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101 Applications for a Binary Table and a Spreadsheet

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The purpose of this paper is to show how to create a binary table in Excel and to give applications of a varied nature using the binary table to generate data on which mathematical investigations can be based. Elementary applications considered are the binary form of an integer, the concept of power set, and sampling distributions.

Keywords

spreadsheets, mathematical investigation, binary table, STEM problems, truth table, binary to decimal conversion, power set, binomial coefficients, sampling distribution, subset sum problem

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Abstract

The purpose of this paper is to show how to create a binary table in Excel and to give applications of a varied nature using the binary table to generate data on which mathematical investigations can be based. Elementary applications considered are the binary form of an integer, the concept of power set, and sampling distributions.

Keywords: spreadsheets, mathematical investigation, binary table, STEM problems, truth table, binary to decimal conversion, power set, binomial coefficients, sampling distribution, subset sum problem.

Background

This paper describes some of the uses of a *binary table*, sometimes called a *truth-table*, in the modern spreadsheet environment. The aim is to generate and to lay bare the structure of several mathematical objects. An example of a binary table is shown in Figure 1. Its purpose there is to simply construct the integers 0 to 15 from their binary forms. In the spreadsheet environment, the potential for learning offered by this fundamental *combinatorial object* [7] (binary table) is vast. In this paper, we consider several examples of the binary table. Each is a mathematical investigation in its own right.

To create a small instance of the table (4 columns and 16 rows), we use the following steps.

1. Rename Sheet 1 as **Binary Table**.
2. The binary powers 8, 4, 2 and 1 respectively are entered into range C2:F2. These powers are given a suitable collective name, such as *bins*, and should be regarded as a heading line for the table, rather than part of the table itself.
3. To create this name, select the range C2:F2 and type the name *bins* in the Name Box, followed by <Enter>.
4. Now put a row of *zeroes* in C3:F3 to create the first row of the table.
5. Fill the range B3:B18 with the integers 0 to 15. This is just a count, but these values should match what we will place in column G in Application 1.

	A	B	C	D	E	F	G
1							
2			8	4	2	1	
3		0	0	0	0	0	
4		1	0	0	0	1	
5		2	0	0	1	0	
6		3	0	0	1	1	
7		4	0	1	0	0	
8		5	0	1	0	1	
9		6	0	1	1	0	
10		7	0	1	1	1	
11		8	1	0	0	0	
12		9	1	0	0	1	
13		10	1	0	1	0	
14		11	1	0	1	1	
15		12	1	1	0	0	
16		13	1	1	0	1	
17		14	1	1	1	0	
18		15	1	1	1	1	
19							

Figure 1: A simple binary table

6. Add the formula¹ (F4) = 1 – F3.
7. Fill this formula down from F4 to F18. This will produce a sequence of alternating zeroes and ones in Column F and give us the rightmost column of the binary table.
8. For each cell in the interior of the table, adopt the following rule: If the value in the cell to your right has not decreased, then just copy the value above, else complement the bit above. To get a prototype of this, add the formula (E4) = IF(F4 >= F3, E3, 1 – E3).
9. Now fill this last formula throughout the range C4:E15.
10. If desired, apply conditional formatting to highlight the 1s in the binary table that has the range C3:F18.

We now have a small instance of a binary table and will extend it to illustrate various applications in the sections to follow. This table and spreadsheets that cover the remaining applications can be found in the accompanying spreadsheet, *101 Uses for a Binary Table.xlsx*.

Application 1: Binary to decimal

A basic distinction exists between the concept of *number*, which is abstract, and a *representation* of that number, which is (usually) a concrete string of characters. Fundamental as this distinction is, few published works make reference to it, with many referring incorrectly to “binary numbers”, instead of the more precise expressions: “binary form”, “binary representation” or even “binary string”. In fact, it is surprisingly difficult to find a satisfactory presentation of these basic concepts; one which comes close is that of Hext [1, p32].

Many representations exist for a given number. For example, the number “fifteen” may be represented as 15 (decimal), 1111 (binary), 17 (octal), F (hexadecimal), XV (Roman), and in many other ways and forms. Thus, to be precise, since a number is independent of any string of characters, glyphs or pictograms used to represent it, we should not speak of a “binary number” or “decimal number”, but rather “binary form” or “decimal form”. Another good alternative is “decimal string” or “binary string”, the latter often abbreviated to “bitstring”.

