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A Pedagogic Demonstration of Attenuation of Correlation Due to Measurement Error

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Abstract
Measurement error is an inescapable reality in most domains of research. Incidental variation in experimental setting, participant motivation and mood are among the multitude of factors that introduce error into measurement. I created a pedagogic spreadsheet that demonstrates the attenuation of correlation resulting from measurement error. The spreadsheet allows students to explore the sensitivity of the Spearman correction for attenuation of correlation to violation of assumptions. The spreadsheet demonstrates that the Spearman correction may not be advisable under certain circumstances. Some students conflate measurement error with sampling error. To clarify this misunderstanding, the spreadsheet uses a Monte Carlo simulation to demonstrate the difference between sampling error and measurement error and how they interact in practice.

Keywords
Reliability, Measurement Error, Attenuation, Correlation, Sampling Error

Cover Page Footnote
I would like to thank Christopher R. Wolfe for his helpful comments on an earlier version of the manuscript.

Introduction

Imagine that you have developed an aptitude test designed to predict employee performance. If this aptitude test works, it could be incorporated into hiring decisions. Visual inspection of your scatter plot reveals a suggestive link between aptitude score and performance. How can you be sure the results are not a fluke due to idiosyncrasies of your sample? Each possible sample will produce a slightly different result—a source of error known as sampling error. Sampling error pertains to random error in the estimation of a parameter, such as a mean or a correlation. In frequentist statistics, sampling error is modeled as a sampling distribution, consisting of all possible statistics that could have been observed [1]. Bayesian methods, by contrast, update a prior distribution with newly acquired data to form a posterior distribution. Several pedagogic spreadsheets are available for teaching sampling error and related concepts [2] [3] [4] [5].

Another important source of error is measurement error [6]. Measurement error refers to inconsistency in measurement. A given individual's score will fluctuate upon repeated measurement, even when measured under similar conditions and no underlying change has occurred. For example, one of your employees may perform worse when given the aptitude test a second time because she is preoccupied with an argument she had with her spouse the previous night. In this case, decreased aptitude would reflect measurement error rather than an underlying decrease in aptitude. Measurement error has important implications for theory and practical application, such as the attenuation of correlation. As measurement error increases, the observed correlation will become more attenuated. As a real-life example, consider the use of the graduate record examination (GRE) for predicting success in graduate school. Its use for graduate admission is controversial, in part, due to its low predictive validity [7]. However, after correcting for measurement error, the GRE showed improved predictability [8]. Thus, without taking measurement error into account, the correlation between two variables will be underestimated, as will the utility of a measurement instrument in applied settings.

Despite the important implications of measurement error, it has received incommensurate attention, both in education and in research. In many statistics courses, the effects of measurement error are given inadequate attention or no attention at all. Thus, it is not surprising that researchers do not typically adjust for the deleterious effects of measurement error in practice [9]. In light of these issues, I developed a spreadsheet to demonstrate the attenuation of correlation resulting from measurement error. Upon entering the true correlation and reliabilities, the effect of measurement error can be seen visually through scatter plots. It is important to note that the attenuating effect of measurement error is predicated on simplifying assumptions of classical test theory, which apply to the population. In any given sample, the assumptions are likely to be violated [10] [12]. For this reason, the spreadsheet allows students to examine how violations of these assumptions affect the attenuation of correlation. When the assumptions are violated, attenuation may be more pronounced, less pronounced or even reverse, resulting in accentuation. Another goal of the present paper is to elucidate the difference between sampling error and measurement error and demonstrate how they interact. Towards this end, the spreadsheet includes a macro that approximates a sampling distribution through Monte Carlo simulation. To visualize the concepts, the resulting simulated distributions are displayed in histograms.

The remainder of the paper is organized as follows. First, classical test theory is introduced as a theoretical foundation for related concepts presented in the following section, including measurement error, reliability and attenuation of correlation. Next, the assumptions of classical test theory are discussed in terms of their implications for attenuation of correlation. In the following section, the concept of a sampling distribution is introduced through Monte Carlo simulation and is distinguished from measurement error. The simulations demonstrate how nuisance correlations arise through sampling error and interact with measurement error. Next, the implementation of the spreadsheet is detailed. Before concluding, several pedagogic questions are provided, including suggested answers.

Classical Test Theory

Before proceeding, it is worth noting that ϱ and σ refer to the true parameters in the population while r and s are sample estimates. According to classical test theory, an observed score can be decomposed into true score and measurement error components as follows [6]:

$$(1)\ X = T_x + E_x$$

where X denotes the observed score, T_x denotes the true score and E_x denotes the measurement error. Sources of measurement error include transient factors, such as mood and alertness, which may interact with wording or response format [11].

