, Materials, and More Mathematical Modeling: The Nuts and Bolts of Let's Make an Error

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Mass, Measurement, Materials, and More Mathematical Modeling: The Nuts and Bolts of Let's Make an Error

Abstract

How can you get students into analyzing and understanding errors? A second semester general chemistry experiment is presented that introduces students to error analysis via simple mass determinations using nuts, bolts, and washers. Students work in groups of four, each group having a triple beam pan balance. The three-hour laboratory consists of four parts: calibrating the balance using standard masses; making errors to discover the behavior of constant and proportional systematic errors via data analysis in Excel; analyzing the effects of an errant nut in a sequential addition of nuts to a bolt, graphical analysis, and sharing data in Google Docs (now Google Drive) for a collaborative online discussion by groups using the chat function, and finally a verification of the errant nut problem by simulation of results in an interactive animated spreadsheet. Each group has a different set of results and must figure out what is possibly wrong with their results using online chat. Students must use some algebraic detective work to find the outlier (the errant nut), and its influence on the regression line. Students are exposed to linear regression in first semester general chemistry.

Keywords

discovery learning, simulations, error analysis

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Cover Page Footnote

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1. Introduction

Measurement and its associated error are fundamental to scientific knowledge and progress. A simple experiment is presented for examining systematic errors by massing nuts, bolts, and washers that builds off an earlier laboratory experiment [1] and a simple numerical simulation to introduce error types [2]. The current experiment is performed in second semester general chemistry laboratory for non-majors; however, it is suitable for any course that discusses measurement. It is expected that students are capable of graphing and performing linear regression using a spreadsheet. With students working in groups of four, the experiment consists of four parts that can be carried out in a three-hour laboratory period. Dealing with measurement error is an important process for students to be exposed to and develop a mindset and skills to handle it.

In our earlier paper [1] we introduced students to mathematical modeling by massing a bolt with from one to five nuts on it, deriving a model, and extrapolating to determine the bolt mass and in this activity we introduce students to tracing systematic errors by discovery and further error analysis, all-the-while using linear modeling and goodness of fit. The four parts of this laboratory include the calibration of the triple beam balance with standard masses; the discovery of the behavior of constant and proportional systematic error using nuts, a bolt, and washers; the massing of an ordered sequence of nuts to a bolt where there is an errant nut in the sequence (we used an aluminum nut among stainless steels nuts); and finally having the six laboratory groups enter data from the errant nut portion into a Google Docs (now Google Drive) form. As part of the experimental design a chat group allows student interactions while examining the projected Google Docs spreadsheet (or on their lab station computer). This last part takes some algebraic detective work by students, and some guided questions posed by the instructor for the chat to be effective. The first thing for students to accomplish is to find the group with no error from an errant nut, and yes there is one. Then they can explore the data and regression results. Their goal is to try to locate the position of the errant nut in the sequence (each group has it in a different position). This is done by looking at the mass differences as the nuts are added in sequence.

After the chat, students go to an interactive Excel spreadsheet to examine the behavior of constant and proportional systematic errors, explore the errant nut position and its influence on the linear regression results, goodness of fit, and experimental error. This allows students to verify their experimental data. Use of

the spreadsheet explores the addition of random errors and the concept of massing by difference. All of this is covered in the post laboratory questions.

In this article, we introduce the experiment, an interactive Excel spreadsheet or simulation, our chat results, and student feedback and assessment. Our goal is to get students exploring systematic errors and to help them develop a mindset to always consider error in measurement [2]. We also briefly discuss the use of Google Docs as an easy collaborative tool for instructors to set up and for students to use. All documents needed by students including Excel help are available on the course webpage [3].

2. Experimental Method

Each student group requires a triple beam balance (readable to 0.01g), a set of standard masses (we used a cheap set from an educational supply company), a bolt, five stainless steel nuts, and at least five washers. Also included in the activity is another bolt with the five numbered (labeled) nuts (four stainless steel nuts and one aluminum nut) in a Ziploc bag with a bag number to track it. This allowed each group to work fairly efficiently at their lab station. Each group needs access to a computer with Excel and Internet access. Upon arriving at lab, students go right to work following the laboratory procedure. They do twenty-five mass determinations in this activity which is why a balance is provided for each group. Each group of students graphs five sets of data and performs linear regression on three different graphs. First students mass a series of standard masses to calibrate the balance. This regression result is usually a straight line with a slope very close to one and a very small, if any, y-intercept, showing that the balance is well calibrated and ready for use. They do this on a plot with a $y = x$ line which would demonstrate perfect agreement (slope = 1 and y-intercept = 0).