Knowing how to compute these various representations and to convert between them is an important part of one’s mathematical education, especially if further studies in computer science, engineering or related fields are planned.

We present two methods of deriving the common decimal form of an integer, given its bitstring representation (binary form). Our typical binary form is in one of the rows 3 through 18, and ranging from column C to column F. Note that the rows of our binary table just constructed enumerate all possible bitstrings of length 4.

¹ We use brackets to indicate the cell in which a formula or data is to be added. For example, (F4) = 1 – F3 should be read as “In cell F4, type the formula =1 – F3”.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2			8	4	2	1	SUMPRODUCT	Horner's Method				
3		0	0	0	0	0		0	0	0	0	0
4		1	0	0	0	1		1	0	0	0	1
5		2	0	0	1	0		2	0	0	1	2
6		3	0	0	1	1		3	0	0	1	3
7		4	0	1	0	0		4	0	1	2	4
8		5	0	1	0	1		5	0	1	2	5
9		6	0	1	1	0		6	0	1	3	6
10		7	0	1	1	1		7	0	1	3	7
11		8	1	0	0	0		8	1	2	4	8
12		9	1	0	0	1		9	1	2	4	9
13		10	1	0	1	0		10	1	2	5	10
14		11	1	0	1	1		11	1	2	5	11
15		12	1	1	0	0		12	1	3	6	12
16		13	1	1	0	1		13	1	3	6	13
17		14	1	1	1	0		14	1	3	7	14
18		15	1	1	1	1		15	1	3	7	15
19												

Figure 2: Binary to Decimal

The steps to build the worksheet of Figure 2 are:

Method 1. We use the SUMPRODUCT function as follows.

1. Right click the Binary Table worksheet tab and select the *Move or Copy ...* option and make a copy of the sheet at (*move to end*). Rename this new worksheet as *Bin to Dec*. Select the range C2:F2 and name these cells as *bins*.
2. Add the formula (G3) = SUMPRODUCT(*bins*, C3:F3). Then fill down to G18 to complete the calculations. Not much effort was required here, and good use was made of the utility function SUMPRODUCT. The values in B3:B18 should correspond to those in G3:G18, namely, the integers 0 through 15.

Method 2. This method does not require the powers of two in row 2, but generates them implicitly and on-the-fly. It is sometimes known as *Horner's method* [8] of polynomial evaluation. Here are the steps.

1. Add the formula (H3) = C3.
2. Add the formula (I3) = 2*H3 + D3 and fill across to K3. This is the fundamental recursion for Horner's method to evaluate a polynomial $P(x)$ whose coefficients are the bits and for $x = 2$.
3. Select the range H3:K3 and fill down to row 18.

The range K3:K18 will now have integers 0 through 15, as before.

Application 10: Power set

The collection of all subsets of a given set (including the empty set and the full set itself) is known as its *power set* [1, p3]. For example, the power set of $\{a, b, c\}$ is:

$$\{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}.$$

In the previous section, we lamented the rarity in the literature of the clear and lucid distinction between the concept of *number* and its representation. A similar situation appears to exist with the strong connection between the concept of *power set* and its close connection with binary tabulation, i.e., not many works dealing with these topics point out the connection. Perhaps it is thought to be too trivial to mention? If so, we strongly disagree! A key property of the binary table as described above is that every row of the table selects a *unique subset* of the numbers above the table. For a given row, the 1s may be regarded as *selector bits*, i.e., if a bit is 1, then the corresponding element of the original set is included in the subset for that row. Instead of using binary powers for our heading row, we could have used a different set of numbers or letters, such as $S = \{a, b, c, d\}$, and the binary table will then generate the power set of S , i.e. the set of all subsets of S . For example, the first four rows of the binary table generate the subsets $\{\}, \{d\}, \{c\}, \{c, d\}$. This feature can be used to make each cell in column G contain some function of a subset (for example, the sum, mean or variance) of the numbers in S and it is this property that is used in the 101 applications that are covered in this article.

The cardinality (number of elements) of the power set of S grows exponentially as a function of the cardinality of S . A set of n elements has a power set of 2^n elements. So, only sets of rather modest size may be investigated. For example, a set of 16 elements will have $2^{16} = 65,536$ subsets. Even though Excel allows for $2^{20} = 1,048,576$ rows, we do not recommend building spreadsheets of that size. Even so, a set of cardinality 16, accompanied by its 65,536 subsets, is more than enough to use the spreadsheet model as a very valuable tool to gain important insight into several fundamental mathematical structures. In the following, we settle for a set of 8 elements, leading to 256 subsets.