Under the assumption of independence between T_x and E_x, the variances are additive:

$$(2)\ \sigma_x^2 = \sigma_{Tx}^2 + \sigma_{Ex}^2$$

According to classical test theory, the expectation of repeated, independent measurements equals the true score in the limit.

$$(3)\ \varepsilon(x) = T_x$$

Reliability is defined as the ratio of true variance and observed variance:

$$(4)\ \rho_{xx} = \frac{\sigma_{Tx}^2}{\sigma_{Tx}^2 + \sigma_{Ex}^2}$$

Several methods exist for estimating the reliability of a measure. One of the most popular methods is test-retest reliability [6]. In test-retest reliability, a sample is measured twice—usually separated by several days or weeks. The correlation between the two measurements serves as an estimate of reliability.

As previously noted, true scores and error scores are assumed to be uncorrelated, allowing for the additive decomposition of true score and error variance in Equation 2. From these definitions, it follows that: the error in x and error in y is uncorrelated ($\rho_{ExEy} = 0$); the error in x and true scores of y are uncorrelated ($\rho_{ExTy} = 0$); the error in y and true scores of x are uncorrelated ($\rho_{EyTx} = 0$). As we shall see later, these properties do not necessarily hold in samples due to sampling error.

Attenuation of Correlation

Measurement error attenuates the correlation between two variables. The logic underlying attenuation of correlation is straightforward. The maximum possible correlation occurs between a variable and itself. This is exemplified in the logic of test-retest reliability. In performing test-retest reliability, a correlation is computed between two test administrations using the same sample. Assuming no systematic changes in the variable occurred between time 1 and 2, test-retest reliability is essentially the correlation between a variable and itself. Thus, reliability sets an upper bound on the possible correlation between two variables. Equation 5 defines the attenuated correlation in terms of the reliabilities and the true correlation [6].

$$(5)\ r_{xy} = r_{TxTy}\sqrt{r_{xx}r_{yy}}$$

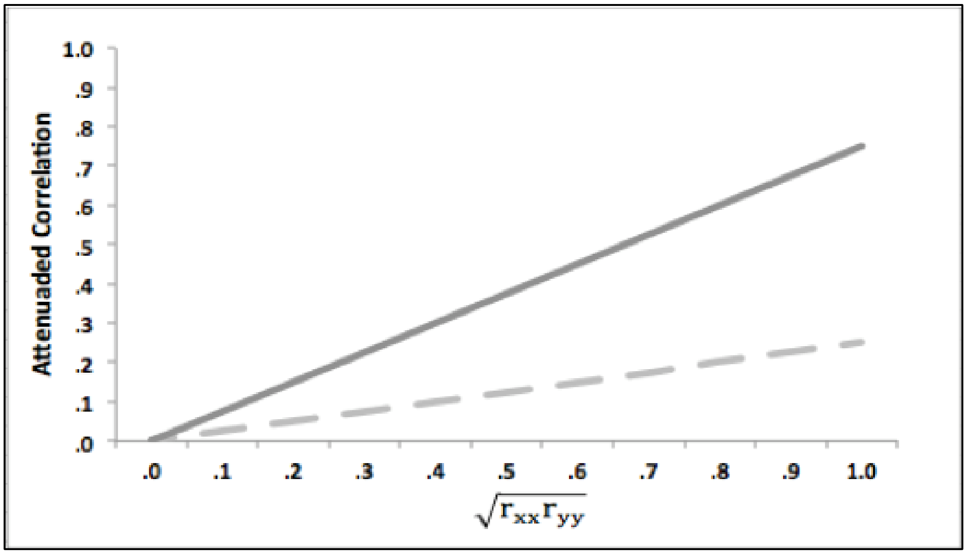


Figure 1: Attenuation of correlation as a function of $\sqrt{r_{xx} r_{yy}}$. The solid line represents a true correlation of ρ_{TxTy} = .75 and the dashed line represents a true correlation ρ_{TxTy} = .25.