Then they perform our “Nuts and Bolts of Extrapolation” in brief [1] to get what we refer to as an “errorless” set of data. If done correctly the results are excellent as students add one to five nuts to the bolt. Next students introduce two different types of errors:

- Constant - Mass with washers on before nuts – Students add a selected number of washers to the bolt before adding any nuts (only adding washers at the beginning) - This induces the constant systematic error and changes only the y-intercept of the line. See left image in Figure 1.

- Proportional - Mass with a specified number of washers per nut – Students add a selected number of washers onto the bolt ahead of each nut (multiple additions of washers) - This induces the proportional systematic error and changes only the slope of the line. See right image in Figure 1.

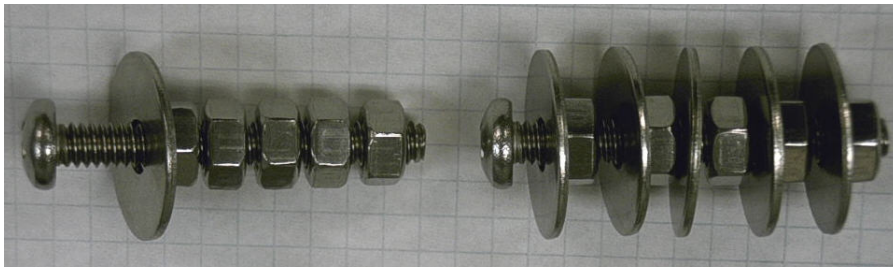


Figure 1. Systematic Errors (left – constant; right – proportional)

Students discover the effect of the above systematic errors and their behavior from graphing and analyzing the data and the comparison to the errorless set of data. Students also examine the differences (absolute error) and percentage error (relative error) behavior.

Next students test their observational and data analysis skills by introducing a bolt with five nuts added in a stated sequence. Five groups will get an errant aluminum nut mixed in with stainless steel nuts (Figure 2). The position of the errant nut is different for each group. One group gets all five stainless steels nuts and is the “errorless group.”



Figure 2. Errant Nut in Sequence

Can you spot the aluminum nut? No student group appeared to spot the errant nut by visual observation. A full set of student data for all of the experimental sections (graphed by the authors) is available in [4].

3. Google Docs Data and Chat

After the students have massed the bolt with the errant nut, they are asked to enter their data and some calculated results into a Google Docs form (the link is provided on the course webpage) to pool the class data as shown on the Google Docs spreadsheet (Figure 3). This spreadsheet illustrates results for three sections of lab (the yellow highlight is the errorless group in each section). The spreadsheet is projected for whole class viewing, and students can access it on their lab station computers. Setting up the data collection form and collecting the data is easy and straight-forward in Google Docs and the data can be exported to Excel [5]. Students now deduce what is wrong, if anything, with their data. They only see their lab section's data (the hide row function is used to remove other sections' from view).

