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Modelling harvesting animal population at high school with spreadsheets – the case of Moby Dick

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Abstract
The paper gives the results of an experiment in which harvesting the world population of the Sperm whale was modelled and analyzed with 27 high school students of age 18-19 by using the Euler method of solving differential equations in Excel. A questionnaire survey was carried out to find out the opinion of the students of the lesson. The results are discussed.

Keywords
differential equation, Euler method, Excel, harvesting model, species extinction

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1 Introduction

Imagine an animal species that lives in a closed territory with renewable but limited resources. Let a constant number of individuals be taken away at times. Will the species survive or become extinct?

The problem is topical. Many animal species became extinct by human activity long time ago, for example the Brown bear in the UK [1], but some have become extinct just recently, for instance the Western black rhinoceros [2]. Some species were saved at the last moment, for example the American bison [3], and most of the whales in 1986 when commercial whaling was banned [4]. However, there are many species threatened by extinction, yet still hunted or caught, for instance more than 40 species of fish in the Mediterranean Sea could disappear in few years [5]. Mathematical models illustrate well how overhunting causes species to become extinct after some time while judicious approach to harvesting enables them to survive.

The problem is governed by the equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1-\frac{x}{K}\right) - h\,,\ x(0) = x_0\,, \tag{1}$$

where $x(t)$ is the number of individuals at time $t > 0$, x_0 is the number at $t = 0$, r is the growth rate, which is the change in the number of individuals per year and capita at unlimited supply of food, K is the carrying capacity, which is the maximum number that the territory can provide with food, and h is the harvesting rate, which is the number of individuals that is annually taken away. If $h = 0$, then Eq. (1) reduces to the logistic growth equation [6] – [10]. Analytical solutions are in [11], [12]. A numerical model in Excel is in [13].

The paper gives the results of an experiment in which a simple numeric Excel model was developed with 27 last year gymnasium (high school) students of age 18-19 in a 90 minute lesson. The model applies the Euler method of solving differential equations, which is simple enough to be understood at that level. Creating the model only requires basic spreadsheet

skills; no programming is used. Harvesting the world population of the Sperm whale was modeled. The species is the largest toothed predator. It is well-known for Moby Dick, a large white male depicted by Hermann Melville in his famous novel "Moby Dick; or, The Whale" [14]. The novel is based on the true story of whaler Essex sunk by a sperm whale in Pacific in 1820 [15]. A questionnaire was given to the students to find out their opinion of the lesson. The results are discussed.

2 Numerical solution

Replacing the differentials in Eq. [1] by differences and rearranging gives

$$\Delta x = \left[rx\left(1-\frac{x}{K}\right) - h \right] \Delta t \,, \tag{2}$$

$$x = x + \Delta x \,,\ x(0) = x_0 \,. \tag{3}$$

Eq. (2) and (3) enable to graph $x(t)$. The Excel application is in Fig. 1.

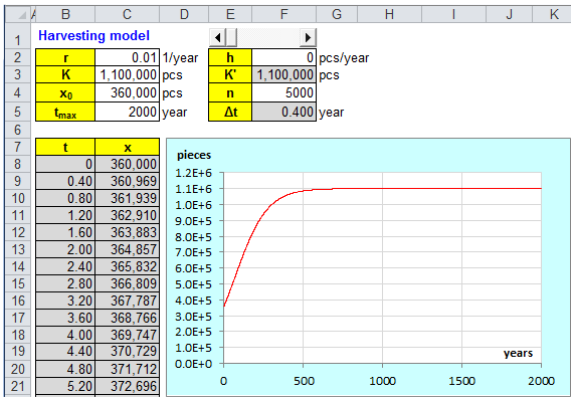


Figure 1 Sperm whale harvesting if x_0 = 360,000 and h = 0

The inputs are in the white cells. The grey cells contain formulas. Growth rate r is in cell C2, carrying capacity K is in cell C3, number x_0 of animals at the start is in cell C4, the time range is in cell C5 and harvesting rate h is in cell F2. Cell F3 contains the formula = C5008 that gives the last calculated value of x. The calculations are made over n = 5,000 points (cell F4). Cell F5 contains the time increment for each calculation Δt = C5/F4. Cell B8 contains =0. Cell B9 contains the formula =B8+$F$5, which is filled down as far as row 5008. Cell C8 contains =C4. Cell C9 contains the formula =C8+($C$2*C8*(1-C8/$C$3)-$F$2)*$F$5, which is filled down as far as row 5008. The harvesting rate is governed with a scrollbar. Its properties Min, Max, SmallChange, LargeChange and LinkedCell are set to 0, 25000, 1, 100 and F2.

