8-6-2015

Another 101 Uses for a Binary Table and a Spreadsheet

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Abstract

The purpose of this paper is to broaden the scope of a previous paper and show applications of a varied nature using the binary table to generate data on which mathematical investigations can be based. Elementary applications considered are the "coins problem", a binary card trick, Killer Sudoku, and expressing integers in terms of others in specified ways.

Keywords

spreadsheets, mathematical investigation, problem posing, problem solving, binary table, truth table, integer partitions, Killer Sudoku

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The purpose of this paper is to broaden the scope of a previous paper and show applications of a varied nature using the binary table to generate data on which mathematical investigations can be based. Elementary applications considered are the "coins problem", a binary card trick, Killer Sudoku, and expressing integers in terms of others in specified ways.

Keywords: spreadsheets, mathematical investigation, problem posing, problem solving, STEM problems, binary table, truth table, integer partitions, Killer Sudoku.

Background

This paper describes further uses of a *binary table* in a spreadsheet environment and is a follow-up to a previous paper [4]. In that paper, the binary table shown in Figure 1 was extended and applied to such topics as power sets, binomial coefficients and statistical relationships. In this paper we further demonstrate that the potential for learning offered by this fundamental *combinatorial object* [9] (binary table) is substantial and each example that we cover is a mathematical investigation in its own right.

There are two spreadsheets associated with this paper that can be downloaded. These are *Another 101 Uses for a Binary Table.xlsx* and *Squares.xlsm*.

	A	B	C	D	E	F	G
1							
2			8	4	2	1	
3		0	0	0	0	0	
4		1	0	0	0	1	
5		2	0	0	1	0	
6		3	0	0	1	1	
7		4	0	1	0	0	
8		5	0	1	0	1	
9		6	0	1	1	0	
10		7	0	1	1	1	
11		8	1	0	0	0	
12		9	1	0	0	1	
13		10	1	0	1	0	
14		11	1	0	1	1	
15		12	1	1	0	0	
16		13	1	1	0	1	
17		14	1	1	1	0	
18		15	1	1	1	1	
19							

Figure 1: A binary table

Application 1: The coins that Jack stole

When writing a series of activities based around the theme of *Jack and the Beanstalk* for a recent National Literacy and Numeracy Week program in Australia [10], we used a context familiar to young students as that gives the problem a meaning that they could engage with.

"One of Jack's daring feats was to steal a bag of gold coins while the giant was asleep and race down the beanstalk to give the bag to his mother, who was in need of consolation after the loss of her pet cow. When the bag was opened, the following coins, shown in Figure 2, fell onto the table.
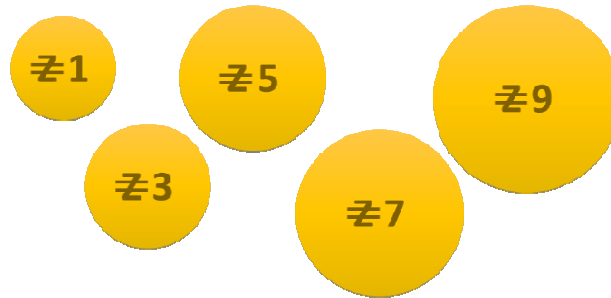

Figure 2: Zekel denominations

There was one each of the 1-zekel, 3-zekel, 5-zekel, 7-zekel and 9-zekel coins that are common currency in Giantland. Jack's mother was anxious to know what amounts could be made with these coins before heading off to buy some much-needed groceries. To find out, Jack opened up a spreadsheet and began to set out a binary table that would help him find the answer to his mother's question. He had just completed the tables (shown in Figure 3) when his younger sister, called Jill (and not Jack's favourite person), peeped over his shoulder to find out what he had discovered.

> "What have you found out, bro?" asked Jill.

Always keen to answer a question with a question, Jack replied by asking her:

> "What is the largest amount that you can make with these coins?"

> "How many different ways can you make an amount of 9 zekels using coins from this collection?"

> "How many different ways can you make amount of 16 zekels using coins from this collection?"

> "What is the smallest amount that you cannot make and so what is the largest amount that you cannot make?"

With that introduction, we would leave the students to work on the problems that Jack posed. We would also add a hint for the teachers noting that having found that the largest amount, which uses all the coins, is 25 zekels, Jack based the next questions on two totals that add to 25. Having found a combination that adds to 9 zekels, the coins not used will add to 16 zekels. And the same property can be used to answer the last question.

	A	B	C	D	E	F	G	H	I	J
1		Zekels								
2		1	3	5	7	9				
3										
4				Binary Table			Zekel Values		Zekel	Occurrences
5		0	0	0	0	0	0		1	1
6		0	0	0	0	1	9		2	0
7		0	0	0	1	0	7		3	1
8		0	0	0	1	1	16		4	1
9		0	0	1	0	0	5		5	1
10		0	0	1	0	1	14		6	1
11		0	0	1	1	0	12		7	1
12		0	0	1	1	1	21		8	2
13		0	1	0	0	0	3		9	2
14		0	1	0	0	1	12		10	2
15		0	1	0	1	0	10		11	1
16		0	1	0	1	1	19		12	2
17		0	1	1	0	0	8		13	2
18		0	1	1	0	1	17		14	1
19		0	1	1	1	0	15		15	2
20		0	1	1	1	1	24		16	2
21		1	0	0	0	0	1		17	2
22		1	0	0	0	1	10		18	1
23		1	0	0	1	0	8		19	1
24		1	0	0	1	1	17		20	1
25		1	0	1	0	0	6		21	1
26		1	0	1	0	1	15		22	1
27		1	0	1	1	0	13		23	0
28		1	0	1	1	1	22		24	1
29		1	1	0	0	0	4		25	1
30		1	1	0	0	1	13			
31		1	1	0	1	0	11			
32		1	1	0	1	1	20			
33		1	1	1	0	0	9			
34		1	1	1	0	1	18			
35		1	1	1	1	0	16			
36		1	1	1	1	1	25			

Figure 3: Jack's calculations

It is easy to see that a 2-zekel total is the smallest that cannot be made, and if that is the case, then a $25 - 2 = 23$-zekel total is the largest that cannot be made.

1.1 Possible Extension

If there were more than one of each coin, the binary table needs to be extended. For example, if there were two of each denomination, the Zekels section would be:

Zekels										
	1	1	3	3	5	5	7	7	9	9

The binary table would also need to be extended to have $2^{10} = 1024$ rows, but that would be well within the scope of a spreadsheet. Even three of each type would be manageable, but after that the size of the binary table would be too large to make it a useful tool for solving Jack's problem.

Application 10: An old trick

In *Mathematical Recreations and Essays* [5], W. W. R. Ball wrote a piece on THE WINDOW READER.