We now use our binary table to generate subsets. Before doing so, we double our number of columns to 8. This raises our row count to 256 but has the advantage of providing a reasonable basis for a later application (sampling distribution).

Here are the steps to create the worksheet of Figure 3.

1. As before, make a copy of the *Binary Table* worksheet and rename it as *Power Set*.
2. Extend the numbers in column B down to 255 (cell B258).
3. Insert 4 extra columns before column C (between columns B and C).
4. Extend the pattern 8, 4, 2, 1 in row 2 back to 128, 64, 32, 16, 8, 4, 2, 1 in C3:J3 and name the range *binaries*.
5. Extend the pattern of zeroes in row 3 to the range C3:J3.
6. Extend the pattern of alternating zeroes and ones in column J down to J258.
7. Replicate the formula in I4 down and across the range C4:I258. You should now have a binary table of 8 columns and 256 rows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1			a	b	c	d	e	f	g	h									
2			128	64	32	16	8	4	2	1	Elements							Subset	
3		0	0	0	0	0	0	0	0	0									{}
4		1	0	0	0	0	0	0	0	1								h	{h}
5		2	0	0	0	0	0	0	1	0								g	{g}
6		3	0	0	0	0	0	0	1	1								g, h	{g, h}
7		4	0	0	0	0	0	1	0	0								f	{f}
8		5	0	0	0	0	0	1	0	1								f, h	{f, h}
9		6	0	0	0	0	0	1	1	0								f, g	{f, g}
10		7	0	0	0	0	0	1	1	1								f, g, h	{f, g, h}
11		8	0	0	0	0	1	0	0	0								e	{e}
12		9	0	0	0	0	1	0	0	1								e, h	{e, h}
13		10	0	0	0	0	1	0	1	0								e, g	{e, g}
14		11	0	0	0	0	1	0	1	1								e, g, h	{e, g, h}
15		12	0	0	0	0	1	1	0	0								e, f	{e, f}
16		13	0	0	0	0	1	1	0	1								e, f, h	{e, f, h}
17		14	0	0	0	0	1	1	1	0								e, f, g	{e, f, g}
18		15	0	0	0	0	1	1	1	1								e, f, g, h	{e, f, g, h}

Figure 3: Creating the Power Set

8. Place the elements of our set in cells C1:J1. At present, just use the letters *a* through *h* to represent the elements of *S*. They can be replaced with numbers later on, when we look at sampling distributions.
9. Now we use the bits of the binary table as selectors to choose our subset elements. Add the formula (K3) = IF(C3 = 1, C\$1,"") and fill it across and down to span the range K3:R258. Note the partial absolute reference here. The column reference is relative so that each potential element is considered for membership, and the row is absolute because we need always to get our elements from row 1.
10. Place the heading *Elements* in K2 and "merge and center" this across the range K2:R2.

In columns K through R, we now have, in each of the rows of the table, elements corresponding to each of the subsets. Perhaps we want to see our subsets written as text in a cell, in the normal manner, as in $\{a, b, c, d\}$. If so, then we need to concatenate across columns K through R. It would be nice if we could put a formula (S3) = CONCATENATE(K3:R3) and then fill down to T258. Unfortunately, Excel does not have a "range concatenate" function. We might also like the eye candy of separating commas for our subset elements. There are several possible workarounds for these problems, but here is one that uses simple text formulas.

Method 1

1. Change the formula in K3 to be

$$(K3) = \text{IF}(C3 = 1, C\$1 \& \text{IF}(\text{SUM}(D3:\$J3) > 0, ", ", ""), "")$$
 where the first part places the symbol *a* in K3 if there is a 1 in C3 of the binary table and a comma is added if there are any bits in table that are to the right of C3.
2. Copy this formula across to Q3 and leave the formula (R3) = IF(J3 = 1, J\$1, "") unchanged.

3. To complete the first row, use the ampersand symbol "&" for joining two strings by adding the formula (S3) ="{" & K3 & L3 & M3 & N3 & O3 & P3 & Q3 & R3 & "}". Select the range K3:R3 and fill this down to S258.