A	B	C	D	E	F	G	H	I	J	K	L
Timestamp	Instructor Code	Bag Number	Mass With 1 Nut	Mass With 2 Nuts	Mass With 3 Nuts	Mass With 4 Nuts	Mass With 5 Nuts	Slope	Y-Intercept	R-Squared	What was the mass of the bolt from the errorless data?
2/13/2012 10:51:07	ss1	4	14.70	15.76	18.89	21.09	24.99	2.672	11.232	0.9783	
2/13/2012 11:02:48	ss1	3	13.90	17.00	20.00	23.10	26.00	3.03	10.91	0.999	errorless data added to help chat
2/13/2012 11:05:30	ss1	1	14.60	17.80	18.80	22.00	25.10	2.52	12.1	0.9789	
2/13/2012 11:08:32	ss1	5	12.91	15.92	19.20	22.30	25.62	3.18	9.65	0.9998	
2/13/2012 11:19:35	ss1	6	15.00	17.90	21.20	24.40	27.60	3.17	11.71	0.9996	
2/13/2012 11:35:16	ss1	2	15.00	18.05	21.65	24.00	25.20	2.635	12.875	0.9708	
2/22/2012 11:06:41	BS	2	14.90	18.00	21.10	24.20	25.20	2.68	12.64	0.976	11.9
2/22/2012 11:12:19	BS	5	12.99	16.00	19.05	22.15	25.15	3.055	9.895	1	11.725
2/22/2012 11:18:30	B.S.	6	14.94	17.93	20.85	22.01	25.12	2.444	12.838	0.9837	11.866
2/22/2012 11:30:54	BS	1	14.98	18.04	19.01	22.21	25.29	2.479	12.469	0.9779	11.8
2/22/2012 11:45:28	BS	3	14.08	18.09	21.00	24.00	27.02	3.179	11.3	0.996	11.07
2/22/2012 11:49:38	BS	4	10.49	10.60	10.90	21.21	21.52	2.967	5.645	0.7784	10.19
2/22/2012 14:33:32	SS2	6	15.00	17.90	20.90	22.00	25.00	2.41	12.93	0.9829	11.87
2/22/2012 14:52:32	SS2	2	14.79	17.90	21.09	24.09	25.25	2.711	12.491	0.9788	11.83
2/22/2012 15:03:21	ss2	5	12.84	15.98	19.21	21.98	25.13	3.058	9.854	0.999	11.89
2/22/2012 15:08:55	ss2	1	14.80	18.00	19.10	22.20	25.30	2.52	12.32	0.9814	11.934
2/22/2012 15:24:41	SS2	4	14.65	17.60	20.90	21.95	24.95	2.495	12.525	0.9804	11.31
2/22/2012 15:30:42	SS2	3	14.69	17.76	20.85	23.95	26.99	3.079	11.611	0.99	11.778

Figure 3. Errant Nut Data in Google Docs Spreadsheet (Key: yellow highlight – “errorless” data; red strike-out - incorrect entry; green highlight – needed for chat)

Collaboration between various organizations is a critical activity for validation of research and experimental results, and associated conclusions. Students' use of the chat feature in this activity for a collaborative discussion provides a real world scenario that they may encounter in almost any career field. To help develop these online collaboration skills [6, 7, 8]; data discussion using the chat feature in Google Docs is included in this activity.

The chat feature appears (upper right of spreadsheet) when two or more people open the spreadsheet. We supplied some questions in the activity to structure the chat; however, we have decided that the instructor must guide the chat by posing questions to catalyze discussion. The first lab section's chat was of little value due to a lack of focus in the chat discussion, and one group that figured things out was annoyed at the rest of the lab for not participating. After that the “What was the

mass of the bolt for the errorless data?" column was added to try to get students to recognize the errorless group. The second lab section had an extensive chat with some prompting that was fairly unproductive (hence more guidance needed). The third lab section was prompted with questions with the instructor acting as a senior project officer facilitating the discussion, and maintained focus by asking questions and for participants to support their statements (This was the authors experimenting to figure out the best method to use). Below is a small excerpt of the beginning of the chat from the third section (arrow is author comment). To save the chat, instructors must block and copy before closing the spreadsheet (Google Docs does not archive) and then paste in to Word.

Instructor: Is any group got a correct bolt mass? If so, based on what?

Instructor: What is the mass of a typical bolt in this activity?

Group 1: We feel the best way to find the correct bolt mass would be to take the average of all of the results and determine which group's value is the closest to the average

Group 2: You cannot say if any group has a 'correct' bolt mass because the data in columns D-K is not related to the data in column L

Instructor: What about column L?

Group 6: no group had a correct bolt mass since there is no "standard" bolt mass used.

Group 2: Hmm... Since the mass of the bolt from the errorless data is determined by the y-intercept, we can compare the y-intercept to the mass of the bolt: columns J and L. → student discovery of correct bolt mass

Group 2: Going by the comparison of y-intercept and mass of the bolt, group 3 has the correct bolt mass

Group 5: The only way you can determine the mass of the bolt is to the range from the set of bolt values. However, it could only give you an estimate of what that value may be because each bolt weights differently given from the errorless data.

Group 6: the typical mass of the bolt is approximately 11.77

It was our expectation that students would identify the errors or lack thereof for each group. Students tended to discover the errant nut by examining the difference on each successive nut addition.

4. Verification using an Interactive Excel Simulation

The last part of the activity which could be done during class or afterwards if needed consisted of first verifying student understanding of constant and proportional systematic errors. This was done by using the "making errors" tab on the Nuts and

Bolts of Let's Make an Error Excelet (Figure 4). Using it students entered their experimental results and checked their understanding. The addition of any washers to the nut(s) and bolt was considered as an error in the mass. The mass of the washer (size of error) can be determined indirectly by how it influences the model compared to the "errorless" model.