> Some years ago, a set of numbered cards was brought out which enabled an operator to state a number chosen by a spectator. Of the 8 cards, each of the first 7 was pierced with window-like openings and contained numbers less than 100 headed by the word *Yes*, with the word *No* at the bottom. The 8th card showed a list of numbers 1 – 100 from which the spectator would choose one number. The first 7 cards were shown to the spectator and if the chosen number appeared on that card it was placed with *Yes* at the top on the 8th card, otherwise it was placed with the *No* at the top.[1]

There is no need to use so many cards to get a deep understanding as to how this trick works. We can restrict ourselves to asking the spectator to choose a number between 0 and 15, and these are shown to them on the first card. And because $2^4 = 16$, we need just four question cards to find the chosen number.

> "Does your number contain a 1/2/4/8 in its binary representation?"

But we disguise the questions by showing the spectator lists of numbers that hide the fact that they share a particular power of 2 in their binary representations. The four window cards that we use are shown in Figure 4 and they are derived from the binary table of Figure 5.

For example, the first question card asks the spectator to choose from the numbers that contain a 1 in their binary representation. If this card is placed on the Choice card, only the odd numbers show through the windows. If this pattern is turned upside down (but the Choice card remain in the same position), the visible numbers are now those that do not contain a 1 in their binary representation, i.e. the even numbers.

Let us suppose that our spectator is called Alex and that she has chosen 9 as her number. If we show Alex the first question card, she will say *Yes*, and we should place our first card with the *Yes* at the top on the choice (first) card. The highlighted section is the cut-out window section of the card and only the odd numbers on the choice card will show though the windows. As we ask the remaining three questions, half of the possible numbers will be eliminated each time and the last number to show through the windows will be the number that Alex chose.

The game can easily be extended to numbers 0 – 100 where seven question cards are needed. And if the operator can add with confidence, the windows aren't needed, as the answers to the questions tell which of 1, 2, 4, 8, 16 32 and 64 should be added to find the chosen number.

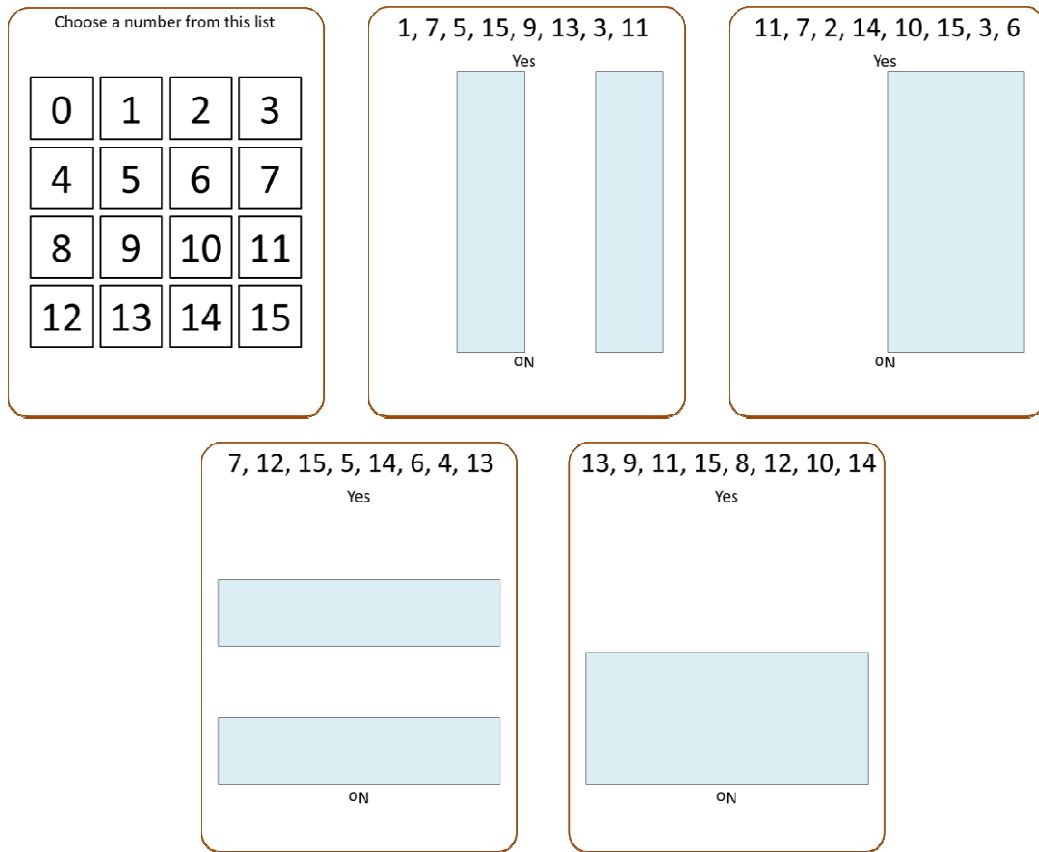[1] This abstract from the book is an adaptation of the original.

Choose a number from this list

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

1, 7, 5, 15, 9, 13, 3, 11

Yes

ON

11, 7, 2, 14, 10, 15, 3, 6

Yes

ON

7, 12, 15, 5, 14, 6, 4, 13

Yes

ON

13, 9, 11, 15, 8, 12, 10, 14

Yes

ON

Figure 4: Old trick game cards

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		Powers of 2																	
2		8	4	2	1														
3																			
4			Binary Table			Decimal			Contains a 1						Contains a 2				
5		0	0	0	0	0			0	1	2	3			0	1	2	3	
6		0	0	0	1	1			4	5	6	7			4	5	6	7	
7		0	0	1	0	2			8	9	10	11			8	9	10	11	
8		0	0	1	1	3			12	13	14	15			12	13	14	15	
9		0	1	0	0	4													
10		0	1	0	1	5													
11		0	1	1	0	6			Contains a 4						Contains an 8				
12		0	1	1	1	7			0	1	2	3			0	1	2	3	
13		1	0	0	0	8			4	5	6	7			4	5	6	7	
14		1	0	0	1	9			8	9	10	11			8	9	10	11	
15		1	0	1	0	10			12	13	14	15			12	13	14	15	
16		1	0	1	1	11													
17		1	1	0	0	12													
18		1	1	0	1	13													
19		1	1	1	0	14													
20		1	1	1	1	15													
21																			

Figure 5: Making the window cards

Application 11: Partitions of integers for Killer Sudoku

Of all the types of Sudoku puzzles that can be found in books, newspapers and on the Internet, the Killer Sudoku is the one that involves both *adaptive reasoning*, a current focus in mathematics syllabuses[2] and *numerical fluency*. Puzzles of this type are readily available from the Killer Sudoku website [7] and they range from easy (don't be fooled by that description) to mind-bending (which is an apt description). The puzzles can be a useful diversion or an opportunity to rehearse number skills in a context that has meaning for students. But most importantly, Killer Sudokus also require the logical reasoning required by the original Sudoku puzzles, and as such, have a role to play in the development of mathematical confidence and capability.

