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## Interest Rate Conversion

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# Interest Rate Conversion

## **Abstract**

The topic of interest rate conversion is part of the time value concepts covered in introductory finance courses. It is about establishing the equivalence of interest rates over different intra-year periods in compound interest settings. From our experience as instructors of such courses, the task involved is far from being a trivial algebraic exercise, and some students do have difficulties in understanding the underlying concepts. By using Microsoft Excel<sup>TM</sup> as a pedagogic tool, this paper is intended to improve the clarity and the effectiveness in the delivery of the materials involved and help students appreciate better some of the corresponding practical nuances.

## **Keywords**

time value of money, stated and effective interest rates, mortgage loan computations

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## **Cover Page Footnote**

The authors wish to thank the two anonymous reviewers for helpful comments and suggestions.

# Interest Rate Conversion

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# Interest Rate Conversion

## 1 Introduction

A fundamental concept of finance is the time value of money. What connects current and future values are discount rates based on the passage of time and risk considerations. In risk-free settings, the materials involved, as covered in standard introductory finance textbooks, are mostly algebraic in nature. For example, the value of a mortgage loan with a fixed interest rate, when stated as the present value of its periodic repayments (in the form of an annuity), can be reduced to a computationally convenient, closed-form expression for a finite geometric series, where the ratio of adjacent terms is constant throughout. Such an expression allows each repayment to be stated explicitly in terms of the dollar amount of the loan, the per-period interest rate, and the number of required periodic repayments.

For a stated annual interest rate, the loan repayments can be weekly, bi-weekly (i.e., fortnightly), monthly, or at other frequencies, depending on the repayment schedules involved. For the above-mentioned closed-form expression to work as intended, the stated annual interest rate must first be converted to the corresponding per-period interest rate. What complicates matters is the market convention that the stated annual interest rate for a loan typically understates the true annual figure. For example, if the stated annual interest rate is subject to semi-annual (monthly) compounding, the corresponding effective semi-annual (monthly) interest rate is one half (one-twelfth) of the stated annual interest rate. In a world of compound interests, the more frequent the compounding for a stated annual interest rate, the higher is the corresponding effective annual interest rate, thus more costly for the borrower.

Notice that the terms *stated annual interest rate*, *quoted annual interest rate*, and *nominal annual interest rate* are often used interchangeably in economics and finance, although the third term can also be used in a context where *nominal* and *real interest rates* are compared. In such a context, a nominal interest rate is where the effect of inflation has not been accounted for, but a real interest rate is where it has. Adding to the confusions to students is that the stated annual interest rate in practical loan settings is known simply as APR, which is the acronym of annual percentage rate. Regardless of whether these alternative terms are used for a stated annual interest rate, the corresponding annual frequency of compounding must be specified, for

the purpose of interest rate conversion.

The need for interest rate conversion is not confined to loan repayment settings; the same also pertains to investment settings. The future cash flows from investing in a coupon bond, for example, include its periodic (usually semi-annual) coupons and its face value at maturity. Bond valuation is based on the present value of such cash flows. To use each bond valuation model properly to establish a relationship of the underlying variables, the effective interest rate over each coupon payment period is required.

Further, while many financial models are discrete-time models, many others are continuous-time models where an infinitesimally short period is used for analytical convenience. The most notable example of the latter models, as covered in standard introductory finance textbooks, is the Black-Scholes option pricing model. [See, for example, Berk, DeMarzo, and Harford (2015, Chapter 21), Berk, DeMarzo, and Stangeland (2015, Chapter 15), and Ehrhardt and Brigham (2014, Chapter 8) for descriptions of the model.] As the derivation of this model requires a risk-free hedge to be maintained continuously between the call option and its underlying stock, regardless of the stock price movements over time, the risk-free interest rate involved must be a continuously compounded figure. Thus, there is a need to convert the available discrete-time risk-free interest rate to its continuous-time equivalent.

From our experience as instructors of introductory finance courses, the task of interest rate conversion is far from being a trivial algebraic exercise. In view of the importance of time value concepts in finance, this paper has its pedagogic focus on interest rate conversion, a crucial component of the topic. As the intended readers include also students, this paper covers the analytical materials involved in considerable detail, with illustrative examples to complement the standard textbook coverage. Microsoft Excel<sup>TM</sup> plays an important pedagogic role here as well.<sup>1</sup> The use of Excel by students, however, is not intended to be a substitute for learning the analytical materials involved. Rather, while benefiting greatly from Excel's computational convenience, students can pay more attention to the corresponding conceptual issues and analytical details.

Although there are many built-in financial functions in Excel, only two of them are for interest rate conversion, and both are confined to discrete-time settings. Specifically, for a

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<sup>1</sup>For the remainder of this paper, whenever the name Excel or any of its computational tools is mentioned, its trademark is implicitly acknowledged.

stated annual interest rate subject to a given annual frequency of compounding, the function EFFECT provides the corresponding effective annual interest rate. The function NOMINAL provides the stated annual interest rate instead, given an effective annual figure.<sup>2</sup> However, there are no built-in Excel functions that can convert an effective per-period interest rate to the corresponding effective interest rates for longer or shorter periods, although for loan repayments in practice borrowers are typically offered alternative periodic repayment schemes. As the assessment of each repayment scheme from a time value perspective requires the corresponding effective per-period interest rate to be deduced first, a pedagogic task of this paper is to show the computations involved.

In an analytical setting of mortgage loan repayments, this paper illustrates how Excel can be used, in conjunction with the displayed results from online mortgage loan calculators as posted by various financial institutions, to help students assess and improve their skills in interest rate conversion. Although time value concepts are universal, market conventions in how annual interest rates for various debt obligations are stated do differ across countries. For example, the stated annual interest rates for mortgage loans at fixed interest rates are compounded semi-annually in Canada, but are compounded monthly in the United States. Further, some financial institutions offer borrowers weekly, bi-weekly, and semi-monthly repayment options, in addition to the traditional monthly repayment schemes. By verifying and comparing, on Excel, the repayment amounts under various repayment options, students will be able to learn some practical nuances of interest rate conversion.

