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Using Spreadsheets to Teach Signal Detection Theory

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Abstract

Signal detection theory (SDT) is a mathematical framework for evaluating the performance of a detection system, which is broadly applicable across many academic and practical domains, including medicine, psychology, statistics and engineering. In this article, I present an interactive spreadsheet that allows students to systematically explore the function of the two key parameters of SDT: the decision criterion and discriminability. Stimulus distributions and ROC curves update in real time according to user input, thereby providing an intuitive visualization of the SDT parameters. The spreadsheet can be used to find an optimal decision criterion for a given level of discriminability, prior probabilities and payoff matrix. The article includes six problems and suggested answers that instructors may use in the classroom. The article concludes with a discussion of simple adaptions of the spreadsheet for teaching frequentist statistics and testing psychologically based models of detection.

Introduction

Many situations can be described as a signal detection task in which the presence or absence of a stimulus must be determined under less-than-ideal conditions. Such situations range in importance from trivial—such as inspecting apples for bruises at the grocery—to highly consequential—such as detecting a tumor on a chest x-ray. What these situations share in common is that error-free performance is not possible and some balancing of different errors is necessary. Signal Detection Theory (SDT) is a mathematical framework for assessing the performance of a detection system when some degree of error is inevitable [1]. SDT has been widely applied in many academic and practical domains, including statistics, psychology, business, engineering and medicine [1]. One benefit of SDT above and beyond simple measurements of accuracy is that it decomposes performance into two independent components: discriminability—the ability of a system to distinguish between two stimuli—and decision criterion—a threshold of evidence required to state that a stimulus has been detected.

Learning SDT can be difficult for students for several reasons. First, students tend to struggle with null hypothesis significance testing [2], a statistical approach that is built upon the same mathematical framework that is used in SDT. Understanding the relationship between conditional probabilities appears to be a primary point of confusion [2]. Moreover, students often fail to understand the important role of prior probabilities in assessing performance [3,4]. Second, many sources provide a rigorous mathematical treatment of SDT that is not accessible to most students. Although more accessible resources exist, they are limited by their static treatment of the material. Several lines of research suggest that allowing students to interact with material in a systematic fashion may lead to better learning than traditional methods [5]. For this reason, students who are learning SDT may benefit from the use of an interactive learning environment. Spreadsheets provide an ideal interface for an interactive learning environment due to their simplicity, flexibility and ubiquity.

Currently, interactive resources for learning SDT are limited if not non-existent. To fulfill this need, I have developed an interactive spreadsheet for teaching SDT. Interactive graphs allow students to systematically explore the effect that model parameters have on performance. The

spreadsheet is designed as a teaching tool that can be used in a computer lab or for students who wish to supplement the curriculum provided by their instructor. The spreadsheet is presented in a ready-to-use form that requires only rudimentary knowledge of spreadsheets. Thus, the spreadsheet is deal for developing a conceptual foundation upon which students can build a more mathematically rigorous understanding of SDT. A description of the technical details of the spreadsheet is provided for interested readers and advanced students, who wish to understand or modify the spreadsheet for specific purposes.

The remainder of the article is organized as follows. First, SDT is introduced formally in the context of an intuitive example. The layout and use of the first spreadsheet is described concurrently. The following section introduces the concept of ideal observer analysis and how it can be used to determine optimal performance when prior probabilities and a payoff matrix can be specified. In addition, the article includes a set of problems with suggested answers that teachers may opt to use in the classroom. For clarity of presentation, the technical details of the spreadsheet are described in the Appendix for interested readers. The paper concludes with a discussion of alternative uses of the spreadsheet for teaching related concepts, such as null hypothesis significance testing and analyzing psychologically plausible models of signal detection analysis.

Signal Detection Theory

SDT was originally developed during World War II to optimize the detection of radio signals in environments perturbed with background interference [6]. SDT has been used more broadly to include human perception among other applications. Perception can be conceptualized as a signal detection task in which the brain must interpret an imperfect signal. The signal may be degraded due to interference from a variety of internal and external sources, including momentary lapses in attention, random thoughts that may come to mind or distractions in the environment. SDT can be intuitively illustrated with phantom vibration syndrome, a phenomenon experienced by the majority of cellphone users [7,8]. Phantom vibration syndrome occurs when a person falsely perceives vibrations from his or her phone. In SDT, random variation in perception

is commonly modeled as a normally distributed random variable as follows [1]:

$$V \sim N(\mu, \sigma^2) \qquad (1)$$

in which V is the perceived vibration, μ is the mean, and σ^2 is the variance. For the sake of simplicity, σ^2 is assumed to be 1. Figure 1 provides a graphical illustration of normally distributed perceived vibration.
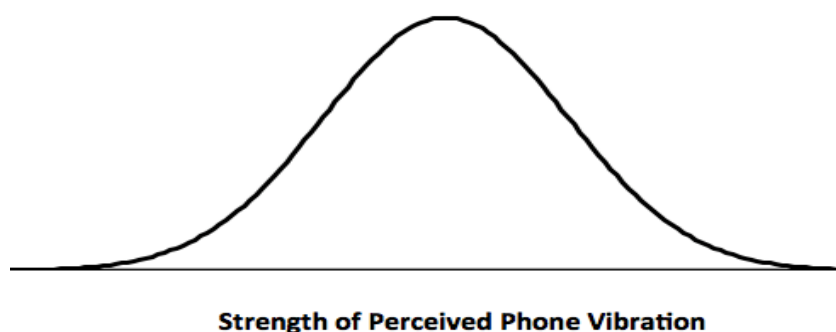

Strength of Perceived Phone Vibration

Figure 1: The strength of perceived phone vibration represented as a normally distributed random variable.

Because a true vibration and a phantom vibration cannot be perfectly distinguished, there are four possible outcomes depicted in the confusion matrix located in the tab 'ROC curves and Distributions', cells C39:F42. (see Figure 2). When the phone actually vibrates, one may correctly respond "Yes", resulting in a hit or "No", resulting in a miss. By contrast, when the phone does not vibrate, a person may incorrectly respond "Yes", resulting in a false alarm or correctly respond "No" resulting in a true negative. As will be discussed shortly, the values in the confusion matrix are controlled by two parameters, corresponding to discriminability and decision criterion.

		State Of the World	
		Vibration	No Vibration
Response	Yes	0.31	0.07
	No	0.69	0.93

Figure 2: A close up of the confusion matrix for phantom vibration syndrome located in the tab "ROC Curves and Distributions"

The confusion matrix can be represented in terms of two normal distributions—one distribution corresponding to each possible state of the world (see Figure 3). Importantly, the placement of the distribution on separate graphs with a common x-axis clearly conveys the conditional relationships used in the SDT analysis. The scale of the perceived vibration is arbitrarily measured in standard units with the "No Vibration" distribution centered with a mean of zero. From the perspective of a person who perceives a vibration, it is impossible to determine which state of the world is true. That is, whether the perceived vibration is real or not. To cope with this uncertainty, one must set a decision criterion to determine whether a "Vibration" response is given or a "No vibration" response is given. When this parameter is varied in cell B45, the distributions, confusion matrix and ROC curve update accordingly (see Figures 3 and 4). In Figure 3, the decision criterion is depicted as vertical black line that separates hits from misses when the vibration is present and separates false alarms from true negatives when no vibration is present. The decision criterion balances the tradeoffs of committing misses and false alarms (and, by extension, hits and true negatives). In comparing Figure 3 to Figure 4, it can be seen that increasing the decision criterion will decrease the rate of false alarms at the expense of increasing misses.

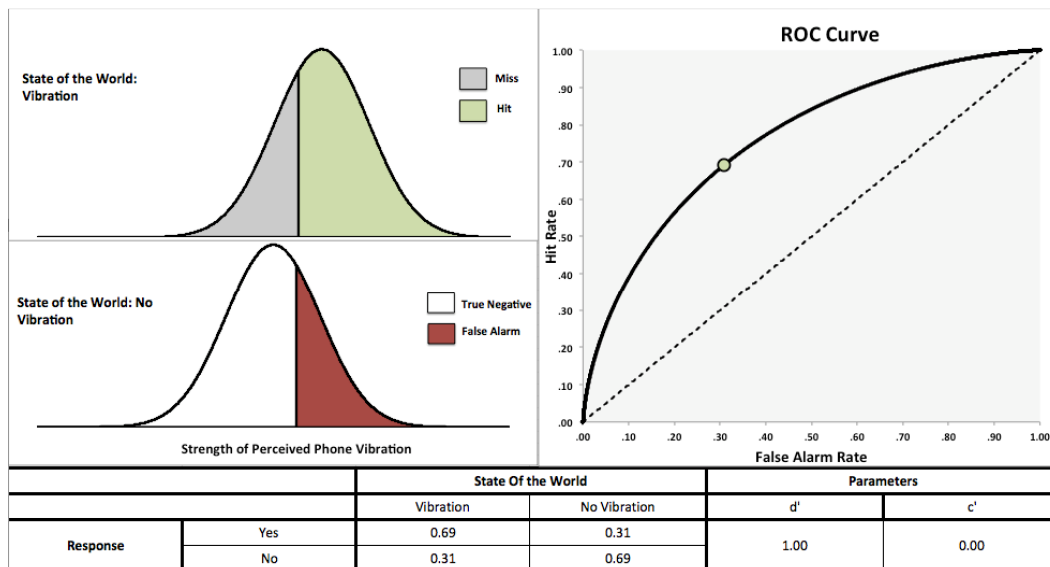|		State Of the World		Parameters	
		Vibration	No Vibration	d'	c'
Response	Yes	0.69	0.31	1.00	0.00
	No	0.31	0.69		

Figure 3: A screenshot of the interface in the tab ROC and Distributions tab. d' = 1 and c' = 0.