As an example of the opportunities that Killer Sudokus provide for the use of numerical fluency, we see that in the puzzle of Figure 6 there are two break lines that enable two numbers to be found by calculation.
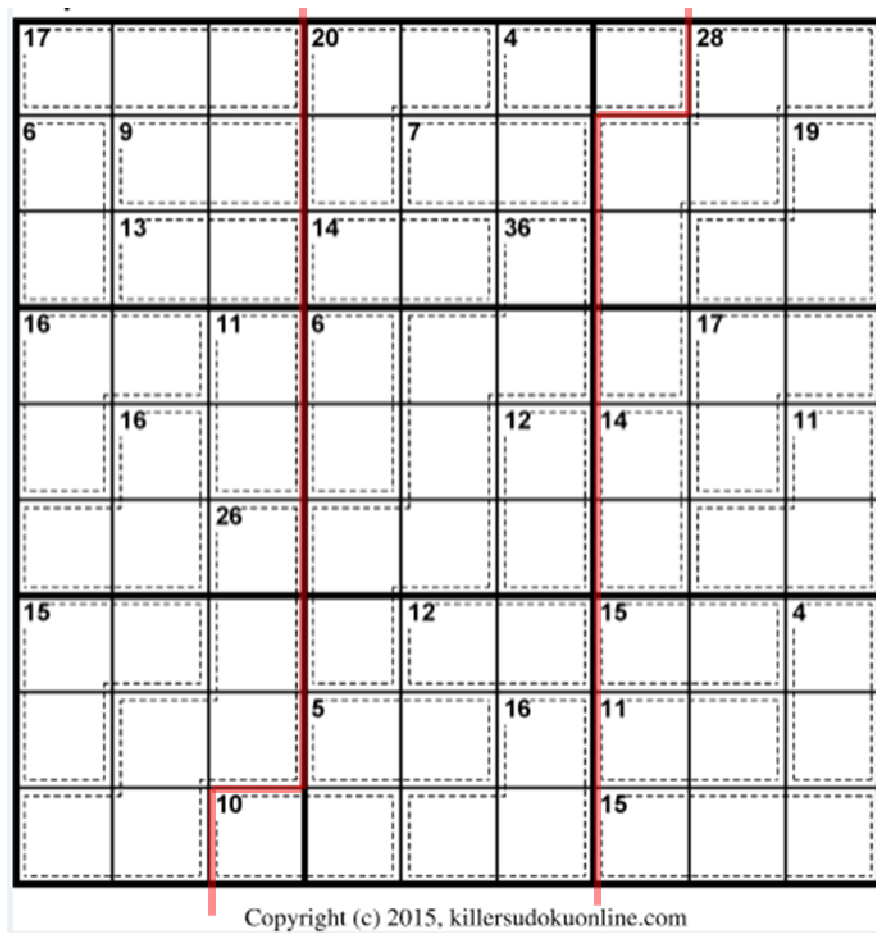

Figure 6: Break lines in a Killer Sudoku

Figure 6 is an example of where the Killer Sudoku rewards clever mental computation, as there is no need to add all the numbers to the left of the left-hand break line. It would be

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[2] The *Australian Curriculum: Mathematics* for example.

sufficient to find the units part only as that can then be used to find the number that should start the 10-pair on the bottom row; similarly for the end number the 4-pair on the top row.

But sometimes is very informative to know what the possible sums are for given numbers. For example if a block that contains four numbers adds to 28, the numbers must be either {9, 8, 7, 4} or {9, 8, 6, 5}. It is in the process of determining what are the possible ways of combining m of the numbers $1 - 9$ to make a total of n that Excel can lend a hand. In our previous article [4], we showed how to enumerate the power sets of a given set. In this case, we would use the same binary table approach. The method is to:

1. Make a binary table with 9 columns that can be used to create the power set of the set $S = \{1, 2, 3, \dots, 8, 9\}$.
2. Set (K5) = IF(B5 = 1, B$2, "") and extend this to the range (K5:S516). Concatenate the values in columns K – S by setting (T5) = K5 & L5 & M5 & N5 & O5 & P5 & Q5 & R5 & S5 and fill this down to T516.
3. Find the total of the numbers in each subset with the formula (U5) = SUM(K5:S5) and fill this down to U516.
4. Use data filtering to select those subsets of S that have a given total and, from that selection, find a subset that has elements that match the constraints of the puzzle.

Figure 7 shows the different ways in which 16 can be made with numbers between 1 and 9.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1		Killer Sudoku Digits																				
2		9	8	7	6	5	4	3	2	1												
3																						
4					Binary Table					▼		Subset Elements							▼	Subset ▼	Sum ⊤	
114		0	0	0	1	0	1	1	1	1				6		4	3	2	1	64321	16	
115		0	0	0	1	1	0	1	1	0				6	5		3	2		6532	16	
116		0	0	0	1	1	1	0	0	1				6	5	4			1	6541	16	
117		0	0	1	0	0	1	1	1	0			7			4	3	2		7432	16	
118		0	0	1	0	1	0	1	0	1			7		5		3		1	7531	16	
119		0	0	1	0	1	1	0	0	0			7		5	4				754	16	
120		0	0	1	1	0	0	0	1	1			7	6				2	1	7621	16	
121		0	0	1	1	0	0	1	0	0			7	6			3			763	16	
122		0	1	0	0	0	1	1	0	1		8				4	3		1	8431	16	
123		0	1	0	0	1	0	0	1	1		8			5			2	1	8521	16	
124		0	1	0	0	1	0	1	0	0		8			5		3			853	16	
125		0	1	0	1	0	0	0	1	0		8		6				2		862	16	
126		0	1	1	0	0	0	0	0	1		8	7						1	871	16	
127		1	0	0	0	0	1	0	1	1	9					4		2	1	9421	16	
128		1	0	0	0	0	1	1	0	0	9					4	3			943	16	
129		1	0	0	0	1	0	0	1	0	9				5			2		952	16	
130		1	0	0	1	0	0	0	0	1	9			6					1	961	16	
131		1	0	1	0	0	0	0	0	0	9		7							97	16	
517																						

Figure 7: Locating the ways of making a given total

As in the first application, we have used a binary table to enumerate possibilities. The next application focusses on the number of possibilities that the binary table creates.