You will see that in row 258, the complete set $\{a, b, c, d, e, f, g, h\}$ is correctly listed. At this point, column S should contain a nicely-formatted list of the 256 subsets of our eight objects in the range C1:J1. Being a spreadsheet, you can now change these values, replacing a with x for example, and everything will be recomputed.

Method 2

A different, but possibly a more elegant, workaround is to write a VBA function tailored to our purpose. The following function does the job, and is found with the Excel model accompanying this paper.

```
Public Function concatRange(rng As Range) As String
Dim c As Range, card As Integer, length As Integer
Const separator = ", "
concatRange = "{"
card = 0
For Each c In rng.Cells
If c.Text <> "" Then
card = card + 1
concatRange = concatRange + c.Text + separator
End If
Next
If card > 0 Then ' remove trailing separator
length = Len(concatRange)
concatRange = Left(concatRange, length - 2)
End If
concatRange = concatRange + "}"
End Function
```

Note: This code cannot be used in the Mac environment as the Mac does not allow VBA macros to be added to a spreadsheet.

Follow these steps to include this code for Excel on a Windows machine.

1. Make a copy of the *Power Set* worksheet and leave it named as *Power Set (2)*.
2. Change the formula in K3 back to (K3) = IF(C3 = 1,C\$1,"") and fill this into the range K3:R258. This will remove the commas in the list of elements, but the subsets in column S will no longer be comma separated. Select and delete column S as we will now use the above code to replace the previous data.
3. Copy the above code.
4. Hit <Alt-F11> to go to the VBA environment and choose **Insert: Module**. Then paste the code into the new module. Choose **File: Close and Return to Excel**.
5. Place the heading "Subset (VBA)" in S2.
6. Place the formula (S3) = concatRange(K3:R3) and fill down to S258.

At this point, column S should contain a nicely-formatted list of the 256 subsets of our eight objects in the range C1:J1 matching those in Column S of the *Power Set* worksheet.

Application 11: Binomial coefficients

Also strongly connected to notions of binary, subsets and power sets are the so-called *binomial coefficients* [2]. Suppose that our original set has size (cardinality) N . We consider subsets of cardinality n , where $0 \leq n \leq N$. It is well-known that the number of such subsets is given by the *binomial coefficient*:

$$\binom{N}{n} = \frac{N!}{n!(N-n)!}$$

This is usually pronounced “ N choose n ” as it corresponds to the number of ways of choosing n objects from N without replacement, i.e., choosing a subset. In Excel, such binomial coefficients may be obtained by using the function COMBIN(N , n).

By building on our power set model, we may also compute the coefficients by counting in Excel. Here are the steps to create the worksheet of Figure 4.

1. Copy the binary table B2:J258 in the *Power Set* sheet and paste it into the same place in a new worksheet named *Binomial Coeffs*.
2. Add a formula to compute the cardinality of each subset in column K. This is as simple as can be: we merely sum the binary values in columns C through J in each row. Add the formula (K3) = SUM(C3:J3) and then fill down to K258. Add the heading *Card* (for cardinality) in K2.
3. Give the name *card* to the range \$K\$3:\$K\$258.
4. Before we begin to accumulate statistics regarding the subsets, add the values 0 through 8 in the range M3:M11. Add a heading *subset cardinality* above this range.
5. Give the name *subsetCard* to the range \$M\$3:\$M\$11.
6. Add the formula (N3) = COUNTIF(card, subsetCard). This counts all subsets of cardinality 0. There is only one (the empty set), so the value returned will be 1. Fill this formula down to N11 to produce the binomial coefficients.
7. We can verify that the subset counts are correct by placing the formula (O3) = COMBIN(8, subsetCard). Fill this down to O11. Add a heading *check COMBIN* above O3. The numerical values in columns N and O should match up exactly. They represent the number of different subsets of each separate cardinality from 0 to 8.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1														subset	# of	check
2			128	64	32	16	8	4	2	1	Card			cardinality	subsets	COMBIN
3		0	0	0	0	0	0	0	0	0	0			0	1	1
4		1	0	0	0	0	0	0	0	1	1			1	8	8
5		2	0	0	0	0	0	0	1	0	1			2	28	28
6		3	0	0	0	0	0	0	1	1	2			3	56	56
7		4	0	0	0	0	0	1	0	0	1			4	70	70
8		5	0	0	0	0	0	1	0	1	2			5	56	56
9		6	0	0	0	0	0	1	1	0	2			6	28	28
10		7	0	0	0	0	0	1	1	1	3			7	8	8
11		8	0	0	0	0	1	0	0	0	1			8	1	1
12		9	0	0	0	0	1	0	0	1	2			256	256	
13		10	0	0	0	0	1	0	1	0	2					
14		11	0	0	0	0	1	0	1	1	3					
15		12	0	0	0	0	1	1	0	0	2					
16		13	0	0	0	0	1	1	0	1	3					
17		14	0	0	0	0	1	1	1	0	3					
18		15	0	0	0	0	1	1	1	1	4					