The remainder of this paper is organized as follows: The time value concepts involved in interest rate conversion and the corresponding algebraic materials are presented in Section 2. An Excel illustration is presented in Section 3; it shows how effective interest rates, over various intra-year periods, can be deduced from given interest rate information. Interactivity, which is achievable via Excel tools such as Scroll Bar and List Box, is a key feature of the Excel illustration. A further Excel illustration, as presented in Section 4, is based on the above-mentioned mortgage loan exercises. Finally, Section 5 provides some concluding remarks.

Notice that there is an Excel file to accompany this paper. The Excel illustration of interest

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<sup>2</sup>Notice that, in the functions EFFECT and NOMINAL, if the annual frequency of compounding for the stated annual interest rate is not an integer, it will be truncated to an integer instead, thus leading to imprecision in interest rate conversion. For example, if weekly compounding is involved, although there are  $365/7 \approx 52.143$  weeks in a 365-day year, the annual frequency is treated as 52 instead in these Excel functions.

rate conversion in Figure 1 of Section 3 is generated by such a file. Given Excel's interactivity and computational convenience, the same file can be used by instructors to generate numerical data for various exercises and examination questions on interest rate conversion, without having to spend much time in performing the corresponding computations themselves. Once thoroughly explained to students, the same file can also be used by students to generate additional numerical exercises for practice and to verify the independently obtained results afterwards.

## 2 Analytical Details and Some Numerical Examples

In a compound-interest setting, the present value of each dollar  $T$  periods from now with certainty, for an interest rate  $r$  each period, is  $\$1/(1+r)^T$ . Here, a period can be a week, a month, six months, or some other time intervals, and  $1/(1+r)^T$  is the present-value factor. In practice, the stated interest rate is a positive annual figure, with compounding performed annually, semi-annually, monthly, or at some other annual frequencies, depending on the investment or loan setting. Thus, except for annual compounding, the stated figure understates the actual annual interest rate. If an annual interest rate is stated as a positive  $i$  with compounding performed  $k$  times a year and if a period is defined as the proportion  $1/k$  of a year, then the effective interest rate over the period is simply  $r = i/k$ , from which the effective annual interest rate can be deduced.

Before presenting the analytical details of interest rate conversion, let us use the case of  $k = 2$  to clarify a subtle point. The case is where the stated annual interest rate  $i$  is compounded semi-annually and the effective semi-annual interest rate is  $i/2$ . When used in conjunction with a stated annual interest rate, the statement that *the interest rate is compounded semi-annually* implies that the corresponding effective semi-annual interest rate is one half of the stated annual figure. However, it does not imply that compounding is confined to semi-annual intervals only. Rather, in a world of compound interests, compounding takes place all the time. The compounded growth of each dollar in a year can be viewed as being achievable not only over two semi-annual periods as the statement implies, but also equivalently over various shorter intra-year periods, such as over four quarters of a year or over 12 months. This equivalence is crucial for the purpose of relating the effective interest rate over a given time interval and the effective interest rates over various other time intervals, in order to accommodate the repayment

patterns involved.

## 2.1 Effective Per-Period Interest Rates: Cases Where the Numbers of Equal Periods in a Year Are Integers

For a stated annual interest rate  $i$  subject to compounding  $k$  times each year, students are taught in introductory finance courses that

$$1 + r_a = \left(1 + \frac{i}{k}\right)^k, \quad (1)$$

where  $r_a$  is the corresponding effective annual interest rate. The idea is that, in a compound-interest setting, if a year is divided into  $k$  equal periods, each dollar at the beginning becomes  $\$(1 + i/k)$  after one period,  $\$(1 + i/k)^2$  after two periods,  $\$(1 + i/k)^3$  after three periods, and so on. The growth from  $\$1$  to  $\$(1 + i/k)^k$  after  $k$  periods is equivalent to the growth from  $\$1$  to  $\$(1 + r_a)$  in a year. This equivalence is captured by equation (1).

Letting  $r_k = i/k$  be the effective interest rate over a time interval that is  $1/k$  of a year, we can write equation (1) equivalently as

$$1 + r_a = (1 + r_k)^k. \quad (2)$$

In a compound-interest setting, the growth from  $\$1$  to  $\$(1 + r_a)$ , as depicted in equation (2), need not be considered as only achievable in  $k$  equal periods of a year. We can also divide a year into  $g$  equal periods, where  $g$  is a finite positive integer, such as 2, 4, 12, 24, 365, or others that have practical relevance. Letting  $r_g$  be the effective per-period interest rate, we can write, in general,

$$1 + r_a = (1 + r_g)^g. \quad (3)$$

For a given  $r_a$ , the shorter the period, the lower is the corresponding effective per-period interest rate, as we have to perform compounding over more periods for each dollar to reach  $\$(1 + r_a)$ .

Let  $r_s$ ,  $r_q$ ,  $r_m$ ,  $r_h$ , and  $r_d$  be the effective semi-annual, quarterly, monthly, semi-monthly, and daily interest rates, respectively, under the assumption that each month is exactly one-twelfth of a 365-day year. As they correspond to  $g = 2, 4, 12, 24,$  and  $365$ , respectively, we must have

$$1 + r_a = (1 + r_s)^2 = (1 + r_q)^4 = (1 + r_m)^{12} = (1 + r_h)^{24} = (1 + r_d)^{365}, \quad (4)$$



satisfying the condition of

$$0 < r_d < r_h < r_m < r_q < r_s < r_a. \quad (5)$$

Equation (4) captures the growth of each dollar to become  $\$(1 + r_a)$  in equivalent ways; the growth is achievable in two half-years, in four quarters, in 12 months, in 24 half-months, or in 365 days. Equation (4) comprises various pairwise relationships among  $r_a$ ,  $r_s$ ,  $r_q$ ,  $r_m$ ,  $r_h$ , and  $r_d$ , from which any unknown effective interest rates can be deduced. The values of these interest rates will always be consistent with each other. Notice that the same idea can be extended to accommodate effective interest rates over other time intervals, such as four months and two months, which correspond to  $g = 3$  and  $6$ , respectively, although repayment schedules based on such time intervals are uncommon in practice. In the following, we illustrate the above analytical materials with some numerical examples.