The factor by which a true correlation is attenuated is equal to the geometric mean of reliabilities: $\sqrt{r_{xx} r_{yy}}$. As shown in Figure 1, attenuation increases as reliability decreases. The slopes indicate the effect is more pronounced for a large correlation (solid line) compared to a small correlation (dashed line). The attenuating effect of measurement error can be demonstrated in the spreadsheet titled “Measurement Error” by entering values in column C, as shown in Figure 2. In this particular example, .70 was entered into cell C1 for the true correlation and .60 was entered in to cells C6 and C7 for the reliabilities. Compared to the true values (black circles) in the top scatter plot in Figure 3, the spread of the attenuated values (pink circles) is more diffuse. In addition, the regression line for the attenuated values is flatter compared to the regression line for the true values, demonstration attenuation.

Measurement Error		
Description	**Notation**	**Value**
True Correlation	ρ_{TxTy}	0.70
Attenuated Correlation	r_{xy}	0.42
Spearman Correction	r_{xyc}	0.70
Reliability of x	r_{xx}	0.60
Reliability of y	r_{yy}	0.60
Correlation true x and error of x	r_{TxEx}	0.00
Correlation true y and error of y	r_{TyEy}	0.00
Correlation true x and error of y	r_{TxEy}	0.00
Correlation true y and error of x	r_{TyEx}	0.00
Correlation error x of and error of y	r_{ExEy}	0.00
Error of x	e_{xx}	0.40
Error of y	e_{yy}	0.40

Figure 2: A screenshot of the user input panel in the "Measurement Error" spreadsheet.

When the reliability of two measures are known, the Spearman correction can be applied, as shown in Equation 6 [6].

$$(6)\ r_{xyc} = \frac{r_{xy}}{\sqrt{r_{xx} r_{yy}}}$$

In the bottom scatter plot in Figure 3, the Spearman corrected values (green circles) completely superimpose the true values, indicating the Spearman correction was successful. The validity of the Spearman correction depends on the tenability of several simplifying assumptions, which do not generally hold in sample data.

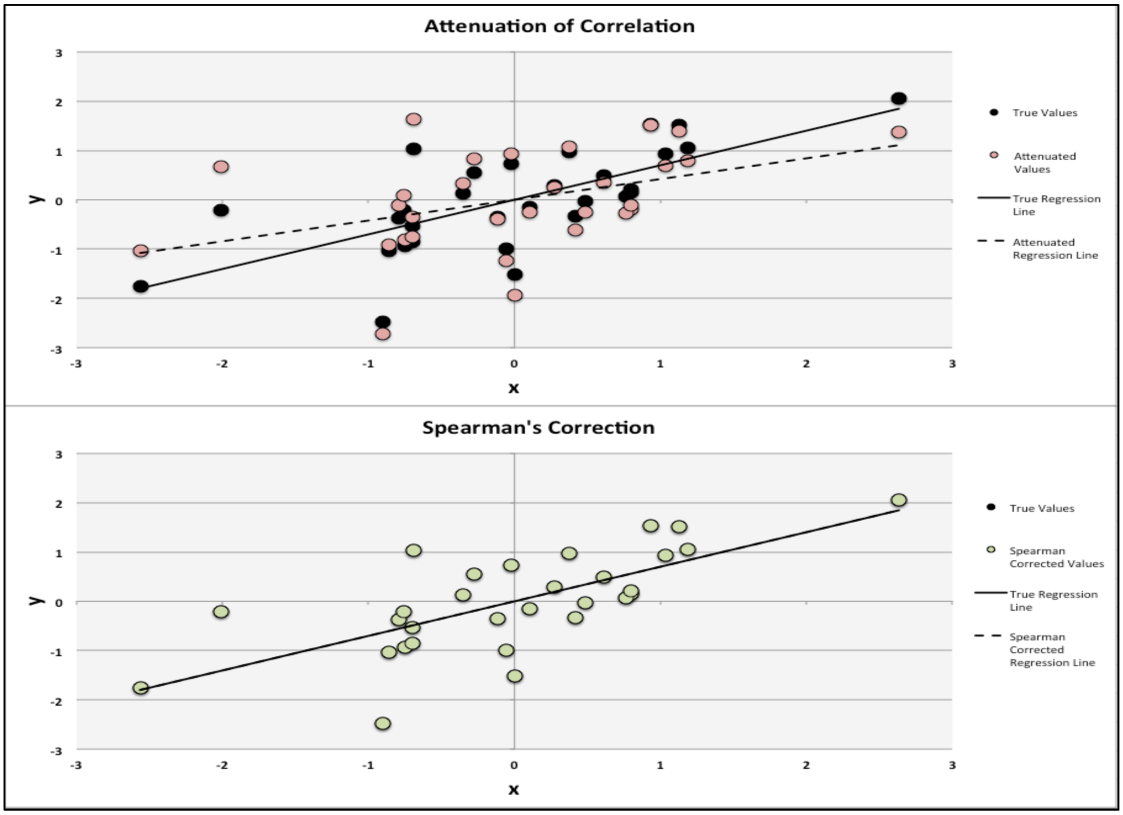


Figure 3: The top graph demonstrates the effect of attenuation of correlation. The attenuated values (pink circles) are more diffusely located around the regression line compared to the true values (black circles). In the bottom graph, the Spearman corrected values (green circles) superimpose the true values when the nuisance correlations are zero.