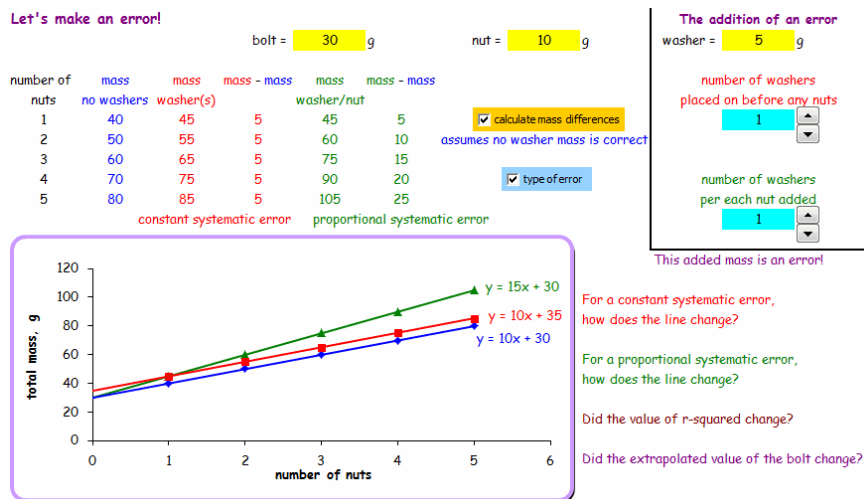


Figure 4. Systematic Errors (making errors tab)

Students should discover that the slope is the average mass of the nuts and the y-intercept is the mass of the bolt. This reinforces the behavior of the mathematical model and understanding of what the slope and y-intercept represent, and how they are influenced by constant and proportional systematic error. The systematic errors have no influence on the goodness-of-fit as measured by the coefficient of determination, R-squared.

The second verification was to examine the behavior of an errant nut (one nut of different density in addition to the four stainless steel nuts). This is accomplished on the "mixed nut" tab (Figure 5). The influence of overall material density was explored earlier [1]. Here students examine the outlier and its influence on the model; and the influence of the material density of the errant nut can also be explored.

Students must use some algebraic detective skills to match the group data to the placement of the errant nut in the sequence using their laboratory data. Students then examine how the errant nut (different density) influences the model. This outlier nut causes a variety of problems depending on its position in the sequence. The key to this spreadsheet simulation is changing the position of the errant nut to see how it influences the slope, y-intercept, and/or the value of R-squared.

Position 1 appears like a constant error (only y-intercept shifts), while the other positions (2, 3, 4, and 5) influence the slope, y-intercept, and the value of R-squared to a varying degree which is more like outlier behavior [9]. If students graphed the residuals ($y_{\text{measured}} - y_{\text{calculated}}$), they show a pattern as well.

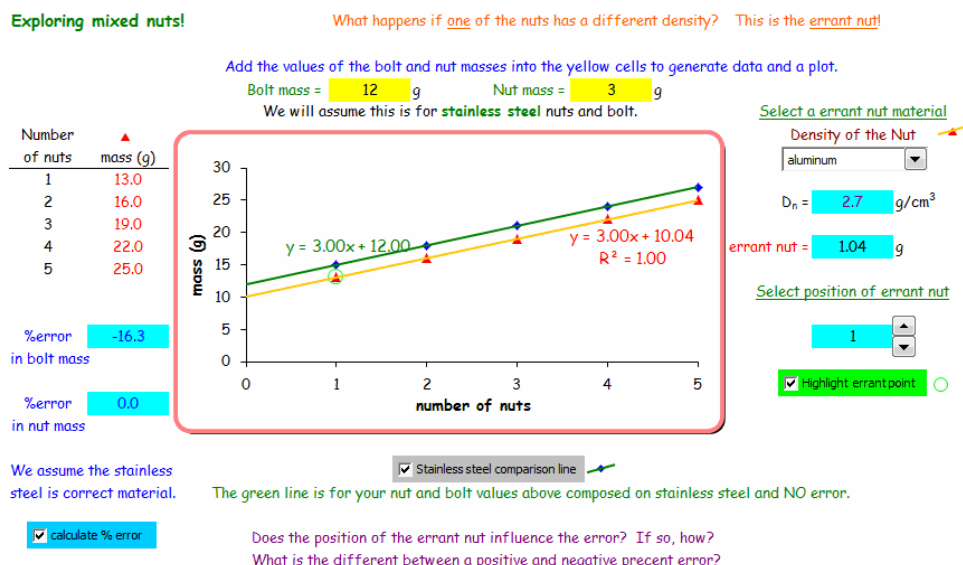


Figure 5. Errant Nut Analysis (mixed nut tab)

5. Further Exploration with the Simulation

This section was done as part of the post-lab questions that students addressed. They explore additional concepts not in the experimental part of the activity.