3 Harvesting the Sperm whale population

According to the IUCN Red List [16], it holds for the Sperm whale that $r = 1\,\%$, $K = 1{,}100{,}000$, which is the estimated number of the species before year 1700 when commercial whaling started, and $x_0 = 360{,}000$, which is the estimated number today. In Fig. 1, there is no harvesting. After long time, number x stabilizes at K. That is the logistic growth model [9]. The model shows that it will take 531 years to reach 99 % of the number before whaling, that is, K. Variations in population size can be explored by changing rate h and time t_{max}. In Figs. 2a and 2b, rate h is small enough to enable the species to survive; number x stabilizes at $K' < K$. In Figs. 2c – 2f, rate h is too large and the species becomes extinct after some time. It can be seen in Figs. 2b and 2c that the maximum h to survive is 2,421 as $h = 2{,}422$ causes extinction after 2366 years. If $h = 5{,}000$, the species would vanish after 104 years. Whaling was particularly intense around 1950 when about 25,000 sperm whales were killed per year [16]. The species would vanish after 15 years, then (Fig. 2f).

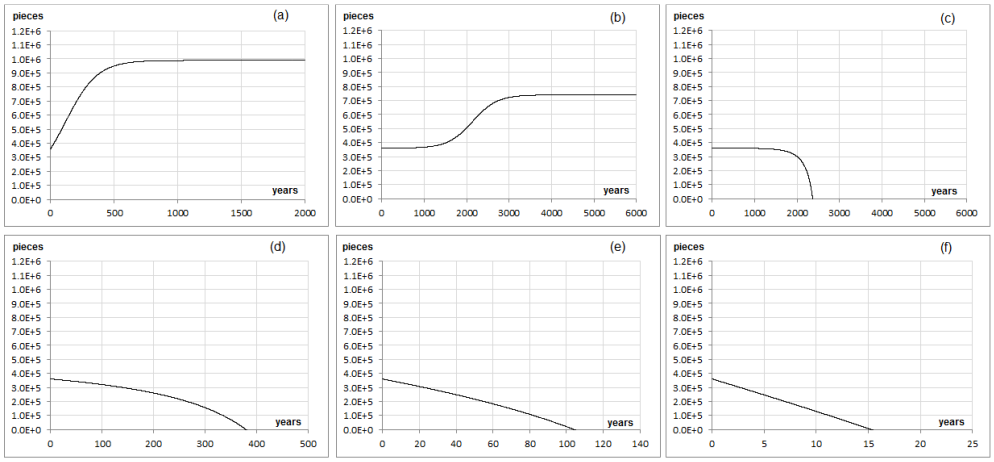


Figure 2 Sperm whale harvesting if $x_0 = 360{,}000$: a) $h = 1{,}000$, $K' = 988{,}748$; b) $h = 2{,}421$, $K' = 740{,}237$; c) $h = 2{,}422$, $t_{ex} = 2366$; d) $h = 2{,}750$, $t_{ex} = 379$; e) $h = 5{,}000$ $t_{ex} = 104$; f) $h = 25{,}000$ $t_{ex} = 15$

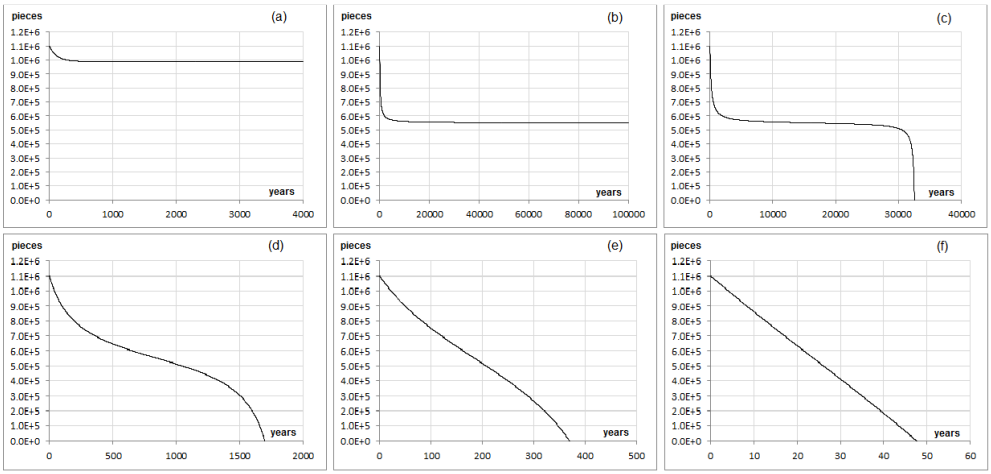


Figure 3 Sperm whale harvesting if $x_0 = 1{,}100{,}000$: a) $h = 1{,}000$, $K' = 988{,}748$; b) $h = 2{,}750$, $K' = 550{,}000$; c) $h = 2{,}751$, $t_{ex} = 32{,}549$; d) $h = 3{,}000$, $t_{ex} = 1695$; e) $h = 5{,}000$, $t_{ex} = 369$; f) $h = 25{,}000$, $t_{ex} = 48$

What if the whaling started now and not in 1700? The case is analyzed in Fig. 3. It holds that $x_0 = K$. The maximum rate h to survive is 2,750 pieces a year as h = 2,751 causes extinction after 32,549 years. If h = 5,000, the species would become extinct after 369 years. If h = 25,000, the species would vanish after 48 years.