Violation of Assumptions

As previously noted, several simplifying assumptions were made in defining attenuation of correlation using Equation 5. Although the assumptions may hold in the population, sampling error will produce nuisance correlations between errors and scores. Complications arise when these assumptions are violated, which can be explored in the spreadsheet "Measurement Error". A more general formulation of the Spearman correction can be specified as follows [12]:

$$(7)\ r_{xyc'} = \frac{r_{xy} - r_{ExEy}\sqrt{e_{xx}e_{yy}} - r_{TxEy}\sqrt{r_{xx}e_{yy}} + r_{TyEx}\sqrt{r_{yy}e_{xx}}}{\sqrt{r_{xx}r_{yy}}}$$

It can be easily seen that Equation 6 is a special case of the full correction in Equation 7 in which $r_{ExEy} = r_{TxEy} = r_{EyTx} = 0$. Each additional term in Equation 7 adjusts for the violation of the assumption to which it corresponds. For example, failing to adjust for a positive correlation between error scores will result in overcorrection. This can be confirmed using the spreadsheet. Building upon the preceding example, a modest violation of independence between the errors was produced by setting C12 to .20. Inspection of Figure 4 indicates that the attenuation is reduced when the error score are correlated. Another important question is to extent is the Spearman correction sensitive to violation of assumptions? As indicated by the steeper regression in Figure 4, the Spearman correction produces a value higher than the true correlation when the errors are correlated. The Spearman corrected values no longer superimpose the true values. Instead, they have moved more tightly around the regression line. This result can be verified numerically by comparing Spearman corrected correlation in cell C5 ($r_{xyc} = .69$) to the true correlation in cell C4 ($r_{TxTy} = .60$). This suggestions that the Spearman correction is no longer valid when r_{ExEy} deviates from zero.

As previously mentioned, one assumption is that true scores and their corresponding error scores are uncorrelated: $r_{TxEx} = 0$ and $r_{TxEx} = 0$. Under what circumstances will a violation of this assumption influence attenuation of correlation? It will influence the attenuation of correlation only if $r_{TyEx} \neq 0$ or $r_{TxEy} \neq 0$ or $r_{ExEy} \neq 0$, as can be verified using the spreadsheet. Conceptually, this means that the correlation between true scores their corresponding error scores (r_{TxEx} , r_{TxEx}) have no bearing on the attenuation of the observed correlation, r_{xy}, unless they are connected indirectly through r_{TyEx}, r_{TxEy} or r_{ExEy}. An indirect relationship of this sort may not be immediately apparent without the use of the spreadsheet. Finally, *accentuation* of correlation occurs when r_{TyEx}, r_{TxEy} or r_{ExEy} are high relative to reliabilities of x and y. Accentuation of correlation is enhanced when r_{TxEx} or r_{TyEy} are negatively correlated. The foregoing examples demonstrate the need to carefully consider the tenability of the assumptions implied by the Spearman correction. When a violation is suspected, the more general formulation in Equation 7 will provide a more accurate correction.