Students can explore random error, which is minimal, if they are careful (Figure 6); this is due to the consistent manufacturing of the nuts and bolts. For 25 bolts, we obtained an average of 11.80 g with a standard deviation of 0.022g. Students should already have a feel for random error and how scatter influences the R-squared value.

Generate Random Error in a Model

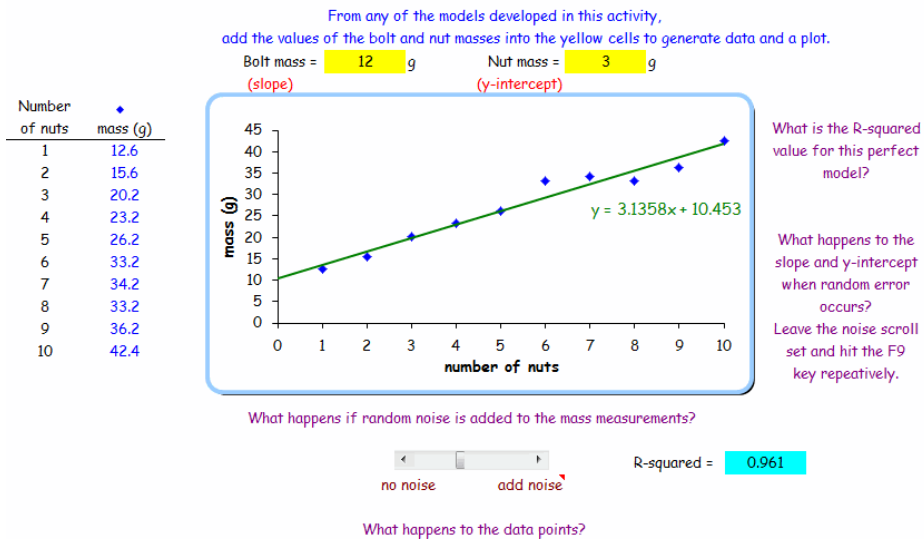


Figure 6. Random Error (random error tab)

Next students discover how mass by difference corrects for a constant systematic error as illustrated in Figure 7 (offset error on balance; not zeroed). They can also

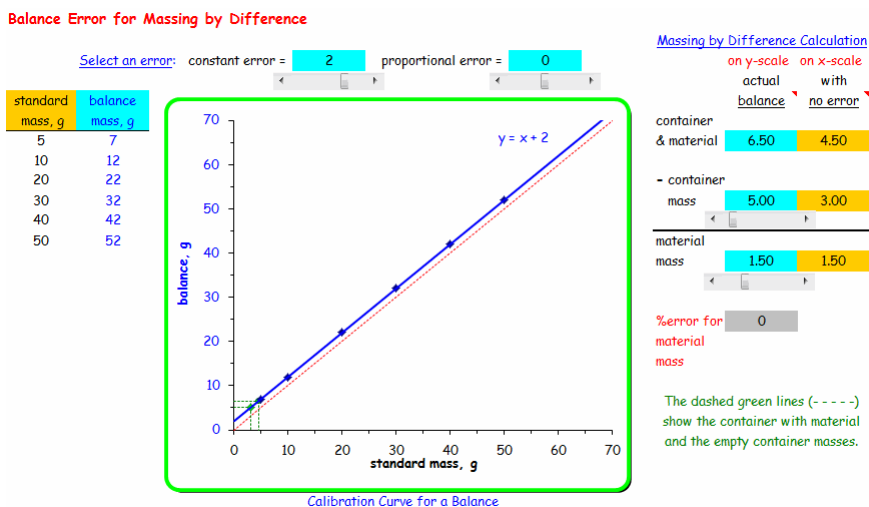


Figure 7. Balance Error (mass by difference tab)

explore balance calibration errors as well.

