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A Bivariate Approximation Spreadsheet Calculator by Lagrange Interpolation

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Abstract

Even though interpolating bivariate data by Lagrange interpolation is straightforward, its repetitive calculations are quite boring and complicated if the number of data is large. Hence, there is a need to have a suitable tool in teaching and learning Numerical Methods for this topic. To simplify things, we have developed an Excel spreadsheet calculator to interpolate the bivariate data with 4 rows by 4 columns using Lagrange interpolation. The spreadsheet calculator can be used by educators and students who need its full solution. In addition, users only need to enter a dataset, two independent variables, and the values of the two independent variables which are not in the dataset to obtain a bivariate approximation solutions table in the respective target cells automatically. Besides, the excel command given helps in the teaching and learning process of this topic using Excel spreadsheet.

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1. Introduction

In many engineering problems, we may have a set of bivariate data (x, y, z) and wish to approximate the value of $z = f(x, y)$ for the given values of x and y which are not in the data set. Here, f is a function that describes the relationship between z and (x, y). We can approximate this function using interpolation techniques such as Lagrange, Newton's forward-difference, Newton's backward-difference, Newton's central-difference or Newton's divided-difference. However, the bivariate approximation is quite tedious, tiring and boring because of the repetitive calculations involved. Hence, there is a need to develop a bivariate approximation spreadsheet calculator in teaching and learning the topic of bivariate approximation. It is hoped that this spreadsheet calculator will be able to attract the attention of learners and develop their interest in doing engineering calculations.

A series of papers working on solving numerical methods in classrooms and examination situations using Excel spreadsheet, which focus on systems of nonlinear and linear equations, approximation of interpolation, computing of eigenvalues, numerical differentiation by Richardson's extrapolation, ordinary differential equations (ODEs) by the Fourth-order Runge-Kutta (RK4) and the Laplace equation, can be seen in [1-10]. However, literature works dealing with bivariate approximation using Lagrange interpolation in Excel have not been explored as yet. Therefore, in this paper, we have developed a bivariate approximation Excel spreadsheet calculator using Lagrange interpolation.

2. Bivariate Approximation

Consider a 4 by 4 bivariate data as given in Table 1.

Table 1: Bivariate Data

	y_j			
x_i	y_0	y_1	y_2	y_3
x_0	$f(x_0, y_0)$	$f(x_0, y_1)$	$f(x_0, y_2)$	$f(x_0, y_3)$
x_1	$f(x_1, y_0)$	$f(x_1, y_1)$	$f(x_1, y_2)$	$f(x_1, y_3)$
x_2	$f(x_2, y_0)$	$f(x_2, y_1)$	$f(x_2, y_2)$	$f(x_2, y_3)$
x_3	$f(x_3, y_0)$	$f(x_3, y_1)$	$f(x_3, y_2)$	$f(x_3, y_3)$

We would like to approximate the value of $z = f(x, y)$ for the given values of x and y which are not in Table 1. To solve the above problem, firstly, we interpolate the data from the first row $(i = 0)$ to the fourth row $(i = 3)$ for the given y by using Lagrange Interpolation to obtain z_0, z_1, z_2 and z_3. Lastly, we interpolate again for the resulted interpolated values (z_0, z_1, z_2 and z_3) for the given x to obtain z by using Lagrange Interpolation as shown in Table 1.

Table 2: Bivariate Approximate Solution Table by Lagrange Interpolation

j	0	1	2	3
y_j	y_0	y_1	y_2	y_3
$L_j(y)$	$L_0(y)$	$L_1(y)$	$L_2(y)$	$L_3(y)$
x_i	x_0	x_1	x_2	x_3
	$z_0 = f(x_0, y)$	$z_1 = f(x_1, y)$	$z_2 = f(x_2, y)$	$z_3 = f(x_3, y)$
$L_i(x)$	$L_0(x)$	$L_1(x)$	$L_2(x)$	$L_3(x)$
$f(x, y)$	z			

where
$$L_0(y) = \frac{(y - y_1)(y - y_2)(y - y_3)}{(y_0 - y_1)(y_0 - y_2)(y_0 - y_3)},$$

$$L_1(y) = \frac{(y - y_0)(y - y_2)(y - y_3)}{(y_1 - y_0)(y_1 - y_2)(y_1 - y_3)},$$

$$L_2(y) = \frac{(y - y_0)(y - y_1)(y - y_3)}{(y_2 - y_1)(y_2 - y_1)(y_2 - y_3)}, \quad L_3(y) = \frac{(y - y_0)(y - y_1)(y - y_2)}{(y_3 - y_0)(y_3 - y_1)(y_3 - y_2)},$$

z_i for $i = 0, 1, 2, 3$ are given as

$$z_i = f(x_i, y) = L_0(y) f(x_i, y_0) + L_1(y) f(x_i, y_1) + L_2(y) f(x_i, y_2) + L_3(y) f(x_i, y_3),$$

$$L_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}, \quad L_1(x) = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)},$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_1)(x_2 - x_3)}, \quad L_3(x) = \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)},$$

and z is given as

$$z = f(x, y) = L_0(x) f(x_0, y) + L_1(x) f(x_1, y) + L_2(x) f(x_2, y) + L_3(x) f(x_3, y).$$

3. Numerical Example

In this section, we provide a four by four bivariate data to be solved using Lagrange interpolation.