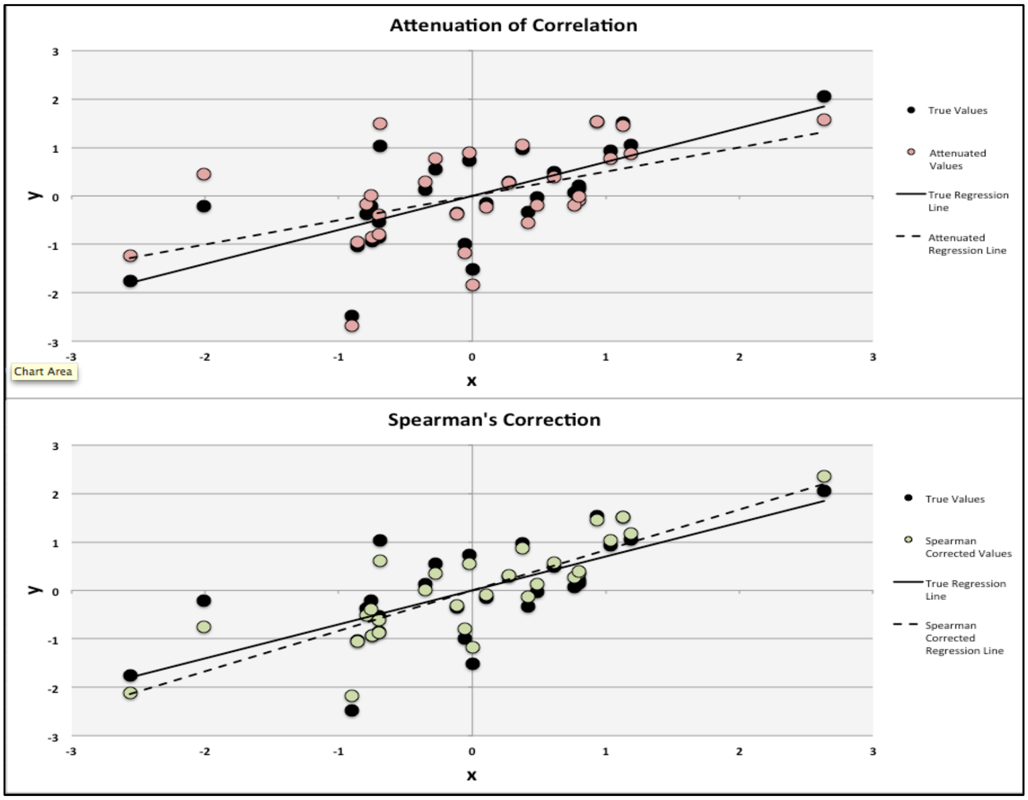
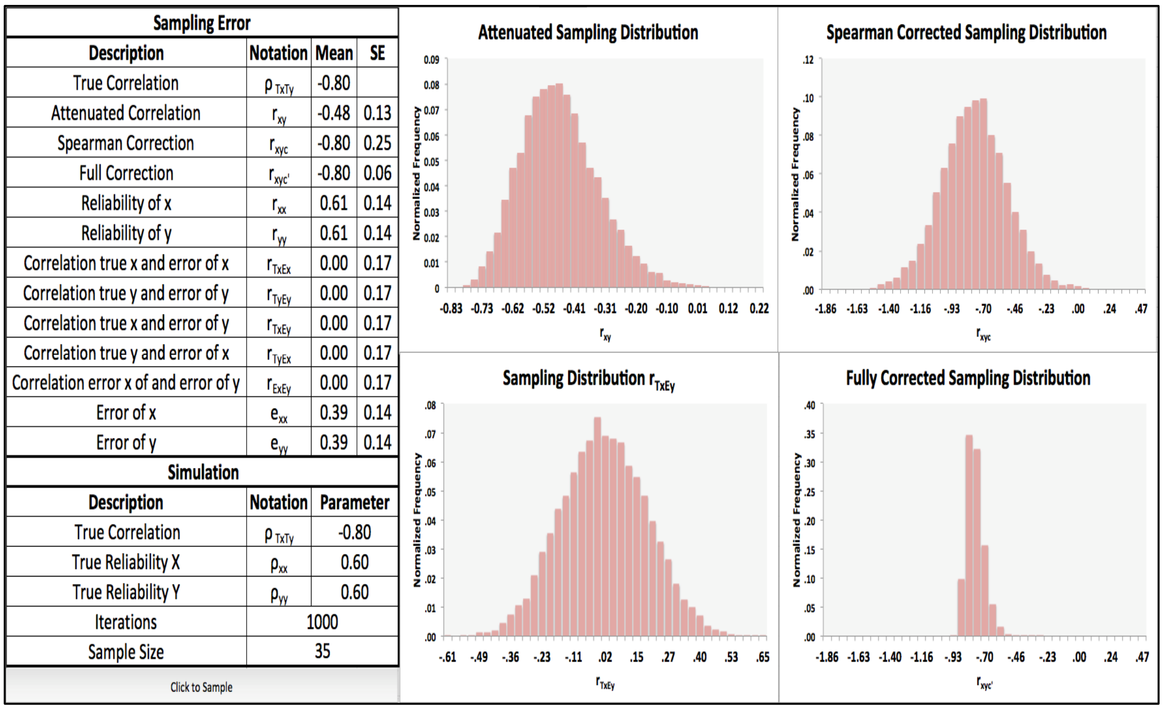


Figure 4: The effect of correlated error scores. The top graph shows decreased attenuation due to correlated error scores. The bottom graph shows overcorrection of the Spearman correction when the error scores are correlated

The Interaction between Measurement Error and Sampling Error

Measuring each member in a population is rarely feasible in practice. Instead, parameters are estimated from a sample drawn from the population. Random sampling reduces systematic errors in parameter estimation. Nonetheless, any given sample will not perfectly represent the characteristics of its corresponding population, resulting in sampling error. A sampling distribution quantifies the uncertainty in estimation by computing every possible statistic for given sample size. This is distinguished from measurement error, which is inconsistency in measurement rather than error in estimating a parameter from a random sample.