The decision criterion can be measured with a variety of metrics. For the purposes of the present article, I will discuss c, which is an absolute measure, and c′, which is a relative measure. The decision criterion in Figure 3 is .50 as measured by c. It is often more informative to measure the decision criterion relative to the standardized differences between the distributions. This standardized difference is known as d′, which is a measure of discriminability (to be discussed below). c′ can be calculated as [1]:

$$c' = .5d' - c \qquad (2a)$$

$$c' = -\left(\frac{\Phi^{-1}(Hit) + \Phi^{-1}(FA)}{2}\right) \qquad (2b)$$

As displayed in cell B45, c′ is zero, A value of zero indicates that the criterion is unbiased in the sense that false alarms and misses are equally likely. In Figure 4, c′ is increased to 1, producing more misses than false alarms.

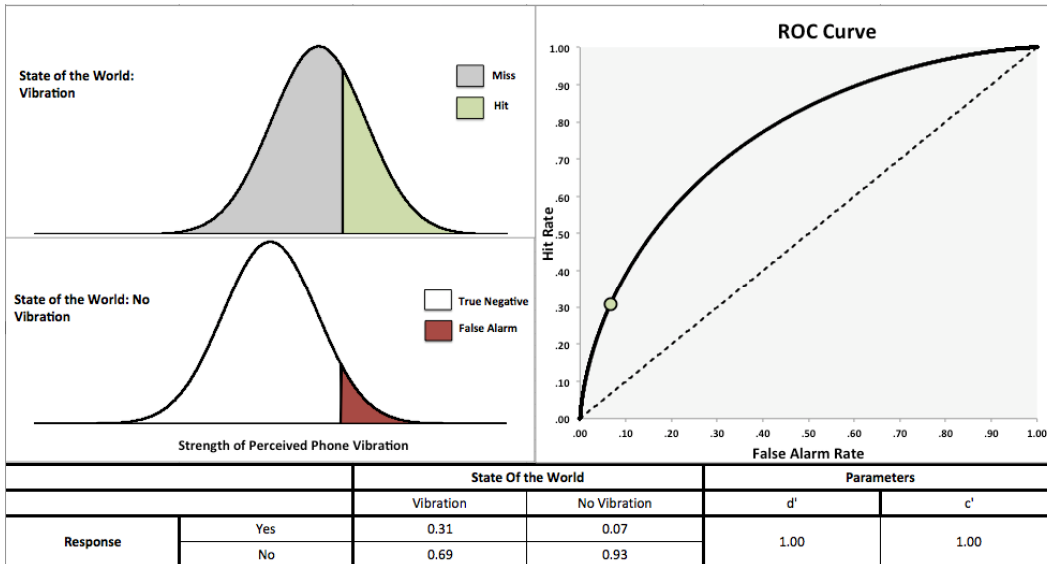|		State Of the World		Parameters	
		Vibration	No Vibration	d'	c'
Response	Yes	0.31	0.07	1.00	1.00
	No	0.69	0.93		

Figure 4: A screenshot of from the ROC and Distributions tab showing an increase in the decision criterion compared to Figure 1. d′ = 1 and c′ = 1.

Errors are due to the fact that the distributions overlap and, thus, are not completely distinguishable from each other. At a conceptual level, the

overlap of the distributions represents discriminability. As such, discriminability serves as an overall measure of accuracy independent of the decision criterion. One of the most common measures of discriminability is d′, which is computed with either of the following formulas [1]:

$$d' = \frac{\mu_v - \mu_{nv}}{\sigma} \tag{3a}$$

$$d' = \Phi^{-1}(\text{hit}) - \Phi^{-1}(\text{FA}) \tag{3b}$$

In the first formula, v and nv subscript the vibration and no vibration distributions, respectively. This formula emphasizes that d′ is the standardized difference between the two distributions. In the second equation, Φ^{-1} denotes the inverse cumulative normal function. The second equation shows how the response probabilities can be transformed into standard units. The effect of increasing d′ with c′ held constant at zero can be seen by comparing Figure 4 to Figure 5. Two important points are worth noting. First, false alarms and misses are less likely when d′ is increased because it is an over all measure of accuracy. Second, discriminability is independent of the decision criterion because false alarms and misses remain equally likely in each case. The reason for independence is that c′ was fixed at zero.
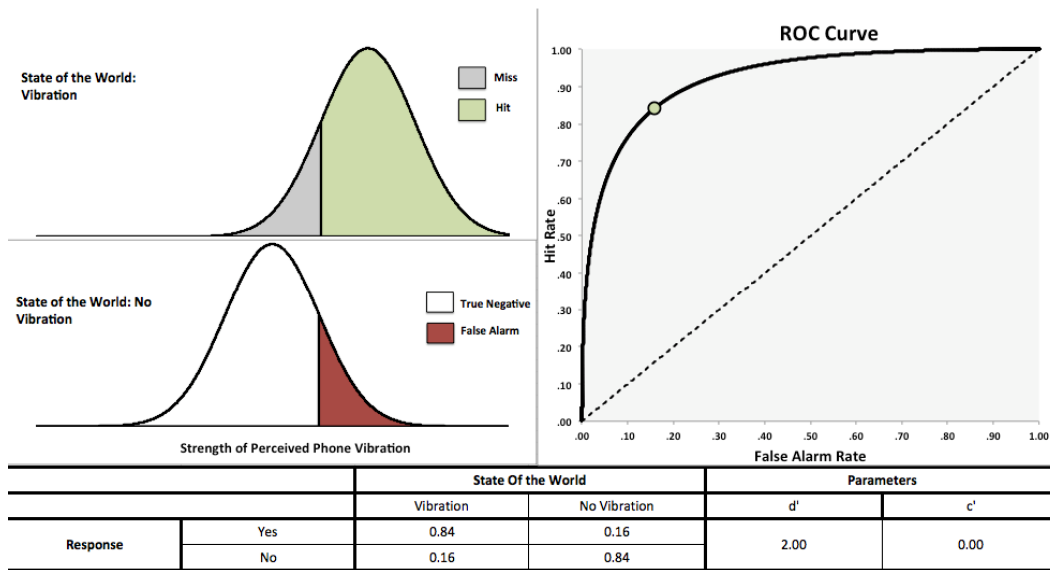

		State Of the World		Parameters	
		Vibration	No Vibration	d'	c'
Response	Yes	0.84	0.16	2.00	0.00
	No	0.16	0.84		

Figure 5: A screenshot of from the ROC and Distributions tab showing an increase in discriminability compared to Figure 1. d′ = 2 and c′ = 0.

Receiver Operating Characteristic (ROC) curves are used in SDT to evaluate performance across a range of decision criteria at a fixed level discriminability. The hit rate is plotted against the false alarm rate as a function of c′ at a fixed d′ (the solid curve). The identity line represents d′ = 0, a case in which a person cannot discriminate phantom vibrations from true vibrations. The green dot represents the hit rate and false alarm rate associated with the user defined parameters d′ and c′ in cells A45 and B45, respectively.

Ideal Observer Analysis

Although perfect performance is not possible, a decision criterion can be selected so as to optimize the balance of false alarms and misses (and true negatives and hits by extension) [1]. The optimal decision criterion will depend on the payoffs associated with each of the four possible outcomes and the prior probability (a.k.a. base rate) of true and phantom vibrations. For instance, suppose you are awaiting an important phone call about a job to which you recently applied. Missing the phone call could potentially be costly and receiving the phone call could be equally beneficial. By contrast, a false alarm and true negative would be relatively less consequential. Figure 6 shows the interface of the tab Ideal Observer Analysis. The payoff matrix in the cells spanning I33:K36 has been configured to map onto the situation described above.
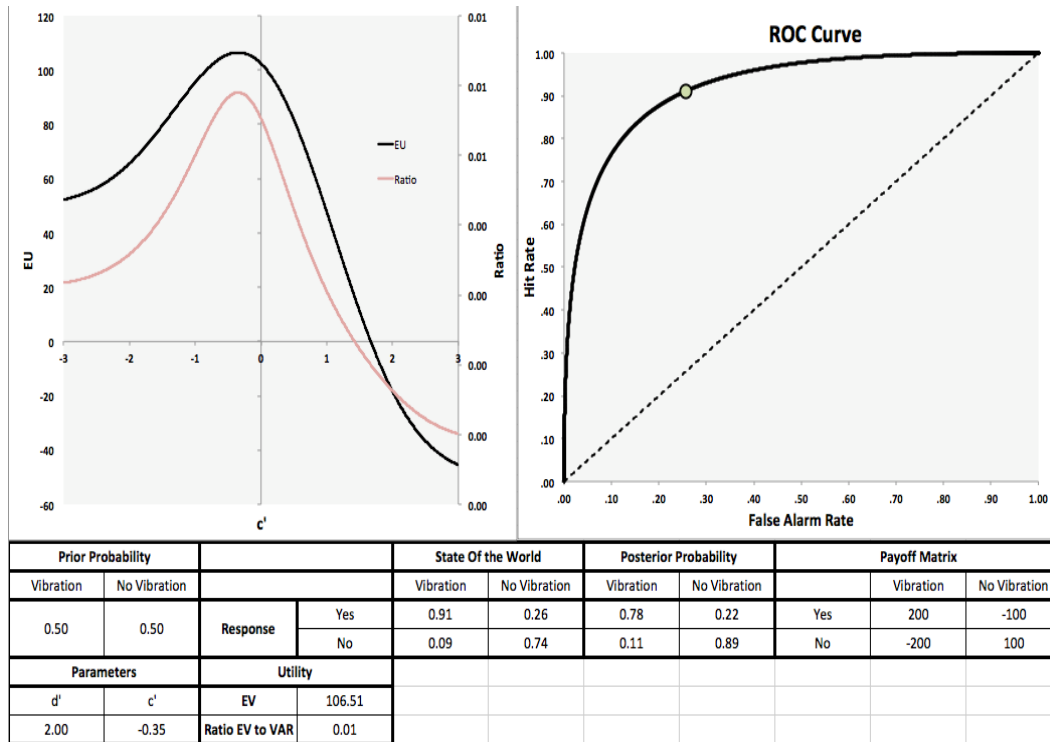
Prior Probability				State Of the World		Posterior Probability			Payoff Matrix	
Vibration	No Vibration			Vibration	No Vibration	Vibration	No Vibration		Vibration	No Vibration
0.50	0.50	Response	Yes	0.91	0.26	0.78	0.22	Yes	200	-100
			No	0.09	0.74	0.11	0.89	No	-200	100