For the statistical diehards, we provide a tab using the Excel function LINEST for calculating the standard errors of the slope and the y-intercept plus confidence intervals.

6. Student Feedback

As part of the post-lab task, students were ask to complete an online form evaluation. From the spring of 2012, 45 students in three lab sections responded.

We did not separate students into cohorts for the sections, since sample size was small. The class is for non-chemistry majors and includes a mix of pre-medicine, pre-pharmacy, biology, and engineering students. The results of the Likert-like questions with 5 being the highest rating are given in Table 1 and a select summary of some comments in Table 2. The activity requires students coming to lab prepared and keeping the large amount of collected data organized and managed. This proved to be a sizable task for some lab groups; yet more than half of the groups did accomplish the task within the lab period.

Table 1. Student Feedback (n = 45 students)

Statement or Question	Ranking out of 5
The instructions for this activity were clear and unambiguous.	3.0
This activity was informative about types of error and their affect on results.	3.8
The size of the workgroup was appropriate for within group discussion and learning.	4.0
How would you rate this laboratory activity as a learning experience?	4.0
What is your overall rating of this laboratory activity?	3.9

Table 2. Selected Student Comments

Activity's Strong Points
<ul style="list-style-type: none"> • Rapid data exchange and analysis in a collaborative environment • Ease of data sharing between peer groups in a structured format • Application of Excel and statistics in a course other than an Excel or statistics course • Enhances critical thinking in a hands-on environment • Emulates a real-world scenario of a collaborative environment among teams in different locations
Activity's Weak Points (with author actions)
<ul style="list-style-type: none"> • Lack of clarity in instructions (adding photos such as Figures 1 & 2) • Dependency on other groups' timing of data entry

- Hard to read numbers on bolts (working on this)
- Discussion sections confusing (adding clarification)
- Chat session hectic & inefficient (now structuring!)
- Insufficient time (come prepared to lab!!!)

Upon a full analysis of the evaluation and points raised in the lab sessions, we are addressing a number of student concerns. A revision of the activity will now include photos (see Figures 1 and 2) and more explanation in a number of places to aid in developing the systematic errors with the washers. We will do the chat as an instructor-guided moderation as well.

Student comments on using the chat feature were overwhelmingly positive for using it for online collaboration.

7. Student Assessment

Upon completion of the activity, students had a week to complete the post-lab questions. These were designed to check their understanding and use the simulation for examining some new concepts or twists not seen in the laboratory. Students scored an average of 19.5 out of 30 points (65%) on the questions. The distribution of scores for 54 students is shown in Figure 8 (scores of zero are included in the mean).

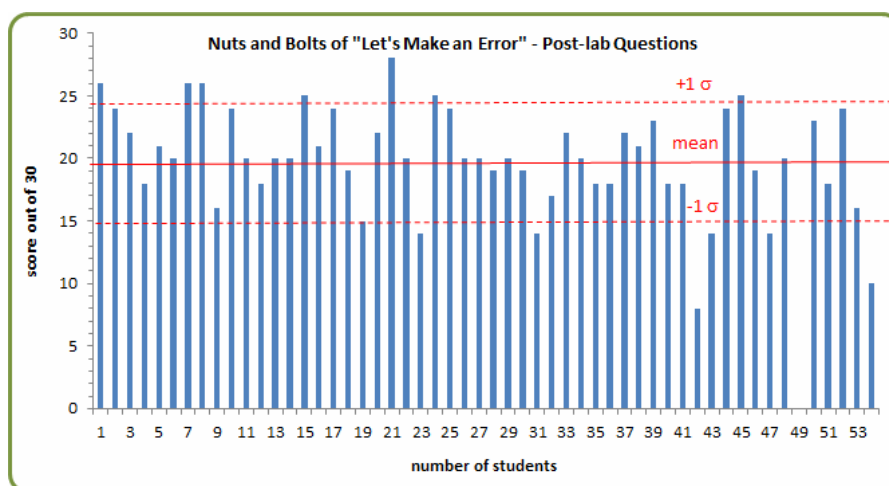


Figure 8. Post-lab Question Results (n = 54 students)

As a follow-up question (about 2-3 weeks later in the term) on an exam done in static fashion without a spreadsheet, this question was posed:

A scientist is going to calibrate a balance with a set of standard masses. On the graph paper provided, illustrate a perfect set of data for the calculation.