Although measurement error and sampling error are conceptually distinct, in practice, however, they interact [12]. The worksheet titled "Simulation" demonstrates the effect of sampling error on the Spearman correction when the nuisance correlations (e.g. ρ_{TxEy} ρ_{TxEx}, ρ_{ExEy}) are zero.



Sampling Error			
Description	Notation	Mean	SE
True Correlation	ρ_{TxTy}	-0.80	
Attenuated Correlation	r_{xy}	-0.48	0.13
Spearman Correction	r_{xyc}	-0.80	0.25
Full Correction	$r_{xyc'}$	-0.80	0.06
Reliability of x	r_{xx}	0.61	0.14
Reliability of y	r_{yy}	0.61	0.14
Correlation true x and error of x	r_{TxEx}	0.00	0.17
Correlation true y and error of y	r_{TyEy}	0.00	0.17
Correlation true x and error of y	r_{TxEy}	0.00	0.17
Correlation true y and error of x	r_{TyEx}	0.00	0.17
Correlation error x of and error of y	r_{ExEy}	0.00	0.17
Error of x	e_{xx}	0.39	0.14
Error of y	e_{yy}	0.39	0.14

Simulation		
Description	Notation	Parameter
True Correlation	ρ_{TxTy}	-0.80
True Reliability X	ρ_{xx}	0.60
True Reliability Y	ρ_{yy}	0.60
Iterations	1000	
Sample Size	35	

Figure 5: A screenshot of the spreadsheet titled "Simulation". The top left panel displays the mean and standard errors of the simulations while the bottom left panel receives user input for the simulations. The histograms to the right illustrate the sampling distributions for the attenuated correlation, the nuisance correlation, r_{TxEy}, the Spearman correction and the full correction.

Although the nuisance correlations are zero in the population, their corresponding sample estimates are generally not equal to zero. One important consequence is that the standard error for the Spearman correction is much larger compared to the analytically derived standard error. This means the Spearman correction can produce anomalous values that fall outside the permissible -1 to 1 range. By contrast, when the full correction based on Equation 7 is applied, the standard error matches the analytically derived standard error and all values are appropriately bounded in the interval -1 to 1.

As shown in Figure 5, the simulation can be configured in the lower left panel titled "Simulation". To perform a simulation, set the true correlation and true reliabilities in cells C18, C19 and C20, respectively. The sample size can be adjusted in cell C22. In addition, the number of iterations in the Monte Carlo simulation can be adjusted in cell C21. Using 1000 iterations is relatively quick and of sufficient precision for most purposes. Once the desired number of iterations is set, click the macro-enabled button in C23 to initialize the Monte Carlo simulation. Upon each iteration, data are sampled from a bivariate normal distribution with the user specified true correlation and reliabilities. For each simulated data set, the reliabilities, attenuated correlation and nuisance variables are recorded. In addition, the Spearman correction and full correction based on Equation 7 are applied to each simulated data set (see Implementation for further details).

The mean and standard errors from the simulation can be found in the panel titled "Sampling Error" under the columns mean and SE. Consistent with the assumption that the nuisance variables are zero in the population, their simulated means are zero (see Figure 5). However, the standard errors are somewhat large, indicating that most sample correlations depart from zero. This can also be verified through visual inspection of the sampling distribution of r_{TxExy} in Figure 5. As expected, the attenuated sampling distribution in Figure 5 shows a systematic underestimation of the absolute magnitude of the true correlation, which is $|-.80|$. The mean of the Spearman corrected sampling distribution is -.80, indicating that it was successful in eliminating the bias resulting from measurement error. However, inspection of the sampling distribution in Figure 5 reveals a glaring anomaly: many of the values exceed the boundary 1. This anomalous behavior indicates the Spearman correction, although unbiased, is not appropriate. The deleterious effects of the nuisance correlations are eliminated through the use of the full correction based on Equation 7. As shown in Figure 5, the full correction produces a much narrower sampling distribution that is appropriately bounded within the range -1 to 1. The standard error for the correlation coefficient can be approximated with the following formula: $\mathrm{SE_r} = \frac{1-r^2}{\sqrt{N}}$ [14]. Applying the formula reveals $\mathrm{SE_r} = .06$, which is the standard error of .06 for the full correction in Figure 5. By contrast, the standard error for the Spearman correction is much larger at .25.