Parameters		Utility								
d'	c'	EV	106.51							
2.00	-0.35	Ratio EV to VAR	0.01							

Figure 6: A screen shot of the tab titled Ideal Observer Analysis.

The optimal decision criterion can by found by maximizing a utility function that weights each outcome by its probability of occurring. Each outcome is associated with an economic measure called a utility that represents a person's subjective preference. In the current example, the negative utility of a miss is twice that of a false alarm. The utility function sums the probability weighted utilities to form a composite measure of the utility or "goodness" associated with a given d' and c'. Formally, the utility function is defined as:

$$EU(X|d', c') = \sum_{s \in S} \sum_{r \in R} P(x_s) P(x_r|s) U(x_{sr}) \tag{4}$$

where EU is the expected utility, $P(x_s)$ is the prior probability of state s, $P(x_r|s)$ is the conditional probability of making response r given state s, $U(x_{sr})$ is a utility function that maps objective units onto subjective units. For simplicity, I assume that utilities are equal to their objective values: $U(x_{sr}) = x_{sr}$. s and r are outcome indicator variables in which

$$S = \begin{cases} s = 1 \text{ if vibration is present} \\ s = 0 \text{ if vibration is absent} \end{cases}$$

$$r = \begin{cases} r = 1 \text{ if respond present} \\ r = 0 \text{ if respond absent} \end{cases}$$

The optimal value of c′ can be approximated through visual inspection of Figure 6, which shows the expected utility as a function of c′ (black line). Alternatively, an exact solution can be obtained using the Solver add-in. Solver uses an efficient algorithm to search for the value of c′ that maximizes the expected utility. The algorithm is efficient because it can generally arrive at a solution without exploring the parameter space in its entirety. A minimum of three settings must be specified in Solver: the objective value, whether the objective value is maximized or minimized and the parameters that are free to vary. As shown in Figure 7, the objective value is the expected utility (cell D38), which is set to be maximized, and c′ (cell B39) is the value Solver will optimize. The optimal c′ = -.35 as shown in Figure 6. Selecting a low decision criterion agrees with the intuition that misses are worse than false alarms. Therefore, more false alarms can be tolerated. One limitation of this analysis is that assuming equal prior probabilities for true and phantom vibrations is not plausible. A more realistic prior probability for the true vibration might be .85, which can be entered into cell A35. When the prior probability for a true vibration is high, a weaker perceptual signal is needed to respond "Yes" to a perceived vibration. Thus, a lower decision criterion is optimal with high prior probabilities. The optimal c′ becomes -1.21 after taking this more realistic prior probability into account.

Once a response has been made, it is possible to update the prior probability that a vibration is real, which is known as a posterior probability. This updating process is formalized through Bayes' theorem. According to Bayes' theorem:

$$P(s = 1|r = 1) = \frac{P(s = 1|r = 1)\,P(s = 1)}{P(s = 1|r = 1)\,P(s = 1) + P(s = 0|r = 1)\,P(s = 0)} \tag{4}$$

In other words, Equation 4 is the probability that a perceived vibration is true, given that one has responded that it is true. In Figure 6, the prior probability increases from .50 to .78 given that one has responded "Yes".