**Example 1** *If the effective annual interest rate is 6%, what is the corresponding effective quarterly interest rate?*

In this example, the effective annual interest rate  $r_a$  and the effective quarterly interest rate  $r_q$  are related by

$$1 + r_a = (1 + r_q)^4. \quad (6)$$

To solve  $r_q$  from equation (6), where  $r_a = 0.06$ , may appear to be a tedious algebraic exercise at first glance, as students have learned in algebra courses that each quadratic equation has two roots, each cubic equation has three roots, each quartic equation has four roots, and so on. However, as effective interest rates must be real and positive, we can simply take the fourth root of each side of equation (6) to obtain

$$1 + r_q = (1 + r_a)^{1/4}. \quad (7)$$

The computed value of

$$r_q = (1 + r_a)^{1/4} - 1, \quad (8)$$

which is  $(1.06)^{1/4} - 1 = 0.01467 = 1.467\%$ , satisfies the condition of  $0 < r_q < r_a$ , as expected. Any remaining solutions of  $r_q$  from equation (6), which are irrelevant in practice, can be ignored.

Equation (7) can be interpreted intuitively. Students are taught that, if we let a dollar grow for  $T$  periods at an interest rate  $r$  each period, it will become  $\$(1 + r)^T$ . This result is easy to explain in a compound-interest setting if  $T$  is an integer. Equation (7) shows that  $T$  can also

be a proportion. Here, we let a dollar grow for three months, at an effective annual interest rate  $r_a$ , to become  $\$(1 + r_q)$ . With a period being a year, three months are only  $1/4$  of the period. According to equation (7), the equivalence of  $\$(1 + r_q)$  and  $\$(1 + r_a)^T$  requires  $T = 1/4$ . The same idea that  $T$  need not be an integer can be extended to various other pairwise relationships of effective interest rates.

**Example 2** *If the effective semi-monthly interest rate is known to be 1%, what is the corresponding stated annual interest rate that is subject to semi-annual compounding?*

In this example, the stated annual interest rate  $i$  is twice the effective semi-annual interest rate  $r_s$ . Given the effective semi-monthly interest rate  $r_h = 0.01$ , we can find  $r_s$  from

$$(1 + r_s)^2 = (1 + r_h)^{24}, \tag{9}$$

as a year can be viewed as having two semi-annual periods or 24 semi-monthly periods. Taking the square roots of both sides of equation (9) leads to

$$1 + r_s = (1 + r_h)^{12} \tag{10}$$

or, equivalently,

$$r_s = (1 + r_h)^{12} - 1. \tag{11}$$

Notice that we can start with equation (10) directly, instead of equation (9), as there are 12 semi-monthly periods in six months. Either way, the result is still  $r_s = (1.01)^{12} - 1 = 0.12683$ , which satisfies the condition of  $0 < r_h < r_s$ . The stated annual interest rate is  $i = 2r_s = 0.25365 = 25.365\%$ .

**Example 3** *If the stated annual interest rate that is subject to semi-annual compounding is 22%, what is the corresponding effective semi-monthly interest rate?*

Given semi-annual compounding, the effective semi-annual interest rate  $r_s$  is one half of the stated annual interest rate  $i$ . With  $r_s = i/2 = 0.11$ , the effective semi-monthly interest rate  $r_h$  can be deduced from equation (10). By writing equation (10) as

$$r_h = (1 + r_s)^{1/12} - 1, \tag{12}$$

we have  $r_h = (1.11)^{1/12} - 1 = 0.00873 = 0.873\%$ , which satisfies the condition of  $0 < r_h < r_s$ .

**Example 4** *If the effective monthly interest rate is 1.5%, what is the corresponding effective daily interest rate for a 365-day year?*

Given the effective monthly interest rate  $r_m = 1.5\%$ , the corresponding effective daily interest rate  $r_d$  can be deduced from

$$(1 + r_d)^{365} = (1 + r_m)^{12}. \quad (13)$$

This equation captures the equivalence of the growth of each dollar in 365 days at an effective daily interest  $r_d$  and the growth of the same dollar in 12 months at an effective monthly interest rate  $r_m$ . As  $r_d$  must be real and positive, we can write equation (13) as

$$1 + r_d = (1 + r_m)^{12/365} \quad (14)$$

or, more directly, as

$$r_d = (1 + r_m)^{12/365} - 1.$$

For  $r_m = 0.015$ , we have  $r_d = (1.015)^{12/365} - 1 = 0.00049 = 0.049\%$ , which satisfies the condition of  $0 < r_d < r_m$ . Notice that, if  $r_m$  is the unknown instead, we can write equation (13) as

$$r_m = (1 + r_d)^{365/12} - 1 \quad (15)$$

for the computation.

## 2.2 Effective Per-Period Interest Rates: Cases Where the Numbers of Equal Periods in a Year Are Not Integers

As indicated in the introductory section, some financial institutions offer weekly, bi-weekly, and semi-monthly repayment options for borrowers of mortgage loans, in addition to the traditional monthly repayment schemes. Analytical connections between the effective semi-monthly interest rate  $r_h$  and interest rates over various other time intervals have been considered in the previous subsection. This subsection is an extension of the same materials, by considering such connections involving also the effective weekly and bi-weekly interest rates, denoted as  $r_w$  and  $r_b$ , respectively. As there are 52 weeks and one day in a 365-day year, resulting in the annual numbers of weekly and bi-weekly periods not being integers, such connections are easier to establish via the effective daily interest rate  $r_d$ , from a pedagogic standpoint.