Figure 4: Binomial Coefficients

- The binomial coefficients should add up to 256. Verify this by adding the formulas (N12) = SUM(N3:N11) and (O12) = SUM(O3:O11). You should see two values of 256 side-by-side.

Application 100: Sampling distributions

In the elementary study of statistics, a *sample* is defined to be a non-empty subset of the *population*, which is the set of all possible values. Any quantity computed from values in a sample is called a *statistic*. For a small set of numbers, we have already seen how to generate its power set in Excel. Regarding this set as a population, it is not too difficult to extend our model to generate statistics like *mean* and *variance* for each of the subsets (samples). For a given statistic, the collection of all such values, for all possible samples of a given size, is called the *sampling distribution* of that statistic [3].

Thus, assuming the population values are all known, we may generate all subsets of cardinality *n* and then compute the sample mean for each possible sample. These are easily tabulated in Excel. Computing subsets implies we are sampling *without replacement*.

One of the basic results of sampling theory is that, regardless of sample size *n*, the mean of the distribution of sample means, called the *sampling distribution*, is equal to the population mean. Our model will verify this result, which is normally written as:

$$\mu_{\bar{x}} = \mu$$

Secondly, the variance of the sampling distribution is that of the population divided by *n*. We do not provide this model, but leave it as an exercise. The result may be written as:

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$$

To look at a particular sampling distribution, we first need to fix a non-zero sample size. We have a set of cardinality *n* = 8, so we may consider separate sampling distributions

corresponding to so-called *proper subsets* which are those that have of cardinality 1 through 8. Here are the steps to create the worksheet of Figure 5.

1. Make a copy of the *Binomial Coeffs* worksheet and name it as *Statistics*. Delete Columns M, N and O.
2. Add the numbers 1 – 8 in cells C1:J1; these will form the elements of our set. These are arbitrary, and other numbers may be used.
3. We now compute the mean of each sample (non-empty subset). In order to do so, we need to exclude the empty set, which has cardinality zero. To facilitate this, define the name *subsetCard* to refer to the range \$K\$4:\$K\$258.
4. Add the formula (L3) = IF(C4 > 0, C\$1, "") and extend this to S3 in order to make a list of the elements in the first non-empty set. Fill this formula down to the range L3:S258 to make a list of the elements of each subset and make (L2) *Elements* which can be centred across L2:S2.
5. Add the formula (T4) =AVERAGE(L4:S4) and fill down to T258. Add a heading *sample mean* above these. Name the range \$T\$4:\$T\$258 as *samplemean*. We now have the mean for each possible sample (non-empty subset) in column T.
6. In Column U, add the numbers 1 – 8 to the range U4:U11 and name these as *nonzeroCard*. Add the title *Subset Cardinality* at the top of Column U.
7. Now we are in a position to compute the mean of each of the sampling distributions of means; one corresponding to each of the positive values in column U. To this end, add the formula (V4) = AVERAGEIF(subsetCard, nonzeroCard, samplemean). The function AVERAGEIF is very useful in that it selects only those sample means corresponding to the current sample size in column U. Above V3 add a heading *mean of sample means*.
8. Add the formula (W3) = AVERAGE(C1:J1) to get the population mean. This should be the same as each of the means of sample means just computed.