Implementation

In this section, the implementation of the spreadsheet is described in detail for the interested reader. Beginning with the worksheet titled 'Measurement Error', the correlations and reliabilities can be entered by the user in cells 3,6-12 under the column heading Value. Each value is labeled with mathematical notation (column heading Notation) and a verbal description (column heading Description) to facilitate communication. The attenuated correlation is computed using the formula $\mathrm{r_{xy}} = \mathrm{r_{TxTy}}\sqrt{\mathrm{r_{xx}r_{yy}}} + \mathrm{r_{ExEy}}\sqrt{\mathrm{e_{xx}e_{yy}}} + \mathrm{r_{TxEy}}\sqrt{\mathrm{r_{xx}e_{yy}}} + \mathrm{r_{TyEx}}\sqrt{\mathrm{e_{xx}r_{yy}}}$, where $e_{xx} = 1 - \mathrm{r_{xx}} - \frac{COV_{TxEx}}{S_X^2}$. Because the worksheet titled "Measurement Error" uses normally distributed data with a mean and standard deviation fixed at 0 and 1, respectively, the last term simplifies to the correlation between T_x and E_x: $\mathrm{e_{xx}} = 1 - \mathrm{r_{xx}} - \mathrm{r_{TxEx}}$. The equation is implemented as: C4 = C3*SQRT(C6*C7)+C12*SQRT(C13*C14)+ C10*SQRT(C13*C14) +C11*SQRT(C13*C14) [12]. Cholesky decomposition was used to generate a scatter plot bearing the exact correlations specified by the True Correlation, Attenuated Correlation and Spearman Correction. For ease of comparison, the data in the scatter plots are transformations of static data in matrix **D**. Using static data allows the correlations to be changed according to user input in cells C3 through C12 without introducing unwanted sampling error. **D** is a 30X2 matrix of bivariate normally distributed data with a correlation exactly equal to zero, a mean exactly equal to zero and the standard deviation exactly equal to one. Data with these specifications were generated in three steps. First, two 30X1 vectors of normally distributed data were generated, **D1** and **D2**. Second, **D2** was regressed onto **D1** and the residuals were recorded in **D2'**, which is uncorrelated with **D1**. Next, **D1** and **D2'** were z-transformed to produce a mean of zero and a standard deviation of one, then concatenated to form matrix $\mathbf{D}$. $\mathbf{D}$ is offset to columns AM and AN for clarity of presentation and are labeled x′ and y′. To produce the desired correlations, Cholesky decomposition was used to decompose the covariance matrix $\mathbf{C} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ [13]. Because the values in $\mathbf{D}$ are standardized, the covariances and correlations are equivalent. Cholesky decomposition produces a matrix, $\mathbf{U}$, such that $\mathbf{U^T U} = \mathbf{C}$. In the simple bivariate case, $\mathbf{U} = \begin{bmatrix} 1 & \rho \\ 0 & \sqrt{1-\rho^2} \end{bmatrix}$. In the next step, $\mathbf{D}$ is multiplied by $\mathbf{U}$ to produce a matrix, $\mathbf{B}$, of data with the desired correlation: $\mathbf{B} = \mathbf{DU}$. Rather than performing matrix multiplication, the operations were applied in a piecewise fashion. First, the true values of x were computed from x′: AG2=AM2. Next, the true values of y were computed from x′ and y′ as follows to produce the desired correlation: AH2=C3*AM2+SQRT(1-C3^2)*AN2. A similar procedure was used to produce data for the attenuated correlation and Spearman correction, except C$4 and C$5, were referenced instead. Finally, the true data and attenuated data are superimposed in the same scatter plot to facilitate comparisons.