Starting with

$$1 + r_b = (1 + r_w)^2 = (1 + r_d)^{14}, \quad (16)$$

where  $r_b$ ,  $r_w$ , and  $r_d$  are all real and positive, we can write

$$1 + r_d = (1 + r_w)^{1/7} = (1 + r_b)^{1/14}. \quad (17)$$

We can also write

$$1 + r_a = (1 + r_d)^{365} = (1 + r_w)^{365/7} = (1 + r_b)^{365/14}, \quad (18)$$

which implies various pairwise relationships among  $r_a$ ,  $r_b$ ,  $r_w$ , and  $r_d$ . It follows from equations (4) and (18) that

$$0 < r_d < r_w < r_b < r_h < r_m < r_q < r_s < r_a. \quad (19)$$

Equation (18) allows us to view the growth of each dollar to  $\$(1 + r_a)$  in a year as being equivalent to the growth of the same dollar in  $365/7$  weeks at the effective weekly interest rate  $r_w$ , or in  $365/14$  bi-weekly periods at the effective bi-weekly interest rate  $r_b$ . The only difference between the algebraic forms of equation (4) and equation (18) is the presence of non-integer exponents in the latter equation. Thus, the examples in the preceding subsection can easily be extended to cases involving weekly and bi-weekly effective interest rates.

Before presenting more examples, notice that the above analytical materials can easily be extended to accommodate cases where each period is three weeks, four weeks, or some other time intervals. For example, letting  $r_t$  be the effective interest rate over a three-week period, we can deduce it from  $1 + r_t = (1 + r_w)^3$  if the effective weekly interest rate is known. As such a period is longer than a half month and shorter than a month, we must have  $r_h < r_t < r_m$ .

Notice also that, in practice, an exactly 52-week year is often assumed for computational convenience. This assumption, which implies  $1 + r_a = (1 + r_w)^{52} = (1 + r_b)^{26}$ , is susceptible to internal inconsistency. To illustrate, suppose that we start with a given effective annual interest rate  $r_a$ . If we deduce the effective weekly, daily, and annual interest rates successively via  $1 + r_w = (1 + r_a)^{1/52}$ ,  $1 + r_d = (1 + r_w)^{1/7}$ , and then  $1 + r_a = (1 + r_d)^{365}$ , the resulting  $r_a$  will differ from its given value. For example, if we start with  $r_a = 5\%$ , these successive computations will result in  $r_a = 5.014\%$ . Such internal inconsistency is nontrivial in practical settings such as computations for weekly or bi-weekly repayments of mortgage loans that are amortized over 20 years or longer.

**Example 5**     *If the effective monthly interest rate is 1%, what is the corresponding effective bi-weekly interest rate for a 365-day year?*

Given the effective monthly interest rate  $r_m = 0.01$ , we can find the corresponding effective bi-weekly interest rate  $r_b$  via

$$(1 + r_b)^{365/14} = (1 + r_m)^{12}, \quad (20)$$

which captures the annual growth of each dollar in two equivalent ways. The result, from

$$r_b = (1 + r_m)^{12 \times 14 / 365} - 1 = (1 + r_m)^{168 / 365} - 1, \quad (21)$$

is  $r_b = (1.01)^{168/365} - 1 = 0.00459 = 0.459\%$ , which is real and positive and satisfies the condition of  $0 < r_b < r_m$ .

**Example 6** *If the stated annual interest rate that is subject to monthly compounding is 6%, what is the corresponding effective weekly interest rate for a 365-day year?*

The effective monthly interest rate  $r_m$  is 1/12 of 6%, which is 0.5%. The corresponding effective weekly interest rate  $r_w$  can be deduced from

$$(1 + r_w)^{365/7} = (1 + r_m)^{12}, \quad (22)$$

or, more directly, from

$$r_w = (1 + r_m)^{12 \times 7 / 365} - 1 = (1 + r_m)^{84 / 365} - 1, \quad (23)$$

by following a similar idea as that in equation (20) of Example 7. The result is  $r_w = (1.005)^{84/365} - 1 = 0.00115 = 0.115\%$ , which is real and positive and satisfies the condition of  $0 < r_w < r_m$ .

### 2.3 Continuously Compounded Interest Rates

Of practical relevance, especially in valuation of various derivative assets, is the interest rate with continuous compounding. This is the case where  $k$ , the frequency of annual compounding, approaches infinity. Analytically, as

$$\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = e, \quad (24)$$

where  $e \approx 2.71828 \dots$  is Euler's number [named after Leonhard Euler (1707-1783)], it can be shown that

$$\lim_{k \rightarrow \infty} \left(1 + \frac{i}{k}\right)^k = \exp i, \quad (25)$$

which is  $e^i$ . However, unless  $i$  is confined to rational numbers, to prove equation (25), based on how  $e$  is defined in equation (24), is beyond the scope of the standard finance curriculum. Thus, instead of formal proofs, numerical illustrations of equation (25) are often provided in introductory finance textbooks. [See, for example, Berk, DeMarzo, and Stangeland (2015, Chapter 5) and Ehrhardt and Brigham (2014, Chapter 4, Web Extension 4C) for some numerical illustrations.]

Given equation (25), to connect the effective annual interest rate to the corresponding continuously compounded annual interest rate is an algebraic exercise. Specifically, combining equations (1) and (25) leads to

$$1 + r_a = \exp i, \tag{26}$$

where  $r_a$  pertains to the case where  $k \rightarrow \infty$ . It follows from equation (26) that

$$i = \ln(1 + r_a). \tag{27}$$

This logarithmic transformation allows the corresponding continuously compounded annual interest rate  $i$  to be deduced from a given effective annual interest rate  $r_a$ .