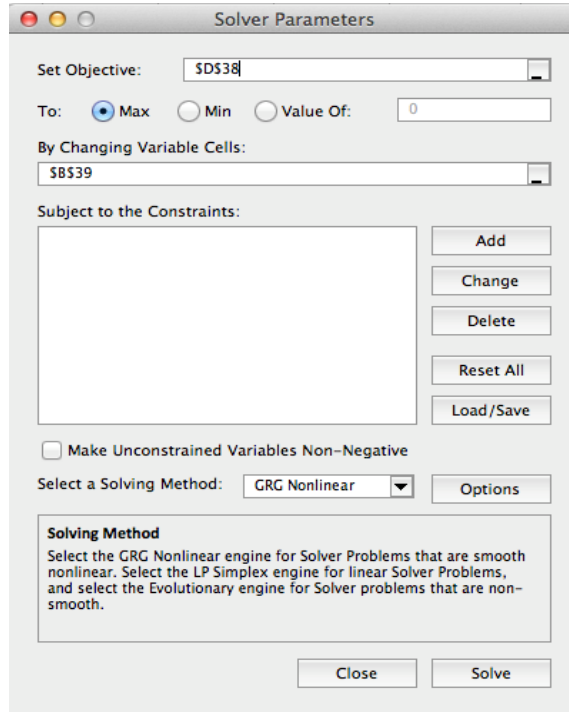Bayes' theorem can be applied in a similar fashion when one responds "No".

Figure 7: A screen shot the setup window for the Solver add-in.

One common finding is that people do not make optimal decisions due to risk aversion [9]. One way to quantify risk is to use the variability in potential outcomes associated with a decision option [9,10]. As an example, many people purchase health insurance even though it increases total healthcare expenditures on average. Purchasing health insurance allows a person to pay a "surcharge" to reduce the variability in potential healthcare expenditures, which could range from nothing to quantities that are several orders of magnitude greater than one's income. A common measure of the variability is the variance:

$$\mathrm{Var}[U(X|d',c')] = \sum_{s \in S}\sum_{r \in R} P(x_s)P(x_r|s)[U(x_{sr}) - EU]^2 \tag{5}$$

One simplified method of accounting for risk aversion is to maximize the ratio of the expected value to the variance (sometimes called the coefficient of variation). In some cases, maximizing the ratio and the

expected utility will produce the same c′, whereas in other cases they may diverge. Figure 8 shows an example in which c′ will diverge for a risk neutral and risk averse person. Simply, changing the payoff for a hit to 50 changes the optimal c′ to -.98 for a risk neutral person. In the general case, a risk averse person would set c′ < -.98. However, if a risk averse person was specifically maximizing the ratio of the expected value to the variance, c′ could equal -1.34.

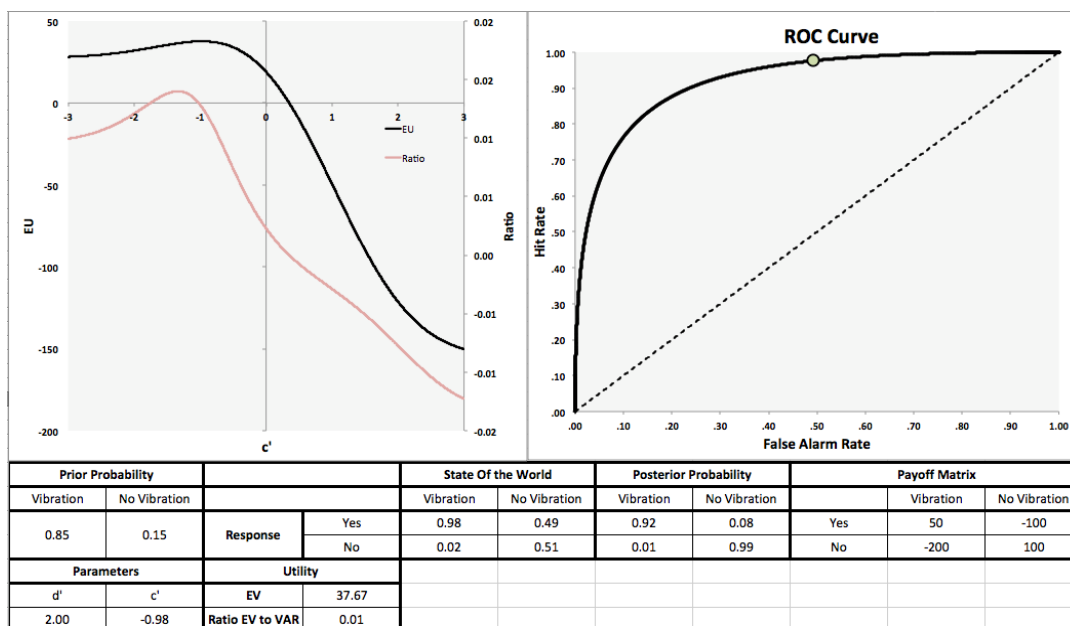| Prior Probability				State Of the World		Posterior Probability			Payoff Matrix	
Vibration	No Vibration			Vibration	No Vibration	Vibration	No Vibration		Vibration	No Vibration
0.85	0.15	Response	Yes	0.98	0.49	0.92	0.08	Yes	50	-100
			No	0.02	0.51	0.01	0.99	No	-200	100
Parameters		Utility								
d′	c′	EV	37.67							
2.00	-0.98	Ratio EV to VAR	0.01							