Application 100: Expressing k as a sum of squares

In a problem solving book for Upper Primary students [1], we included the following:
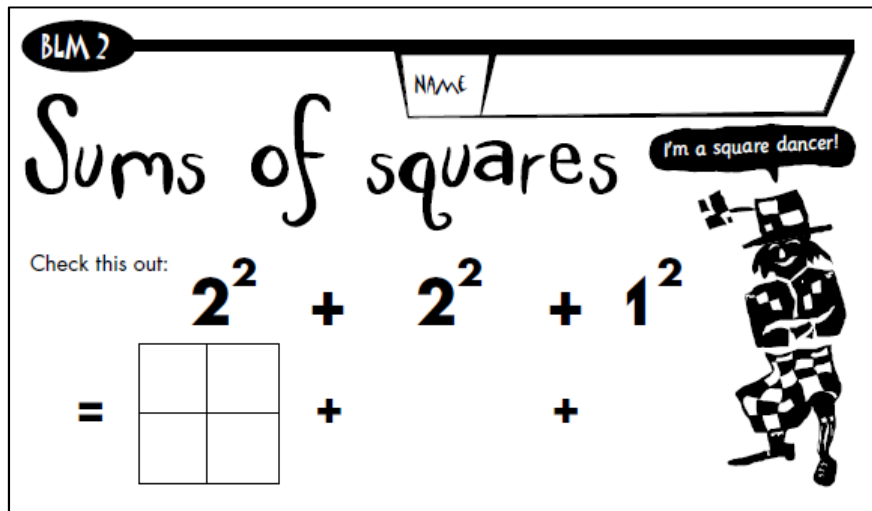

Figure 8: Sums of squares

The problem asked the students to investigate what other square numbers between 1 and 100 can be expressed as the sum of squares. With the help of a binary table, the investigation can be extended to find:

> *In how many ways can a natural number k be expressed as the sum of **distinct** square numbers?*

The table in Figure 9 shows how a binary table can be used to select subsets of the square numbers 1, 4, 9, … and to find the sum of the squares in those subsets. But a few more columns are needed to enable the binary table to generate some data that can be investigated. The spreadsheet that accompanies this article includes a binary table to calculate the number of ways in which the number k can be expressed as the sum of square numbers that range from 1 to 12^2. To be sure that all possibilities are covered, namely that we have found all the numbers whose sum can be expressed as the sum of the first 12 square numbers, we need a binary table with 12 columns and $2^{12} = 4096$ rows. This is just the sort of calculation that Excel handles very quickly and the table in the accompanying spreadsheet is large, but quite within its calculation power.

Let us define a function $sq(k)$ = the number of different ways in which k can be made as the sum of squares. Columns Q and R of the spreadsheet find the number of occurrences of k in the list of subset sums, using the formula (R5) = COUNTIF(sumsquares, Q5), where *sumsquares* is the name given to the subset sums in column N. The graph of Figure 10, derived from the binary table, shows a possible trend, even though there can quite a variation in the differences between the values of $sq(k)$ and $sq(k + 1)$. For example $sq(129) = 5$ whereas $sq(128) = 0$, and the graph suggests that the largest k for which $sq(k) = 0$ is 128, i.e. 128 is the largest number that cannot be expressed as the sum of distinct squares.

To extend the range of values for k, it is clear that the binary table becomes unwieldy and that a better approach is to write a VBA program in which much larger values of k can be obtained. The program can be found in the spreadsheet *Squares.xlsm*, and Figure 11 shows the result of extending k to 2000. The program generated enough data to be able to plot the values of $sq(k)$ for $1 \le k \le 2000$ and, for that range, the trend for $sq(k)$ becomes a lot clearer.

|	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1						Square Numbers								
2		144	121	100	81	64	49	36	25	16	9	4	1	
3														
4						Binary Table								subset sum
5		0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	1	1
7		0	0	0	0	0	0	0	0	0	0	1	0	4
8		0	0	0	0	0	0	0	0	0	0	1	1	5
9		0	0	0	0	0	0	0	0	0	1	0	0	9
10		0	0	0	0	0	0	0	0	0	1	0	1	10
11		0	0	0	0	0	0	0	0	0	1	1	0	13
12		0	0	0	0	0	0	0	0	0	1	1	1	14
13		0	0	0	0	0	0	0	0	1	0	0	0	16
14		0	0	0	0	0	0	0	0	1	0	0	1	17
15		0	0	0	0	0	0	0	0	1	0	1	0	20
16		0	0	0	0	0	0	0	0	1	0	1	1	21
17		0	0	0	0	0	0	0	0	1	1	0	0	25
18		0	0	0	0	0	0	0	0	1	1	0	1	26
19		0	0	0	0	0	0	0	0	1	1	1	0	29
20		0	0	0	0	0	0	0	0	1	1	1	1	30
21		0	0	0	0	0	0	0	1	0	0	0	0	25
22		0	0	0	0	0	0	0	1	0	0	0	1	26