Combining equations (4), (18), and (27) leads to

$$\begin{aligned} i &= \ln(1 + r_a) = 2 \ln(1 + r_s) = 4 \ln(1 + r_q) = 12 \ln(1 + r_m) \\ &= 24 \ln(1 + r_h) = \frac{365}{14} \ln(1 + r_b) = \frac{365}{7} \ln(1 + r_w) = 365 \ln(1 + r_d), \end{aligned} \tag{28}$$

which allows continuously compounded interest rates for various intra-year periods to be deduced. For example, as

$$\frac{i}{2} = \ln(1 + r_s) \tag{29}$$

implies

$$1 + r_s = \exp\left(\frac{i}{2}\right), \tag{30}$$

$i/2$  can be interpreted as the continuously compounded semi-annual interest rate (given that  $i$  is the continuously compounded annual interest rate). Using equation (28) in an analogous manner, we can also interpret  $i/4$ ,  $i/12$ ,  $i/24$ ,  $14i/365$ ,  $7i/365$ , and  $i/365$  as the continuously compounded quarterly, monthly, semi-monthly, bi-weekly, weekly, and daily interest rates, respectively. That is, to find these per-period interest rates, we simply divide the continuously compounded annual interest rate  $i$  by the number of the corresponding periods in a year, which

need not be an integer.<sup>3</sup> The following are two examples pertaining to continuously compounded interest rates:

**Example 7** *If the stated annual interest rate that is subject to semi-annual compounding is 8%, what is the corresponding continuously compounded monthly interest rate?*

The effective semi-annual interest rate is  $r_s = 8\%/2 = 0.04$ . The corresponding effective annual interest rate  $r_a = (1 + r_s)^2 - 1 = 0.08160$ . The continuously compounded annual interest rate is  $\ln(1 + r_a) = \ln(1.08160) = 0.07844$ . The continuously compounded monthly interest rate, which is  $1/12$  of  $\ln(1 + r_a)$ , is  $0.00654$  or, equivalently,  $0.654\%$ . Alternatively, we can first find the continuously compounded semi-annual interest rate, which is  $\ln(1 + r_s) = \ln(1.04) = 0.03922$ . As there are six months in a semi-annual period, the continuously compounded monthly interest rate is  $1/6$  of  $\ln(1 + r_s)$ , which is also  $0.00654$  or, equivalently,  $0.654\%$ .

**Example 8** *If the continuously compounded semi-annual interest rate is 4.5%, what is the corresponding effective bi-weekly interest rate for a 365-day year?*

The continuously compounded annual interest rate, which is twice the continuously compounded semi-annual interest rate, is  $i = 9\%$  or, equivalently,  $0.09$ . The corresponding effective annual interest rate is  $r_a = \exp i - 1 = \exp(0.09) - 1 = 0.09417$ . As  $1 + r_a = (1 + r_b)^{365/14}$ , where  $r_b$  is the effective bi-weekly interest rate, we have  $r_b = (1 + r_a)^{14/365} - 1 = (1.09417)^{14/365} - 1 = 0.00346 = 0.346\%$ .

### 3 Excel Exercises on Interest Rate Conversion

The interactivity feature of Excel is particularly useful for making the analytical materials of interest rate conversion more meaningful to students. To illustrate, this section presents an Excel-based example that shows the corresponding values of stated and effective annual interest rates, as well as various intra-year per-period interest rates, based on some input information. Figure 1 shows a sample Excel worksheet for the task. Although Excel 2007 is used, the saved file is in its 1997-2003 version for ready access by more readers. (See also the Excel file accompanying this paper.)

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<sup>3</sup>Equation (28) can be stated more concisely as  $i = g \ln(1 + r_g)$ , where  $r_g$  is the effective interest rate over a time interval that is  $1/g$  of a year. The term  $\ln(1 + r_g)$  can be interpreted as the continuously compounded interest rate over such a time interval. The case of  $g = 1$  is where each period is a year, and the corresponding  $r_g$  is  $r_a$ ; in such a case, the right hand side of the equation is simply  $\ln(1 + r_a)$ . The remaining cases, where  $g = 2, 4, 12, 24, 365/14, 365/7,$  and  $365$ , correspond to  $r_g$  being  $r_s, r_q, r_m, r_h, r_b, r_w,$  and  $r_d$ , respectively.

	A	B	C	D	E	F
1						
2		Scroll Bar (0-1000)	250			
3		Stated Annual Interest Rate (Min)	0.0000%			
4		Stated Annual Interest Rate (Max)	20.0000%			
5						
6		Interest Rate	5.0000%			
7						
8		Specify the Type of Interest Rate Data	Indicator			
9		Stated Annual Interest Rates ▲	2			
10		Effective Interest Rates				
11		Continuously Compounded Interest Rates ▼				
12						
13		For Stated Annual Interest Rates, Select One	Indicator		Frequency of Compounding	
14			5		12	
15						
16		Annual Compounding ▲				
17		Semi-Annual Compounding				
18		Quarterly Compounding				
19		Bi-Monthly Compounding				
20		Monthly Compounding				
21		Semi-Monthly Compounding				
22		Bi-Weekly Compounding				
23		Weekly Compounding				
24		Daily Compounding ▼				
25						
26						
27		For Other Interest Rates, Specify the Period	Indicator		Number of Periods in a Year	
28			2		2	
29						
30		One Year ▲				
31		Six Months				
32		Three Months				
33		Two Months				
34		One Month				
35		Half Month				
36		Two Weeks				
37		One Week				
38		One Day ▼				
39						
40						
41		Effective Annual Interest Rate	10.2500%			
42		Stated Annual Interest Rate	9.7978%			
43						
44		Effective Semi-Annual Interest Rate	5.0000%			
45		Effective Quarterly Interest Rate	2.4695%			
46		Effective Bi-Monthly Interest Rate	1.6396%			