Figure 8: A screen shot of the tab titled Ideal Observer Analysis showing that optimal c′ can differ for risk neutral and risk averse persons.

Problems

This section provides five problems that instructors may use to teach SDT. A suggested answer is given for each problem. Problem 1 is designed to exemplify the difference between discriminability (d′) and decision criterion (c′). Problem 2 demonstrates that decision criterion (c′) balances tradeoffs between false alarms and misses. Problems 3 and 4 require students to identify which SDT parameters are likely to play a key role in different situations. The goal of Problem 5 is to provide students with practice constructing plausible payoff matrices for realistic situations. Problem 6 demonstrates that maximizing accuracy (such as posterior

probability correct) is not necessarily an ideal criterion for good performance.

Problem 1

In the tab "ROC and Distributions", set $d' = 1$ and $c' = 0$. What happens to the false alarm and miss rate in the distribution when d' is increased? What happens to the ratio of the false alarm to the miss rate?

Suggested Answer: Increasing d' decreases the false alarm and miss rates. However, the ratio remains the same when $d' = 1$ and $d' = 2$. The ratio is about 1 by visual inspection. The reason the ratio is the same is that c' is constant and represents a decision criterion that balances the tradeoff between false alarms and misses.

Problem 2

In the tab "ROC and Distributions", set $d' = 1$ and $c' = 0$. What happens to the false alarm and miss rate in the distribution when c' is increased?

Suggested Answer: Increasing c' decreases the false alarm but at the expense of more misses because c' balances false alarms and misses. When adjusting the criterion, false alarms cannot be improved without a corresponding increase in misses and vice versa.

Problem 3

Suppose that you are at a store and see a person who might be a former classmate. It has been several years since you have see your classmate and so it is unclear whether this person is your classmate or a stranger. Further suppose that you normally use glasses but you forgot to wear them. How would that change d' and c'?

Suggested Answer: Not wearing your glasses will make it more difficult to distinguish you classmate from a strange. Thus, d' would decrease. However, this would not change c'.

Problem 4

Imagine that you must adjudicate an alleged case of racial profiling involving airport security. According to the allegation, airport security has used racial profiling when deciding to conduct searches for suspected contraband. Suppose you want to use SDT to compare the security performance for searching a racial minority group and a racial majority group. Which SDT parameter(s) would indicate whether airport security is using racial profiling?

Suggested answer: lower d' for the racial minority group would simply indicate poorer discriminability. In other words, performance may be worse for detecting contraband on the racial minority group, perhaps due to cultural variation in the display of suspicion. This would not provide evidence of profiling. A decreased c' for the racial minority group would provide evidence of racial profiling because it implies that security sets a lower decision criterion for searching.

Problem 5

Suppose that you work in quality control at a company manufactures toasters. The toaster may malfunction or cause a small fire if the electrical components are faulty. For this reason, missing a defective toaster can be very costly. While false alarms are less costly, the company loses some money when functional toasters are thrown away. Construct a plausible payoff matrix for this situation. Note that the scaling of the matrix entries is somewhat arbitrary. What is most important is the relative values.

Suggested answer: Answers may vary from student to student. However, misses should be more costly than false alarms to reflect the higher cost of potential fire damage. The problem did not provide any information about hits and true negatives. One reasonable approach is to set those values equal to each other.

Problem 6

Building upon your previous answer, suppose the prior probability of a defective toaster is 1%. Find the optimal decision criterion (c') for your payoff matrix. What happens when you increase the prior probability to 5%? What happens when you maximize the posterior probability of detecting a defective toaster given a "Yes" response (cell G35) and why is that not necessarily a suitable method to optimize performance?

Suggested answer: Answers will vary from student to student but the qualitative results should be similar. Assuming the utility of a hit = true negative = 100, false alarms = -50 and miss = -1,000, the optimal decision criterion is c′ = 1.30. When the prior probability of a defective toaster increase from 1% to 5%, the decision criterion will decrease (c′ = .48). The reason is that a person should be more willing to say a toaster is defective when they tend to be defective more often in general. The c′ associated with maximizing the posterior probability of detecting a toaster is 6.57, which is much higher than the c′ = 1.30. This is a poor benchmark because it does not take into account that misses are more costly than false alarms and the prior probability of a defective toaster is low.