Question

The wind chill is the effective temperature of the skin from heat loss due to winds at cold temperatures. Wind chill temperatures less than -20 °F are considered low, temperatures less than -40°F are extreme and temperatures less than -60°F are considered dangerous [11]. Consider wind-chill index (WCI) for certain wind speed $v\,(mi/h)$ and temperature $T(°F)$ as shown in Table 3.

Table 3: Wind-chill index, WCI

Wind Speed $v\ (mi/h)$	Temperature $T^{\circ}(F)$			
	20	25	30	35
5	16	22	27	32
10	3	10	16	22
15	-5	2	9	15
20	-11	-3	4	11

(Data source: [12]: Pg 922)

Estimate (WCI) if the temperature is 33 $°F$ and the wind speed is 17 mi/h.

Solution

This problem is solved using the bivariate approximation spreadsheet calculator as given in Section 4.

4. Bivariate Approximation Spreadsheet Calculator

Figure 1 illustrates the bivariate approximation spreadsheet calculator based on Lagrange interpolation. Users need to enter the dataset with its label into the cells A3:E8. The two independent variables, x and y are entered into cells A10 and C10, with their respective values in cells B10 and D10. Then its bivariate approximation solution table based on Lagrange interpolation together with the final interpolated value in cell B18 will be calculated automatically. The spreadsheet calculator is simple and very easy to use.

	A	B	C	D	E
1		Bivariate Approximation			
2					
3	v	Temperature (F)			
4	mi/h	20	25	30	35
5	5	16	22	27	32
6	10	3	10	16	22
7	15	-5	2	9	15
8	20	-11	-3	4	11
9					
10	V	17	T	33	
11					
12	j	0	1	2	3
13	T	20	25	30	35
14	L	0.0640	-0.3120	0.8320	0.4160
15	V	5	10	15	20
16	Interpolated data	29.9360	19.5360	12.7840	8.1360
17	L	0.0560	-0.2880	1.0080	0.2240
18	Answer	10.7588			

Figure 1: Bivariate spreadsheet calculator and its solution

The numerical solution that has been obtained in Figure 1 results from the following steps and the respective Excel commands below:

Step 1 Select B13:E13, then type in **=B$4:E$4**, followed by pressing Ctrl, Shift and Enter concurrently.

Step 2 Type in L_0 in cell B14 as
=(D10-C13)*(D10-D13)*(D10-E13)/((B13-C13)*(B13-D13)*(B13-E13)).

Step 3 Then copy L_0 to get L_1. To achieve this, drag the formula in cell B14 to C14 and due to the relative column concept, the copied formula becomes
=(D10-D13)*(D10-E13)*(D10-F13)/((C13-D13)*(C13-E13)*(C13-F13))
Since F13 is outside the list, it should be dragged back to the first cell (T_0) or B13 as follows:
=(D10-D13)*(D10-E13)*(D10-B13)/((C13-D13)*(C13-E13)*(C13-B13)).

Step 4 Repeat by copying L_1 formula to L_2 and edit F13 to B13 to get the correct formula as:
=(D10-E13)*(D10-B13)*(D10-C13)/((D13-E13)*(D13-B13)*(D13-C13)).

Step 5 Repeat by copying L_2 formula to L_3 and edit F13 to B13 to get the correct formula as:
=(D10-B13)*(D10-C13)*(D10-D13)/((E13-B13)*(E13-C13)*(E13-D13)).

Step 6 Select cells B15 to E15, type in **= TRANSPOSE(A5:A8)** followed by pressing Ctrl, Shift and Enter concurrently

Step 7 To get the interpolated row data, type in cells
B16= SUMPRODUCT(B14:E14,B5:E5)
C16= SUMPRODUCT(B14:E14,B6:E6)
D16= SUMPRODUCT(B14:E14,B7:E7)

E16= SUMPRODUCT(B14:E14,B8:E8)

Step 8 Type in L_0 in cell B17 as

=(B10-C15)*(B10-D15)*(B10-E15)/((B15-C15)*(B15-D15)*(B15-E15))

Step 9 Then copy L_0 to get L_1 and edit F15 to B15 to get the correct formula as:

=(B10-D15)*(B10-E15)*(B10-B15)/((C15-D15)*(C15-E15)*(C15-B15))

Step 10 Repeat by copying L_1 formula to L_2 and edit F15 to B15 to get the correct formula as:

=(B10-E15)*(B10-B15)*(B10-C15)/((D15-E15)*(D15-B15)*(D15-C15))

Step 11 Repeat by copying L_2 formula to L_3 and edit F13 to B15 to get the correct formula as:

=(B10-B15)*(B10-C15)*(B10-D15)/((E15-B15)*(E15-C15)*(E15-D15))

Step 12 To get $z = f(x, y)$ for given x and y values which are not in the dataset, type in cell

B18=SUMPRODUCT(B17:E17,B16:E16)

5. Conclusion

In conclusion, a four by four bivariate approximation spreadsheet calculator based on Lagrange interpolation has been developed to approximate $z = f(x, y)$ for given x and y values which are not in the dataset. The Excel commands used in building the spreadsheet calculator are given in Section 4 and its layout is shown in Figure 1. The bivariate spreadsheet calculator is very user-friendly and it provides an alternative tool for bivariate data approximation. It can be used as a marking scheme for educators and a self-learning tool for students who need its full solutions. Besides, the Excel commands given in Section 4

will help educators and students in teaching and learning the bivariate approximation topic by using the Excel spreadsheet. Hopefully, it will be able to develop students' interest and enhance their performance.

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