Figure 1 An Excel Worksheet for Illustrating Interest Rate Conversion



	A	B	C	D	E	F
41		Effective Monthly Interest Rate	0.8165%			
42		Effective Semi-Monthly Interest Rate	0.4074%			
43		Effective Bi-Weekly Interest Rate	0.3750%			
44		Effective Weekly Interest Rate	0.1873%			
45		Effective Daily Interest Rate	0.0267%			
46						
47		Continuously Compounded Annual Interest Rate	9.7580%			
48						
49		Continuously Compounded Semi-Annual Interest Rate	4.8790%			
50		Continuously Compounded Quarterly Interest Rate	2.4395%			
51		Continuously Compounded Bi-Monthly Interest Rate	1.6263%			
52		Continuously Compounded Monthly Interest Rate	0.8132%			
53		Continuously Compounded Semi-Monthly Interest Rate	0.4066%			
54		Continuously Compounded Bi-Weekly Interest Rate	0.3743%			
55		Continuously Compounded Weekly Interest Rate	0.1871%			
56		Continuously Compounded Daily Interest Rate	0.0267%			
57						
58						
59		Cell Formulas				
60						
61		E14: =CHOOSE(C14,1,2,4,6,12,24,365/14,365/7,365)				
62						
63		E25: =CHOOSE(C25,1,2,4,6,12,24,365/14,365/7,365)				
64						
65		C35: =IF(C9=1,(1+C6/E14)^E14-1,IF(C9=2,(1+C6)^E25-1,EXP(C6)^E25-1))				
66		C36: =((1+C35)^(1/E14)-1)*E14				
67						
68		C38: =(1+C35)^(1/2)-1				
69		C39: =(1+C35)^(1/4)-1				
70		C40: =(1+C35)^(1/6)-1				
71		C41: =(1+C35)^(1/12)-1				
72		C42: =(1+C35)^(1/24)-1				
73		C43: =(1+C35)^(14/365)-1				
74		C44: =(1+C35)^(7/365)-1				
75		C45: =(1+C35)^(1/365)-1				
76						
77		C47: =LN(1+C35)				
78						
79		C49: =LN(1+C38)				
80		copied to C49:C56				

Figure 1 An Excel Worksheet for Illustrating Interest Rate Conversion (Continued)

Interactivity here is achieved via a Scroll Bar and three List Boxes. Both Excel tools are available from the menu item **Developer | Insert**. The Scroll Bar, which is preset (via Format Control) for its minimum and maximum values, the incremental change, and the cell link, enables the user to specify an interest rate. In Figure 1, as the linked cell is C6, the specified interest rate is shown there.<sup>4</sup> The first List Box is preset (also via Format Control) to have three items, with the input range being B9:B11. It allows the user to specify whether the interest rate provided in C6 is (1) a stated annual interest rate, (2) an effective interest rate, or (3) a continuously compounded interest rate, with the selected item number displayed in C9. For better appearance of the worksheet, this List Box is placed over B9:B11, which uses a white font and a white fill for each listed item (in order to make B9:B11 appear as blank cells).

The second List Box is for specifying the annual frequency of compounding, with regard to the stated annual interest rate. It is preset to have nine items (from annual compounding, semi-annual compounding, quarterly compounding, and so on, down to daily compounding), with the input range being B14:B22. Accordingly, the function CHOOSE associated with this List Box has ten arguments. In the function CHOOSE(C14,1,2,4,6,12,24,365/14,365/7,365) for E14, the first argument is the selected item indicator, which can be one of 1, 2, . . . , 9, as provided by C14, and each of the remaining nine arguments is the corresponding annual frequency of compounding.

The third List Box, which also has nine items (labeled as one year, six months, three months, and so on, down to one day), is for effective interest rates and continuously compounded interest rates instead. The selected item number is displayed in C25. The function CHOOSE(C25,1,2,4,6,12,24,365/14,365/7,365) in E25 provides the number of periods in a year. Also for better appearance of the worksheet, these two List Boxes are placed over B14:B22 and B25:B33 in a manner similar to that for the first List Box as described above.

The effective annual interest rate is computed first. Given the information in C6, C9, E14, and E25, the effective annual interest rate can be deduced directly by using the cell formula =IF(C9=1,(1+C6/E14)^E14-1,IF(C9=2,(1+C6)^E25-1,EXP(C6)^E25-1)) for C35. The use of two nested IF functions is intended to accommodate all potential cases, where the interest rate in C6 can be a stated annual interest rate, an effective per-period interest rate, or a continuously compounded per-period interest rate. Once the effective annual interest rate is known, all

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<sup>4</sup>The provided interest rate can also be entered directly to C6 instead, without involving a scroll bar.

related interest rates can be computed subsequently.

In Figure 1, the cells C6, C9, C14, and C25, which contain the input data, are shaded. The use of the Scroll Bar provides a 5.0000% interest rate in C6. As the selection in the first List Box is the second item (with the indicator in C9 being 2), this 5.0000% is an effective interest rate. The selection in the third List Box is for specifying the period involved. Given that the second item is selected (with the corresponding number of periods in a year being 2), this 5.0000% is for six months; that is, it is an effective semi-annual interest rate. Thus, the part of the cell formula for C35 pertaining to C9=2 is relevant. The effective annual interest rate based on  $(1+C6)^{E25-1}$  is  $(1 + 5.0000\%)^2 - 1 = 0.102500 = 10.2500\%$ , as displayed in C35.