4 School experiment

The Excel application was developed with three groups of last year gymnasium (high school) students of age 18-19. There were 27 students altogether. The first group were 9 students of Gymnazium Parovska 1 in Nitra in optional subject Informatics, which is a follow-up to the compulsory subject Informatics taught in the previous years. The second group were 3 students of Gymnazium in Nove Zamky, members of the school computer club. The third group were 14 students of Gymnazium in Surany in subject Applied Informatics, which is another optional follow-up to compulsory subject Informatics. All students were familiar with writing formulas and graphing in Excel. A 90 minute lesson was taught to each group. The goal was to let the students experience a lesson of computer modelling, and promote STEM disciplines. The lessons were taught by the first author.

The lesson started with a discussion about whales and whaling [17], [18]. The students were surprised to hear that the story of Moby Dick is based on the true story of whaler Essex that was sunk by a sperm whale in the southern Pacific in 1820 [15], [19]. Then, the Excel application was shown by a projector to motivate the students. Equation (1) was presented. Parameters r, K, x_0 and h were explained. Equations (2) and (3) were presented. It was pointed out that they allow calculating the new value of x by using the previous one, that is, by using recursion. The high number of steps, 5000, was argued by the fact that the accuracy of the solution depends on the length of the time step – a smaller time step gives higher accuracy. The algorithm of the numerical solution was discussed in detail. The students downloaded the template (the application in Fig. 1 with empty grey cells and no chart) from the author's website and developed the application following the steps written in section 2. Each step of creation was discussed. The author was developing the application along with the students and projecting it to give feedback. The author worked with a delay so that the clever student could work individually.

After completing the application, the students were given the following tasks to solve:

1) *When will be reached 99% of K (suppose no harvesting)?*
2) *What is the maximum h to survive if there is harvesting?*

3) *How long would the species live if that maximum h was exceeded?*
4) *When would the species become extinct if h = 5000?*
5) *What would be the maximum h to survive if whaling started now, that is,* $x_0 = K$ *?*
6) *How long would the species live if that maximum h was exceeded?*
7) *When would the species become extinct if h = 5000?*
8) *When would the species become extinct if h = 25,000 just like at the peak of whaling?*

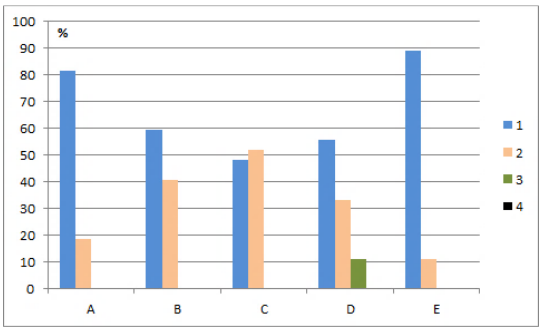
The solutions are in Figs. 1, 2 and 3. At the end of the lesson, the students were given the following questionnaire, and they were asked to mark the options that meet their opinion:

A) The lesson was (1: very; 2: quite; 3: little; 4: not) interesting
B) I understood (1: everything; 2: majority; 3: minority; 4: nothing)
C) I learned (1: very much; 2: quite much; 3: little; 4: no) new skills in Excel
D) I learned (1: very much; 2: quite much; 3: little; 4: no) new in biology/ecology
E) I would like to model a predator-prey system (1: yes; 2: no)

The absolute frequency, average and standard deviation of the answers is in Tab. 1. The relative frequencies are graphed in Graph 1.

Table 1. The absolute frequency, average and standard deviation of the answers

	1	**2**	**3**	**4**	**Avg**	**Std**
A	22	5	0	0	1,2	0,39
B	16	11	0	0	1,4	0,49
C	13	14	0	0	1,5	0,50
D	15	9	3	0	1,6	0,68
E	24	3			1,1	0,31



Graf 1. Relative frequency of the answers

The result is that: (A) 81% of the students found the lesson very interesting and 19% quite interesting; (B) 59% understood everything while 41% understood the majority; (C) 48% learned very much new in Excel while 52% learned quite much; (D) 56% learned very much new in biology/ecology, 33% learned quite much and 11% learned little; (E) 89% would like to continue with the predator-prey model while 11% not.

5 Conclusions

The paper gave the results of an experiment in which a numeric model of harvesting the world population of the Sperm whale was developed with 27 gymnasium (high school) students of age 18-19. The model applied the Euler method of solving differential equations. A questionnaire was given to the students to find out their opinion of the lesson. The main result is that 100% of the students found the lesson interesting and 89% would like to continue with developing a predator-prey model. That implies that modelling dynamic systems with spreadsheets by using the Euler method of solving differential equations has a potential in promoting sciences and applied mathematics at the pre-university level.

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