We mentioned the result for variances. Unfortunately there is no STDEVPIF function in Excel, but the veracity of the formula can be checked if the binary table is cleared of formulas and then sorted by cardinality.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1			1	2	3	4	5	6	7	8										sample	subset	mean of	pop	
2			128	64	32	16	8	4	2	1	Card	Elements								mean	cardinality	sample means	mean	
3		0	0	0	0	0	0	0	0	0	0													4.5
4		1	0	0	0	0	0	0	0	1	1								8	8	1		4.5	
5		2	0	0	0	0	0	0	1	0	1								7	7	2		4.5	
6		3	0	0	0	0	0	0	1	1	2								7	8	7.5	3	4.5	
7		4	0	0	0	0	0	1	0	0	1								6		6	4	4.5	
8		5	0	0	0	0	0	1	0	1	2								6	8	7	5	4.5	
9		6	0	0	0	0	0	1	1	0	2								6	7	6.5	6	4.5	
10		7	0	0	0	0	0	1	1	1	3								6	7	8	7	4.5	
11		8	0	0	0	0	1	0	0	0	1								5		5	8	4.5	
12		9	0	0	0	0	1	0	0	1	2								5	8	6.5			
13		10	0	0	0	0	1	0	1	0	2								5	7	6			
14		11	0	0	0	0	1	0	1	1	3								5	7	8			
15		12	0	0	0	0	1	1	0	0	2								5	6	5.5			
16		13	0	0	0	0	1	1	0	1	3								5	6	8	6.3333		
17		14	0	0	0	0	1	1	1	0	3								5	6	7			
18		15	0	0	0	0	1	1	1	1	4								5	6	7	8	6.5	

Figure 5: Calculating Statistics

Application 101: The subset sum problem

One of the classical problems in mathematics and computing is the *subset sum problem* in which the goal is, given a set of numbers $S = \{a_1, a_2, \dots, a_n\}$, to find a subset of S that has a given sum [4]. Not only is this well-known within the academic world, but solutions of the problem as frequently used in real-world settings. For example, to find an efficient way to cut a given number of boards (whose lengths form the subset S) of chipboard from sheets of a fixed size (whose dimensions fix the given sum) calls for repeated solutions of the subset sum problem.

We use a binary table to create all possible subsets of the set $S = \{2, 3, 5, 7, 9, 11, 13, 17\}$ using the SUMPRODUCT function as before. The steps for creating the spreadsheet of Figure 6 are:

1. Copy the binary table worksheet and rename it as Subset Sum.
2. Remove the numbers 0 – 15 from in front of the table and move it so that it the last column is in column K.
3. Extend the table back to column C, and make it fill the range C2:J258.
4. Change the numbers in Row 2 to be the values 2, 3, 5, 7, 9, 11, 13, 17, or whatever values you choose. Name the range C2:J2 as *subset*.
5. Add the title Subset Sum to K2, and add the formula (K3) = SUMPRODUCT(subset, C3:J3).

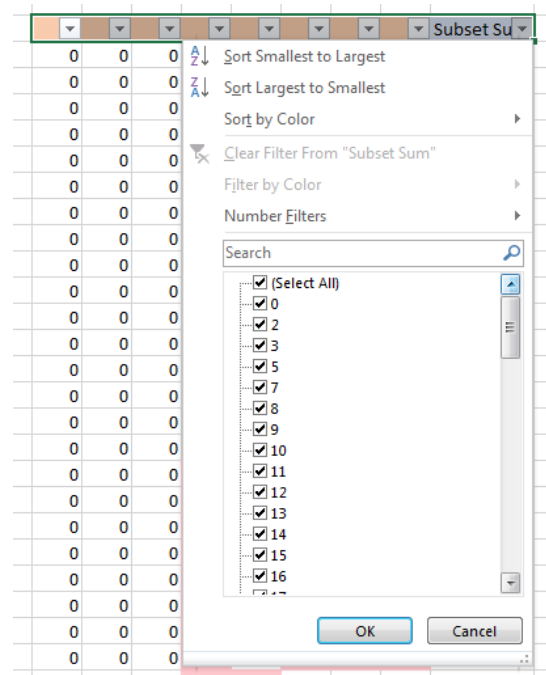


Figure 7: Using the Filter

Fill this formula down to K258.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2			2	3	5	7	9	11	13	17	Subset Sum	
3			0	0	0	0	0	0	0	0	0	0
4			0	0	0	0	0	0	0	1	17	
5			0	0	0	0	0	0	1	0	13	
6			0	0	0	0	0	0	1	1	30	
7			0	0	0	0	0	1	0	0	11	
8			0	0	0	0	0	1	0	1	28	
9			0	0	0	0	0	1	1	0	24	
10			0	0	0	0	0	1	1	1	41	
11			0	0	0	0	1	0	0	0	9	
12			0	0	0	0	1	0	0	1	26	
13			0	0	0	0	1	0	1	0	22	
14			0	0	0	0	1	0	1	1	39	
15			0	0	0	0	1	1	0	0	20	
16			0	0	0	0	1	1	0	1	37	
17			0	0	0	0	1	1	1	0	33	
18			0	0	0	0	1	1	1	1	50	

Figure 6: Subset Sum

Having found the sums of all possible subsets in this way, we now need a way to find that subset (or subsets) whose sum is a given number.