The selection in the second List Box allows the stated annual interest rate, which corresponds to a 5.0000% effective semi-annual interest rate, to be deduced. As the fifth item in the second List Box selected, the stated annual interest rate is subject to monthly compounding. The Excel function CHOOSE in E14 returns a 12 as the annual frequency of compounding. Thus, the stated annual interest rate, as displayed in C36, according to the cell formula  $=((1+C35)^{(1/E14)-1})^{E14}$ , is  $[(1 + 10.2500\%)^{1/12} - 1] \times 12 = 0.097978 = 9.7978\%$ .

Other per-period interest rates corresponding to the effective annual interest rate of 10.2500% in C35 are displayed in C38:C56. (See the bottom of Figure 1 for the corresponding cell formulas.) For example, the effective weekly interest rate in C44, based on the cell formula  $=(1+C35)^{(7/365)-1}$ , is  $(1 + 10.2500\%)^{7/365} - 1 = 0.1873\%$ , as displayed. The corresponding continuously compounded weekly interest rate in C55, based on the cell formula  $=LN(1+C44)$ , is  $\ln(1 + 0.1873\%) = 0.1871\%$ , also as displayed.

To benefit more from computational exercises based on the same Excel worksheet, students can first use the Scroll Bar and the three List Boxes to generate other displays and then use any of the displayed interest rates in C35:C56 to compute the corresponding interest rate elsewhere in the same blocks of cells. For example, for the displayed interest rates in Figure 1, suppose that we start with the effective bi-weekly interest rate in C43, which is 0.3750%. To find the corresponding continuously compounded annual interest rate, we can first deduce the effective annual interest rate as  $(1 + 0.3750\%)^{365/14} - 1 = 0.102500 = 10.2500\%$ . The corresponding continuously compounded annual interest rate is  $\ln(1 + 10.2500\%) = 0.097580 = 9.7580\%$ , as shown in C47.

The above Excel worksheet can assist instructors of introductory finance courses in gener-

ating various exercises, assignments, and examination questions on the topic of interest rate conversion. What is convenient is that they do not have to perform the corresponding computations themselves. Their teaching assistants can also use some of the generated exercises for tutorial sessions. Further, the analytical materials in the preceding section can easily be extended to accommodate various intra-year periods that are not covered in the above Excel worksheet. Such an extension is a good exercise for students as well.

## 4 Excel Exercises Involving Practical Mortgage Problems

Computations involving the time value of money, as covered in introductory finance courses, typically include periodic repayments of mortgage loans at fixed interest rates. For a  $\$P$  mortgage loan that requires  $n$  periodic repayments of  $\$M$  each period, under a per-period interest rate  $r$ , students are taught that

$$P = \frac{M}{1+r} + \frac{M}{(1+r)^2} + \frac{M}{(1+r)^3} + \cdots + \frac{M}{(1+r)^n}, \quad (31)$$

where the right hand side is a geometric series, with the ratio of adjacent terms being  $1/(1+r)$ . After some algebraic manipulations, equation (31) can be written equivalently as

$$P = M \left[ \frac{1 - (1+r)^{-n}}{r} \right], \quad (32)$$

for computational convenience. [See, for example, Berk, DeMarzo, and Harford (2015, Chapter 4), Berk, DeMarzo, and Stangeland (2015, Chapter 4), and Ehrhardt and Brigham (2014, Chapter 4) for derivations.]

In practice, monthly repayments of mortgage loans are most common. A  $\$P$  mortgage loan that is amortized over  $T$  years requires  $12T$  monthly repayments of  $\$M$  each month, equations (31) and (32) hold for  $n = 12T$  — which is an integer — and for  $r$  being the effective monthly interest rate  $r_m$ . However, equation (32) can accommodate other periodic repayment schemes; the equation still holds, regardless of how a time period is defined. It is easy to solve numerically, on Excel, any of  $P$ ,  $M$ ,  $r$ , and  $n$  given the remaining three variables in equation (32). [See, for example, Sugden and Miller (2010) for some Excel-based illustrations.]

To compute the periodic payment  $M$  in equation (32) on Excel, for example, suppose that the cells containing the given  $P$ ,  $r$ , and  $n$  are named **Loan**, **Interest**, and **NumPeriods**,

respectively. (The naming of cells in Excel 2007 can be achieved via the menu item **Formulas** | **Define Name**.) Suppose also that the cell containing the present value of annuity factor, which is the bracketed term on the right hand side of equation (32), is named **PVAF**. This cell can be computed by using the formula  $= (1 - (1 + \text{Interest})^{-\text{NumPeriods}}) / \text{Interest}$ . The formula  $= \text{Loan} / \text{PVAF}$ , when used for the cell containing the periodic payment, will complete the computations.

The Excel function PMT can also be used to compute periodic payments in the same loan setting. The function has five arguments, with the first three being mandatory. For the cell names as defined above, these three arguments are **Interest**, **NumPeriods**, and **Loan**. The fourth argument is the cash balance after the final payment; if omitted, it is assumed to be 0. The fifth argument can be 0, 1, or omitted. If 0 or omitted, payments are due at the end of each period; otherwise, payments are due at the beginning of each period instead. For computations based on equation (32), these two optional arguments of the function PMT can simply be omitted. Notice that the function PMT will return a negative number, indicating a payment. Given **Interest**, **NumPeriods**, and **Loan**, an advantage of using two simple cell formulas to compute the corresponding periodic payments based on equation (32), instead of relying on the function PMT directly, is that the analytical underpinnings of the task need not be sacrificed, for the sake of computational convenience.<sup>5</sup>

There are many online mortgage calculators, as posted by various financial institutions, to compute the required monthly repayments, thus allowing potential borrowers to assess the affordability of different repayment schemes at prevailing or expected interest rates. Besides the dollar amount of the loan, which is from user input, the amortization — which is the number of years for the loan to be fully repaid — and the corresponding annual interest rate either are part of user input or are provided directly by the financial institution involved. In the United States, for example, the calculator posted online by Bank of America does not ask for user input on interest rates. For its offered annual interest rates pertaining to amortizations of 15, 20, and 30 years, the corresponding monthly repayments are displayed. In contrast, user input to the online mortgage calculator from Wells Fargo Bank includes both the amortization and the

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<sup>5</sup>In the same loan setting, suppose that the cell containing the periodic payment is named **Payment**. If **NumPeriods**, **Payment**, and **Loan** are known, the Excel function RATE will provide the corresponding **Interest**, which is the effective per-period interest rate pertaining to the loan. However, this function is not intended for the task of interest rate conversion as detailed in Section 2.

annual interest rate.<sup>6</sup> However, in these calculators, as well as many others like them, what the annual interest rate represents is left unexplained.