The worksheet titled ‘Simulation’ is embedded with macro that approximates the sampling distributions through Monte Carlo simulation. The simulation is based on the methods described in [13]. To provide a conceptual overview, the Monte Carlo simulation proceeds with the following steps. First, T_x and T_y scores are sampled from a standard normal distribution according to the sample size specified by the user in cell B22. The desired correlation between T_x and T_y was produced using the Cholesky decomposition procedure previously described. Second, corresponding error scores E_x and E_y are sampled from a normal distribution with a mean of 0 and a standard deviation defined in terms of the reliability: $\sigma_{Ex} = \sqrt{\frac{(1-\rho_{xx})}{\rho_{xx}}}$. Observed scores, x and y, were formed by adding the true scores and error scores according to Equation 1. Similarly, the reliabilities were computed according to Equation 4. The nuisance correlations were computed using the CORREL function. The Spearman correction and full correction based on an alternative form of Equation 7

[10] were computed from the aforementioned values. This process is repeated as specified by the user in cell B21. Upon completion, the resulting values are recorded in the worksheet in columns AG through AR. To provide appropriate scaling, a dynamic binning system is used. The bins range from the min to the max of the simulated correlations and increase incrementally using the following recursive formula. AZ3 =(MAX(AG:AG)-MIN(AG:AG))/40+AZ2.

Problems

In this section, six problems are provided to help students understand the issues associated with measurement error. Suggested answers are included to verify comprehension.

Problem 1

What happens to the correlation between two variables when measurement error is introduced? To answer this question, increase measurement error by decreasing the reliabilities (cells C6 and C7) in the worksheet titled 'Measurement Error'.

Suggested answer: The correlation attenuates as the reliability decreases. This is reflected by greater dispersion in the scatter plot and a flatter regression slope.

Problem 2

What happens when the assumptions of classical test theory are violated? Under what conditions will the correlation become accentuated and how does that impact the Spearman correction?

Suggested answer: In worksheet 'Measurement Error', the attenuation of correlation is mitigated when r_{TyEx}, r_{TxEy}, or r_{ExEy} increase. Once they become sufficiently high, accentuation will occur. When this happens, the Spearman correction overcorrects for attenuation because it assumes $r_{TyEx} = r_{TxEy} = r_{ExEy} = 0$.

Problem 3

Under what conditions will the correlation between the true score and measurement error (e.g. r_{TxEx}) influence attenuation of correlation?

Suggested answer: the correlation between true scores and measurement error only occur when $r_{TyEx} \neq 0$ or $r_{TxEy} \neq 0$ or $r_{ExEy} \neq 0$

Problem 4

Explain the difference between sampling error and measurement error. How are they related? To help you answer this question, examine the sampling distributions under two conditions perfect reliability and imperfect reliability (e.g. .7) in the worksheet. What happens to the sampling distribution of r_{TxEy} under each of these conditions?

Suggested answer: Measurement error refers to inconsistency in measurement, whereas sampling error is error in estimation due to a mismatch between the characteristics of a sample and the characteristics of the population from which it was taken. Sampling error can be seen in the variability in the sampling distributions. When reliability is perfect, there is no bias in estimation. However, when reliability is lower, the correlation is systematically underestimated due to attenuation (see histogram of Attenuated Sampling Distribution). When the reliabilities are perfect (e.g. 1), there is no variability in the sampling distribution of r_{TxEy}. All the values $r_{TxEy} = \rho_{TxEy} = 0$, as assumed by classical test theory.

Problem 5

What happens to the sampling distribution for the Spearman correction when reliability is less than 1? How does it compare to the sampling distribution for the full correction? What accounts for their differences?
Suggested Answer: Both distributions have the same mean. However, the sampling distribution for the Spearman has a larger standard deviation and exceeds 1. The fully corrected sampling distribution, on the other hand, has a smaller standard deviation and remains within the -1 to 1 range. The reason for this difference is that the Spearman correction does not adjust for nuisance correlations (e.g. r_{TxEy} r_{TxEx}) that emerge as a result of sampling error.

Problem 6

Under what conditions is the Spearman correction most likely to exceed 1? Adjust the parameters in the simulation to help you answer this question. Suggested Answer: The Spearman correction is more likely to exceed 1 as the true correlation increases, the reliabilities decrease, and the sample size decreases.

Conclusions

The pedagogic spreadsheet demonstrates that measurement error attenuates the observed correlation between two variables. Students can adjust the reliability of measurement to observe its attenuating effects in the scatterplot. In addition, students can explore the effects resulting from violations of classical test theory that are typical in sample data. This underscores the importance of considering potential violation of assumptions when applying the Spearman correction. Understanding the difference between measurement error and sampling error can be difficult for many students initially. To underscore this difference, the spreadsheet includes a macro that simulates the sampling distributions and displays the results graphically in real time. The simulations demonstrate how nuisance correlations contaminate the Spearman correction, thereby causing its sampling distribution to exceed the acceptable -1 to 1 boundary.

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