6. Select the range C2:K2 and choose the Filter option in the Sort & Filter menu. This will create filters for the whole table.
7. Select the dropdown menu for K2, which is the one that allows you to select rows from the table whose values match the selected filter.
8. Click the Select All option as this will remove all the selections in the list in one go, and then scroll down the list looking for a particular subset sum to find.
9. Click the select box for your 'given sum' and click the OK button.

The values that were used to create the subsets are hidden by the filter arrows, but these can be revealed by selecting Row 2 and setting the alignment to Left Align.

We chose 49 to be the given sum and the filter showed us that, using only the numbers 2, 3, 5, 7, 9, 11, 13, 17, the subset sum of 49 can be made in 5 ways, as shown in Figure 7.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2			2	3	5	7	9	11	13	17	Subset Sum	
64			0	0	1	1	1	1	0	1	49	
94			0	1	0	1	1	0	1	1	49	
106			0	1	1	0	0	1	1	1	49	
224			1	1	0	1	1	1	0	1	49	
238			1	1	1	0	1	0	1	1	49	
259												

Figure 7: Subsets of the set {2, 3, 5, 7, 9, 11, 13, 17} with element sum 49.

Conclusion

A binary table is quick to generate and can be used to analyse data from varied sources. It is a fundamental combinatorial object and gives useful insight into the structure of a variety of fundamental combinatorial and number-theoretic problems. Use of the binary table to illustrate mathematical principles, structures and patterns has been limited in the past due to the tedious nature of tabulation and also because the size of the table grows exponentially with the number of parameters (columns). Now that computer hardware is so cheap and tools like Microsoft Excel are so ubiquitous, we believe that binary tabulation approaches should be shared with students of mathematics as useful problem-solving tools and that all students of STEM subjects would benefit from exposure to it, beginning in secondary school and onwards to university undergraduate studies.

We have followed up this article with another 101 uses of a binary table in which we build on articles [5] and [6], both of which made use of this analysis tool.

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Appendix A – some classroom exercises & activities

To the student: The file *101 Uses for a binary table.xlsx* is provided to help you avoid much of the tedium associated with enumeration of binary tables, partitions and subsets, etc. The following exercises assume you know how to use this model to generate the binary tables and then ask you to extend the model to compute various objects of interest. These include sets, power sets, partitions and their construction in Excel is a powerful way to illustrate various mathematical fundamentals. You are in control! Use and extend the models along the lines suggested in the exercises to build and strengthen your knowledge the building blocks of discrete mathematics and number theory: sets, sequences, binary representation of integers, partitions of integers, primes and other essential concepts.

Exercises

1. Explain the “101” used in the title and throughout this paper.
2. Extend the Binary Table worksheet to cater for 5 bits, i.e. from 00000 to 11111.
3. Extend the Power Set worksheet to create a list of the subsets of the set $S = \{1,2,3,4,5,6,7,8,9\}$.
4. Use the Horner method to evaluate the polynomial $f(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$ when $x = 4$.
5. Use the model of sheet *Binomial Coeffs* to generate a binary table with 10 columns and 1024 rows. Then extend the model to generate the binomial coefficients for $n = 10$.
6. Extend the sampling distribution model to illustrate that the variance of a sampling distribution of means is the population variance divided by the sample size. This is a non-trivial exercise and requires that the note about clearing the binary table of formulae be heeded.
 - a. Make a copy of the Statistics worksheet and remove columns T onwards.
 - b. Select C3:J258, copy the range and paste values. Remove Row 3.
 - c. Sort the data in C3:S257 using the Card column as the sort key. This will bring the subsets of the same cardinality together and from there the variance calculations can be made.
7. Constrained subset sum problem. Determine the minimal and maximal cardinality subsets of $\{1,2,3,4,5,6,7,8,9,10,11,12\}$ both having element sum 30.