Figure 9: a binary table for sum of squares
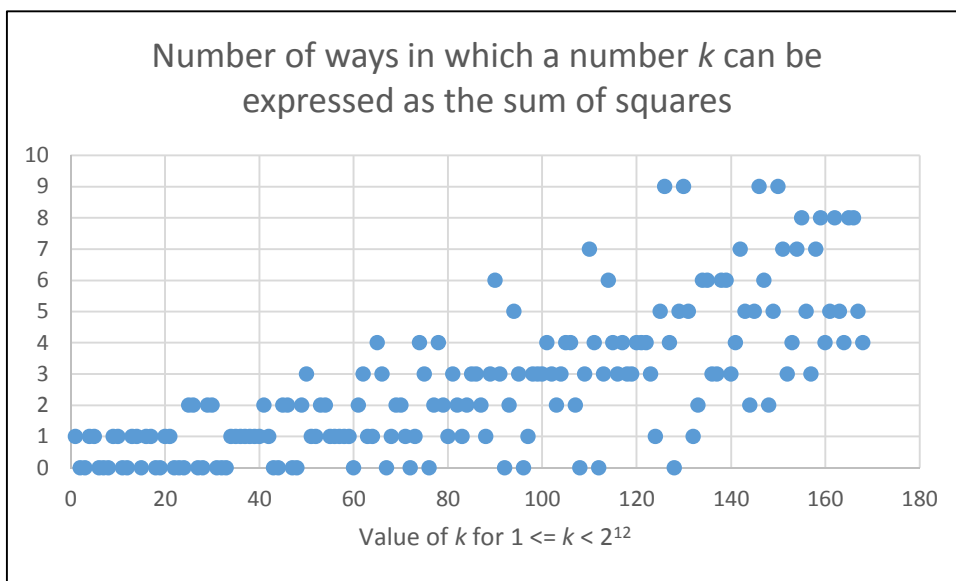
Figure 10: Graph of $sq(k)$ derived from a binary table
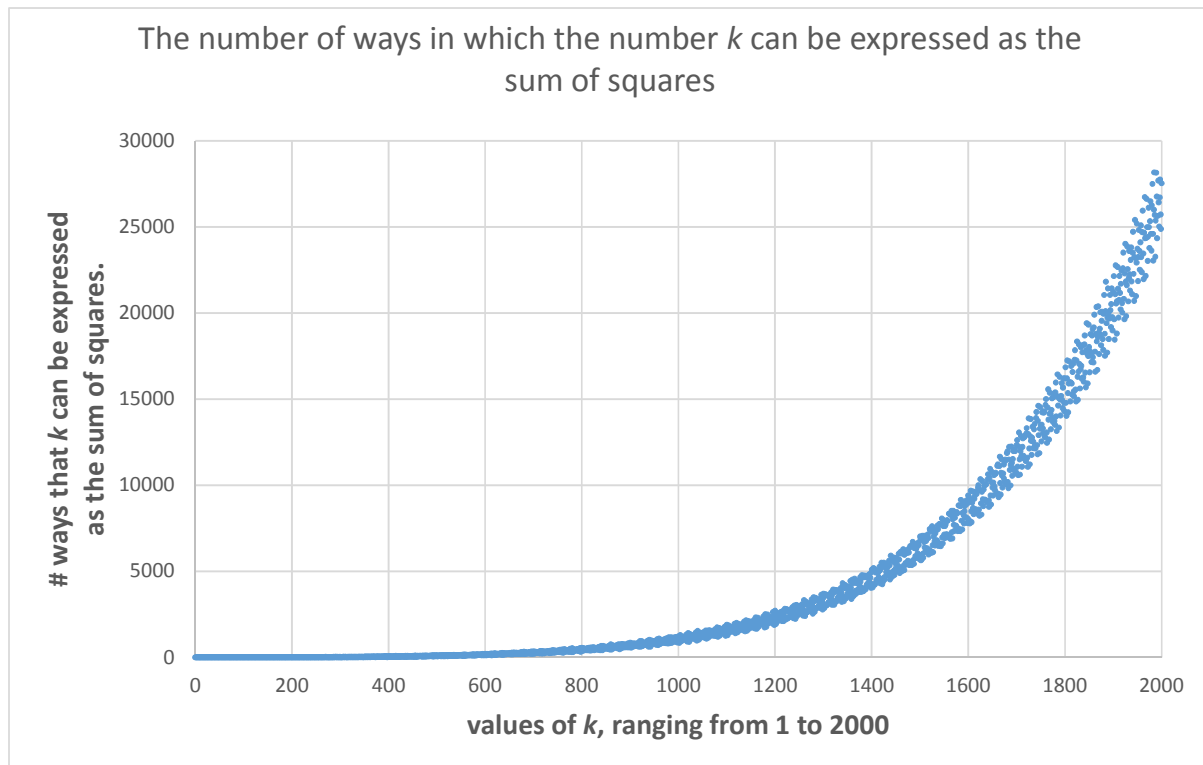
Figure 11: Graph of $sq(k)$ extended for squares

Application 101: Extensions to other generating sets

In the previous example, a binary table headed by the square numbers was used to generate data that showed the number of ways in which a number k can be expressed as the sum of (distinct) square numbers. As well as generating data with a binary table, we also looked at what happens when the data set is expanded by means of a VBA program. In this section we apply the same methods to two other generating sets, the natural numbers and the prime numbers.

When the numbers in the heading of the binary table for square numbers are replaced by the numbers 1, 2, … , 15 we have a binary table that shows in how many ways a number $k: 1 \leq k \leq 15$ can be expressed as the sum of (distinct) natural numbers[3]. We are restricted to that range for k because the next natural number, 16, will affect the count of all subsequent numbers. The graph of $nat(k)$, the number of different ways in which k can be made as the sum of natural numbers, is given as Figure 12 and it shows the trend to be quite regular.

When the VBA program used for square numbers is adapted to the natural numbers, the range for k can be increased quite considerably. Figure 13 shows that the counts reach very high values for as k increases towards 100 the counts are getting close to half a million,