Then illustrate on the graph a positive constant systematic error and a negative proportional systematic error.

The answer is shown in Figure 9 along with the results in Figure 10 (scores of zero are included in the mean). About a third of the students had a score of 60% or above. Many of the errors were careless mistakes or lack of details including unlabeled axes and no scales to support “perfect calibration” as the $y = x$ line.

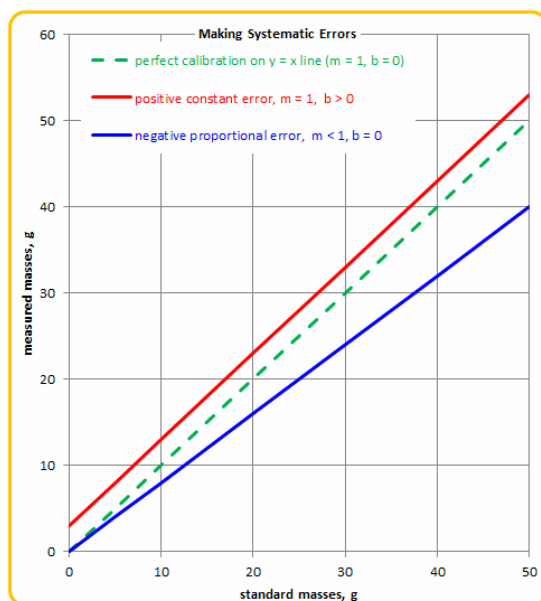


Figure 9. Exam Question Answer

The initial results show how collaborative group work does not always lead to immediate learning.

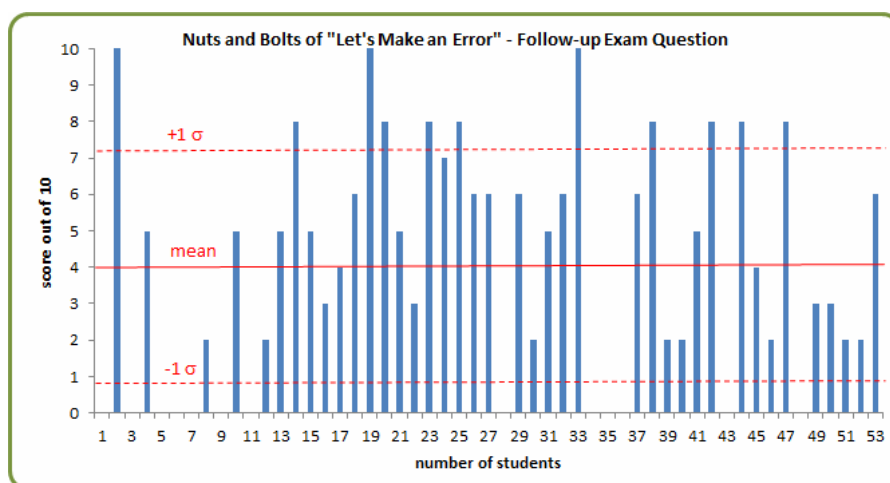


Figure 10. Follow-up Exam question Results (n = 53 students)

Again at the end of semester, an exam question (at 15 weeks) considering systematic error for ruler measurements based on an interactive spreadsheet did yield a 72%

score (data not shown). We also developed three off-shoot interactive spreadsheets or Excelets to support concepts in this activity. These include the following – balance calibration, ruler measurement and its error, and the comparison plot ($y = x$ line).

8. Some Final Thoughts

This activity using hands-on manipulatives and simple mass measurements helped to develop the mindset in students to deal with errors. Students must manage a variety of data sets and deal with data analysis (computations, graphing, and linear regression) in Excel. Mathematical models are developed and the influence of the errors on the models examined; and a number of higher-order thinking skills must be applied by students. The included spreadsheet simulation allows students to further manipulate variables to enhance their understanding of concepts. In addition, we introduce an emulation of online collaboration skills, a vital in the 21st century technology. The chat, being semi-anonymous (i.e. - by groups) when moderated by the instructor, leads to discourse that showed student engagement and thinking plus how easy the technology was to accomplish it. Osborne [10] has suggested that argument and debate are a needed element in science education as now recommended by the Framework [8]. We are working on expanding online collaboration using chat in other experiments.

For those interested in developing spreadsheet simulations, see the Developer's Guide to Excelets website [11]. The supporting materials developed for this activity can be found at the MatSci Excelets website [12].

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