Conclusions

SDT provides a flexible mathematical framework for assessing the performance of a detection system (human or otherwise) and has applications in many academic and practical domains. The interactive spreadsheet provides an intuitive way to understand and visualize performance in terms of the model's key parameters: discriminability and the decision criterion. The ready-to-use format ensures that students with little knowledge of spreadsheets can benefit from the pedagogic spreadsheet presented in this paper. The spreadsheet can be easily modified to serve other purposes, such as teaching statistics in the frequentist framework. Concepts such as false alarms, hits, decision criterion and discriminability in SDT correspond to type 1 errors, power, alpha and effect size in frequentist statistics, allowing an easy transition between topics. In addition, psychologically based models, such as Prospect Theory, could be subsumed within SDT to better describe actual performance of humans [11]. For example, risk aversion could be easily incorporated into SDT with the addition of one parameter, α. For the sake of simplicity, I assumed risk neutrality in the utility function and used the properties of the payoff matrix to measure risk aversion (e.g. the ratio of the expected value to the variance). However, risk aversion can be modeled with a concave function representing diminishing sensitivity to the magnitude of the outcomes, i.e. $U(x) = x^{\alpha}$, where $0 < \alpha < 1$. This model of risk aversion is more flexible than the simple model presented in the paper, which contains no adjustable parameters. Prospect Theory also suggests that people are loss averse. In other words, losses are weighted

more than gains of equivalent magnitude: $U(x) = \lambda x^{\alpha}$ for x < 0. Solver could be used to find the value of α and λ that optimally fits the data. In conclusion, the interactive spreadsheet provides an interactive interface for learning SDT that can easily be adapted for advanced purposes.

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Appendix 1 Construction Details

ROC and Distributions

The data for the distributions are offset to columns BJ through BN for clarity of presentation. The values for the X-axis range form -5 to 5 in column BJ. The distributions were divided at the criterion, c' (Cell B45), into hits and misses and false alarms and true negatives. This results in four columns—one for each of the possible outcomes starting in column BK. For example, a true negative is defined by the area under the curve of the "No Vibration" distribution starting at c'. This was implemented with the following function: = IF(BJ2<=$CJ$2,NORM.DIST(BJ2,$CC$2,$CD$2,0)," "), which suppresses values above the absolute criterion c. c is found by solving for equation 2a: CJ2 = 0.5*E41+F41. The mean is 0 in cell CC2. The distributions were partitioned for hits, false alarms and misses in a similar manner. d' in cell E41 controls the standardized difference between the distributions. The ROC curve was found by plotting hits against false alarms across varying levels of c' in column BJ. For example, the false alarm values were computed as BO2 =1-NORM.DIST(BJ2,$CC$2,$CD$2,1) across the full range of values in column BJ. Coordinates for the identity line can found in columns BQ and BR, which range from 0 to 1. The green circle in the ROC plot indicates the hit and false alarm rate associated with d' and c' in cells E41 and F41, respectively. The hit rate and false alarm rate were computed with the cumulative normal distribution functions: C41 = 1-NORM.DIST(CJ2,E41,1,1) and D41 = 1-NORM.DIST(F41+E41*0.5,0,1,1), respectively.

Ideal Observer Analysis

The ROC curve was generated using the same procedures described in the previous section. The other graph shows the expected utility and its standard deviation as a function of c'. The data for this graph is offset to columns BT through BX. Column BT contains c' values ranging from -3 to 3. Using a similar procedure as described in the previous section, columns BW and BX contain false alarm and hit rates as a function of c' in column BT. The following formula computes the expected utility based on Equation 3 as a function of the c', the false alarm rate and the hit rate: BV2 = $A$35*($J$35*BX2+$J$36*(1-BX2))+$B$35*($K$35*BW2+$K$36*(1-BW2)).

This formula references the prior probabilities (A35 and B35) and appropriate values of the payoff matrix (J35:K36). Column BU contains the ratio of the expected utility and its variance as a function of c′, the false alarm rate and the hit rate and can be computed with the following formula based on Equation 4: BU2 = BV2/($A$35*BX2*(BV2-$J$35)^2+$A$35*(1-BX2)*(BV2-$J$36)^2+$B$35*BW2*(BV2-$K$35)^2+$B$35*(1-BW2)*(BV2-$K$36)^2)). Cell AG2 computes the exceed utility associated with the user defined d′ and c′ in cells A39 and B39, respectively. The expected utility can be maximized by optimizing cell B39 in the Solver Add-in. The prior probabilities can be entered in cell A35. The posterior probabilities in cells G35:H36 are computed through Bayes' theorem, e.g. =(A35*E35)/(A35*E35+B35*F35).