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[3] The order in which the numbers are listed does not affect the sum and hence does not affect the value of $nat(k)$.
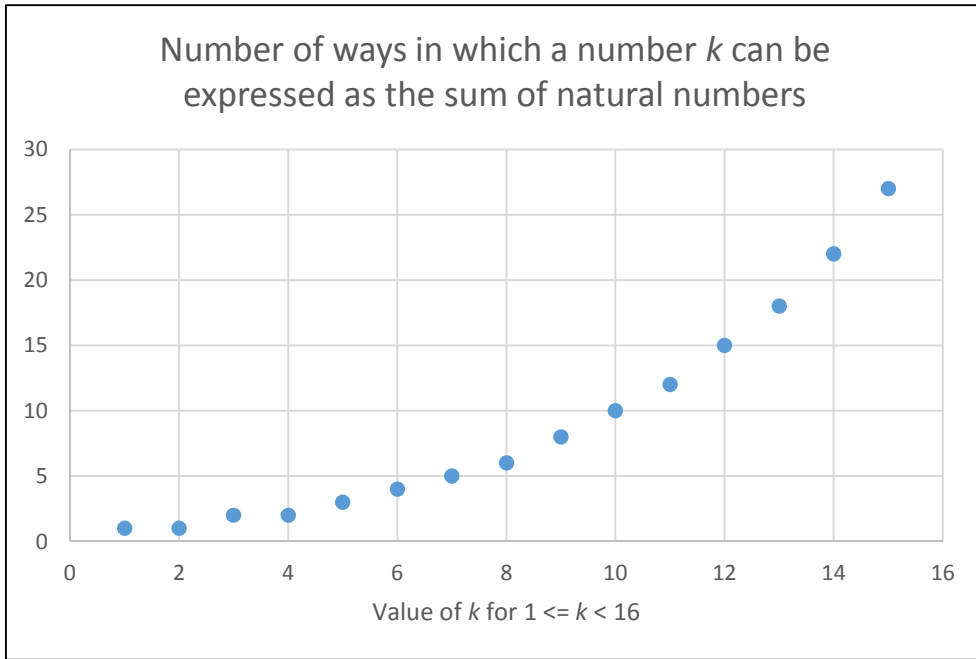
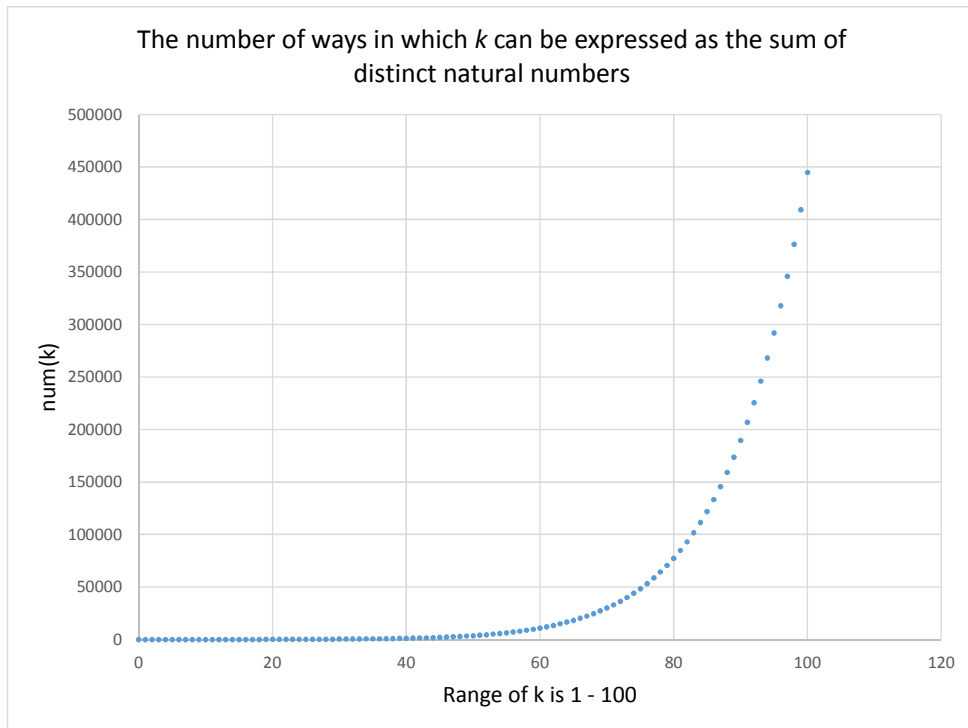Figure 12: Using the natural numbers as a generating set
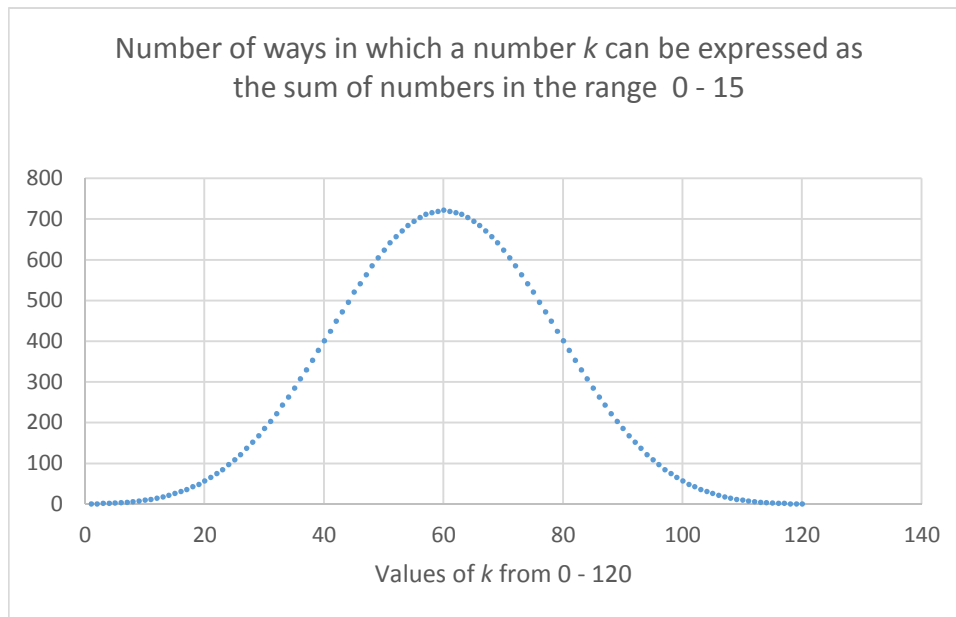

Figure 13: Extending the range of *k*.

Figure 14: Solution to the subset sum problem

Using the binary table in a different way

There is another way in which, we can make use of the data that has been generated by the binary table. This can be expressed in the form of a problem to be investigated.

> Let $nat(n) =$ the number of subsets of the set $S = \{k : 1 \leq k \leq 15\}$ whose subset sum is n. What range of values does n take and what does the graph of $nat(n)$ show?

The largest number that can be generated by a subset of S is 120 which is the sum of all the numbers from 1 to 15 and the chart in Figure 14 shows that every number in the range $1 \leq k \leq 120$ can be expressed as a subset sum. This distribution of counts has the appearance of a normal curve, however it can be shown that this is not the case by plotting the logarithm $ln(nat(n))$.

A second extension

We mentioned earlier that the process used to investigate square numbers would be extended to prime numbers. In a previous article [4], Excel was used to explore Goldbach's conjecture that every even number greater than 2 can be expressed as the sum of two prime numbers. The graphs generated in that investigation are strikingly different from those that have been generated here, but that is due to the fact that the subset sums in this context are not subject to being subsets of just two elements of the generating set. What happens, then, when the head row of the binary table contains the first fifteen prime numbers? These are:

> 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

In particular, will the results mimic those of the square numbers, where the graph has a pattern that spreads as k gets larger, or is the density of primes sufficient to produce an outcome closer to that of the natural numbers?

With very little change, the binary table and VBA program of the squares investigation were adapted to a generating set of prime numbers. The charts of Figures 15, 16 and 17 show that it is the behaviour of subsets of the natural numbers that are similar to those created when the generating set is the set of primes. There are differences, however, the main ones being:

1. The chart of Figure 12 for natural numbers is much smoother than the equivalent chart of Figure 15 for primes. But as the number of primes included increases, this difference is less marked.

2. The number of counts for the natural number is much greater than that for primes in Figures 13 and 16. This should be expected as there are $O(n)$ numbers less than n for natural numbers but only $O(p/ln(p))$ prime numbers[4] less than the prime number p.

3. In Figure 17, the range of k-values for the first 15 primes is much greater than that for the first 15 natural numbers (Figure 14), but then the first 15 primes add to 328 whereas the first 15 natural numbers add to 120.
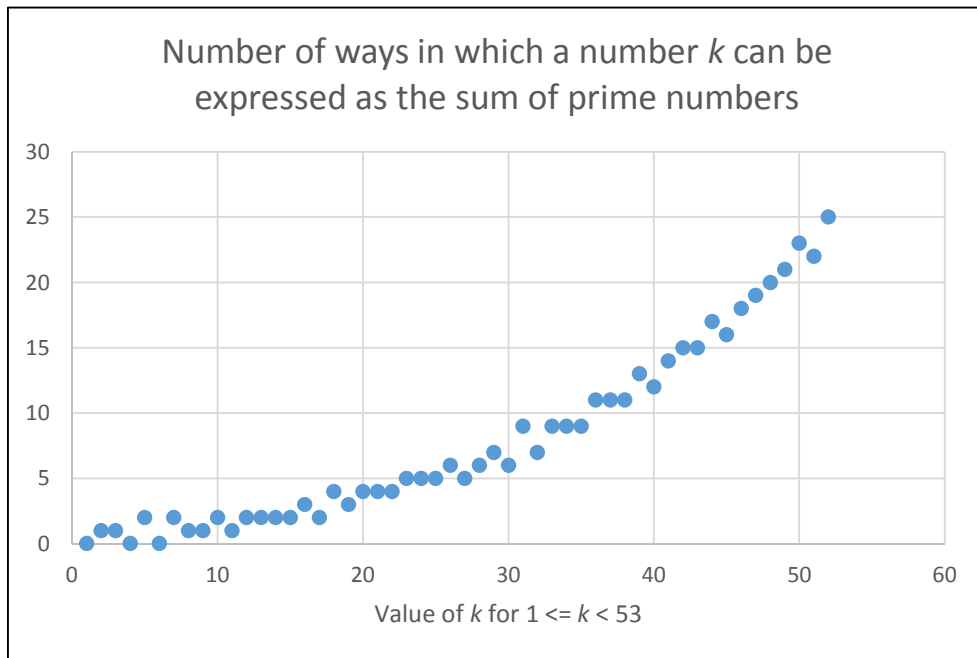
Figure 15: Binary table outcomes for the first 15 prime numbers.

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[4] This is the essence of the famous "prime number theorem", proved independently by both Jacques Hadamard and Charles Jean de la Vallée-Poussin in 1896.
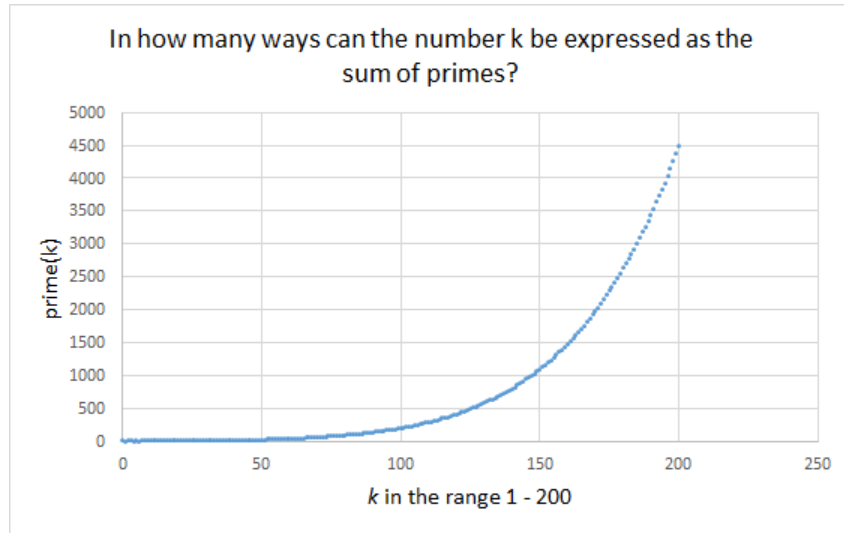
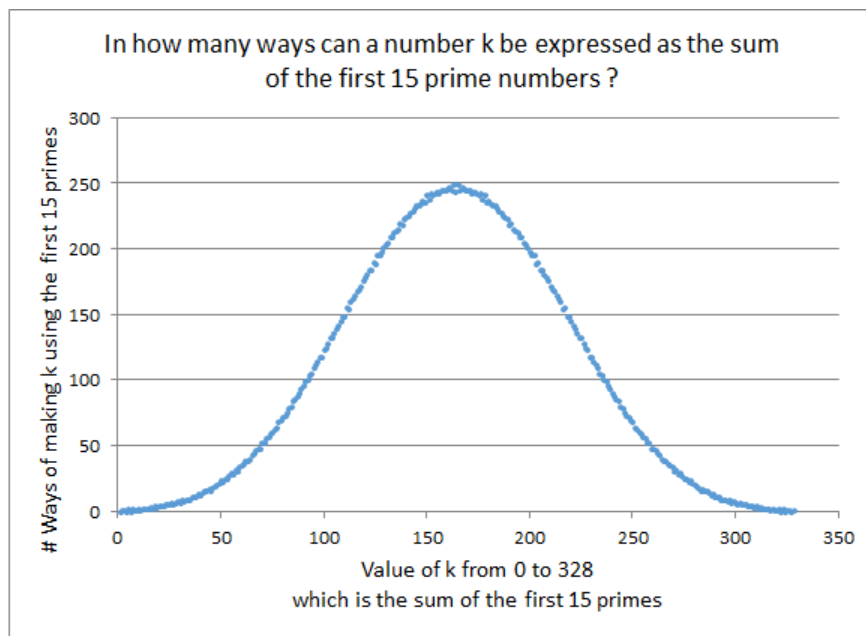Figure 16: Extending the range of   with a VBA program.

Figure 17: Solution to the subset sum problem with primes.

A further extension of these features would be to make the generating set of cube numbers where similar trends continue to be displayed. And we can pose questions such as:

What is the largest number that cannot be expressed as the sum of distinct cubes?

Answer: 15,278.

An unexplained difference

In earlier papers, [2] and [3], a Zeckendorf expansion of k was defined as:

$$k = \sum_j e_j f_j \text{ where } f_i \text{ is a Fibonacci number and } e_j = 0 \text{ or } 1$$

This is equivalent to saying that a Zeckendorf expansion is the same as a subset sum of Fibonacci numbers that add to k. The function $Z(k)$ was defined as the number of different ways in which k can be expressed as a Zeckendorf expansion. Having adapted the squares spreadsheet to make the heading row comprised of the first fifteen Fibonacci numbers, a few adjustments were needed to generate the Fibonacci charts that are equivalent to Figures 12 and 15. Because the binary table goes up to $f_{15} = 610$, it was found that a VBA program was not needed to generate a larger dataset. Figure 18 shows that the binary table enables the range of k to be extended to $1 \leq k \leq 986$, because $f_{16} = 987$. The range of the chart that shows the counts for all numbers that can be expressed as the sum of the first 15 Fibonacci numbers also has an increased range. Using the fact that $\sum_j f_j = f_{16} - 1 = 1596$, the range for k needed to be adjusted accordingly and the chart is shown in Figure 19.
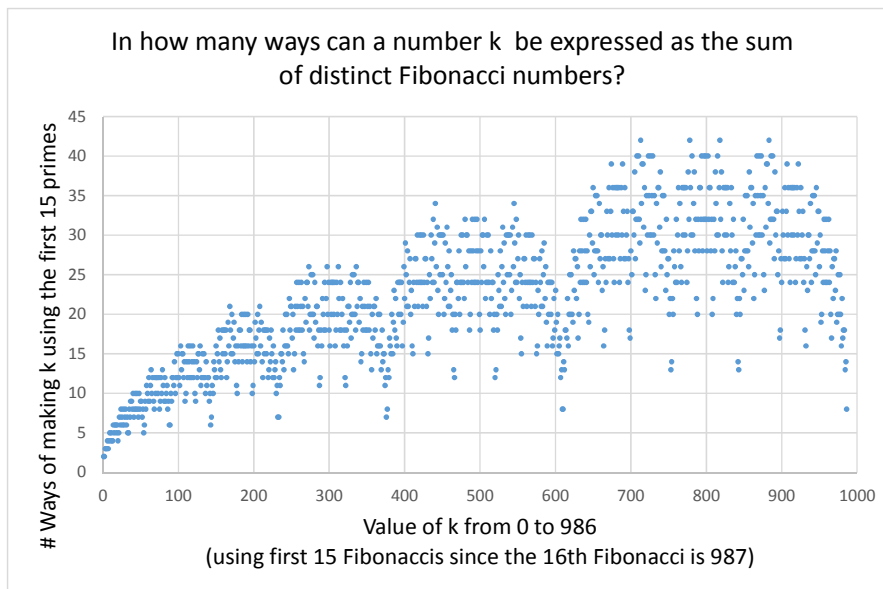

Figure 18: The chart for Fibonacci numbers

There are two substantive differences between the above charts and those obtained for squares, natural numbers and primes.

1. In Figure 18, the one derived from the binary table, the trend does not show an exponential increase; rather, the increase is logarithmic.
2. The chart of Figure 19 shows none of the normal curve characteristics that the previous three charts showed. Instead, the logarithmic growth of Figure 18 is repeated for $k \leq 798$ and then replicated in mirror form for $k \geq 798$.

An explanation as to why the Fibonacci numbers should give such different results to squares, natural numbers and primes has yet to be found.
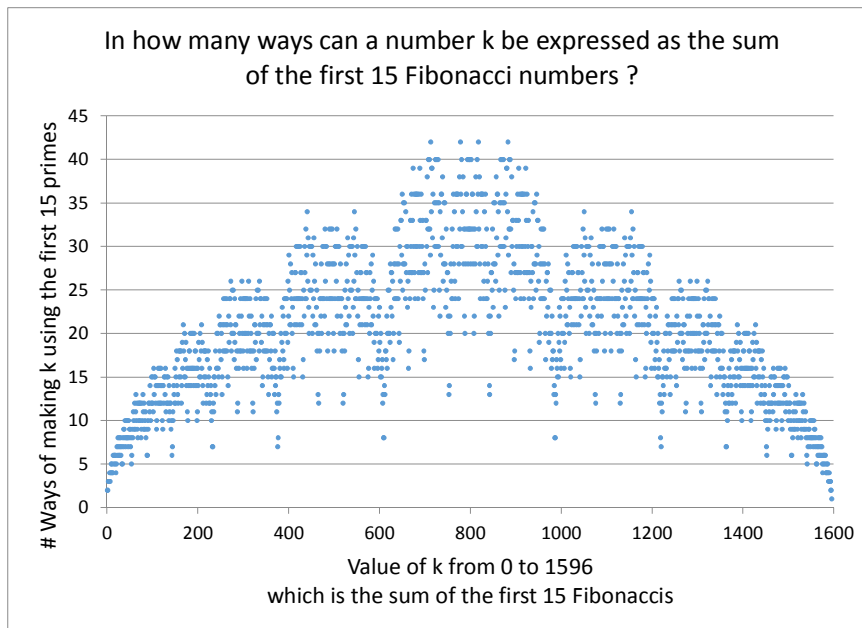
Figure 19: Counts for the first 15 Fibonacci numbers

The reader will also find a similar outcome for Lucas numbers, which use the same recurrence as the Fibonacci numbers, but which have different starting numbers, 2, 1, 3, 4, 7, … rather than 1, 1, 2, 3, …. The graphs for Lucas numbers show a similar dispersion as well as the grouping of counts between successive Lucas Numbers.

Conclusion

A binary table is quick to generate and can be used to analyse data from varied sources. It is one of the standard ways in which to enumerate solutions of small instances of a wide range of fundamental mathematical problems.

The binary table is also fundamental combinatorial object. It gives useful insight into the structure of a variety of combinatorial and number-theoretic problems. Use of the binary table to illustrate mathematical principles, structures and patterns has been limited in the past due to the tedious nature of tabulation and also because the size of the table grows exponentially with the number of parameters (columns). Now that computer hardware is so cheap and tools like Microsoft Excel are so ubiquitous, we believe that binary tabulation approaches should be shared with students of mathematics as a useful problem-solving tool and that all students of STEM subjects would benefit from exposure to it. The examples described herein are deemed by the authors to be suitable for investigation at secondary and tertiary levels of mathematics education.

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Appendix – some classroom exercises & activities

To the student: The file *Another 101 Uses for a Binary Table.xlsx* is provided to help you avoid much of the tedium associated with enumeration of binary tables, partitions and subsets, etc. The following exercises assume you know how to use this model to generate the binary tables and then ask you to extend the model to compute various objects of interest. These include sets, power sets, partitions and their construction in Excel is a powerful way to illustrate various mathematical fundamentals. You are in control! Use and extend the model along the lines suggested in the exercises to build and strengthen your knowledge the building blocks of discrete mathematics and number theory: sets, sequences, binary representation of integers, partitions of integers, primes and other essential concepts.

1. Explain the "101" used in the title and throughout this paper.
2. Before they became almost worthless, Australia used to have one-cent and two-cent coins. In those days, the coins available were worth 1, 2, 5, 10, 20, and 50 cents.
   a. What is the largest amount that you can make with one of each of these coins?
   b. How many different ways can you make an amount of 9 cents using coins from this collection?
   c. How many different ways can you make amount of 86 cents using unlimited coins from this collection?
   d. What is the smallest amount that you cannot make and so what is the largest amount that you cannot make?
3. For Killer Sudoku, sometimes we need a constrained partition of the numbers from 1 to 9, that is, we know that certain numbers cannot be included. Discuss how to modify the Excel model described here for this case.
4. While writing this article, both authors received the following Brainteaser via the CSIRO *Stats and Maths* email (highly recommended).

   > Tom doesn't like touching money, so he keeps all his cash in bags. Right now, he has $15 spread among four different bags. When he's buying things, Tom can make any whole number of dollars from $1 to $15 by selecting some of his bags of money, without taking any money out of the bags. How has Tom distributed his money?

   This problem can be solved in many ways, but using a binary table makes reduces the stress level in the problem solver to an absolute minimum!

5. In the diagram you will see that there are two lines in each of three directions. In how many ways can 6 counters be placed if the hexagons such that there are two counters on each line?

   Note: There are 53 possible ways if rotation and reflection is allowed, but many fewer if two placements are considered the same if one is a rotation/reflection of the other.