Similar features can also be found in online mortgage calculators that are posted by Canadian financial institutions. For example, the calculators from TD Canada Trust and HSBC Bank Canada allow user input on amortization and annual interest rates, besides their offered annual interest rates.<sup>7</sup> The calculators from both banks offer weekly and bi-weekly repayment options, in addition to the standard monthly repayment scheme; the semi-monthly repayment option is also available in the HSBC calculator. Likewise, in these and various other calculators that are posted by Canadian financial institutions, what the annual interest rate represents is left unexplained.

By performing mortgage loan computations on Excel and then comparing the computed results with those displayed by various online mortgage calculators, students can gain valuable experience in some practical nuances of interest rate conversion. Take, for example, a \$400,000 mortgage loan that is amortized over 20 years. Bank of America offered a 4.000% annual interest rate, as of July 3, 2014. The posted monthly repayment was \$2,424. In contrast, for the same information (with the 4.000% annual interest rate being part of user input), the posted monthly repayment was \$2,416.99 instead, by the online calculator from TD Canada Trust. The approximately \$7 difference between the two posted monthly repayments, though very small at times of low interest rates such as now, does reveal that the stated interest rates in the above two countries do not correspond to the same effective monthly interest rate.

In the U.S., the stated annual interest rate for mortgage loans is subject to monthly compounding; in Canada, it is subject to semi-annual compounding instead. Thus, the effective monthly interest rate in the former case is  $1/12$  of 4.000%, which is 0.3333333%. In the latter case, as the effective semi-annual interest rate is  $1/2$  of 4.000%, or simply 2.000%, the effective monthly interest rate is  $(1 + 2.000\%)^{1/6} - 1 = 0.330589\%$ , which is lower. For a 0.3333333% effective monthly interest rate, each monthly repayment according to equation (32) is \$2,423.92, which, upon rounding, is the same as the posted figure by Bank of America. For a 0.330589% effective monthly interest rate, the use of equation (32) leads to a \$2,416.99 monthly repayment,

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<sup>6</sup>The two electronic links are <https://www.bankofamerica.com/home-loans/mortgage/mortgage-payment-calculator-results.go> and [http://partners.leadfusion.com/tools/wellsfargo/pathway/pw\\_home02/tool.fcs](http://partners.leadfusion.com/tools/wellsfargo/pathway/pw_home02/tool.fcs).

<sup>7</sup>The two electronic links are <http://www.tdcanadatrust.com/docs/mortCalc/MortgageCalculator.jsp> and <http://www.hsbc.ca/1/2/personal/borrowing/mortgages/knowledge-centre/calculators/payment>.

which is the same as the posted figure by TD Canada Trust.

Excel is ideal for computations involving equation (32). Although any calculator with the  $y^x$  function can be used for the same task, the use of Excel does simplify the computational task greatly, especially for repeated computations. Once  $M$  is expressed in terms of the  $P$ ,  $r$ , and  $n$  via an Excel formula, any change in them will allow the corresponding  $M$  to be updated automatically. This computational convenience will make it easier for students to explore market conventions in stating interest rates in various countries. For example, just like the posted result by the online mortgage calculator from Bank of America, as mentioned above, the one from National Australia Bank also shows a \$2,424 monthly repayment for a 20-year \$400,000 loan at a 4.000% annual interest rate.<sup>8</sup> Posted results for various sets of user input, when compared with the corresponding results from Excel-based computations for equation (32) will enable students to confirm that, for Australian mortgage loans requiring monthly repayments, the stated annual interest rate is subject to monthly compounding.

Computations of weekly and bi-weekly mortgage repayments will also help students gain more experience with interest rate conversion in practice. To illustrate, let us consider a Canadian \$400,000 mortgage loan that is amortized over 25 years, for a 5.000% stated annual interest rate and weekly repayments. As the stated annual interest rate is subject to semi-annual compounding, the effective semi-annual interest rate is  $1/2$  of 5.000%, which is 2.500%. The corresponding effective weekly interest rate is  $(1 + 2.500\%)^{14/365} - 1 = 0.094756\%$ . As there are  $365/7$  weeks in a year without considering any leap years, a 25-year amortization is over  $25 \times 365/7 (= 1303.57)$  weeks. If all leap years are accounted for, the amortization is over approximately  $25 \times 365.25/7 (= 1304.46)$  weeks instead. In either case, as the number of repayments can only be an integer, there are 1304 weekly repayments, upon rounding. The use of a 0.094756% effective weekly interest rate and 1304 weekly repayments for equation (32) will lead to weekly repayments of \$534.46. This result is the same as the displayed result by the online mortgage calculator from TD Canada Trust.

For the same input information, however, the online mortgage calculators from HSBC Bank Canada and Canada Mortgage and Housing Corporation (CMHC) both show weekly repayments

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<sup>8</sup>The corresponding electronic link is <http://www.nab.com.au/personal/loans/home-loans/loan-calculators/loan-repayments-calculator>.

of \$536.02 instead; this amount is higher than the above \$534.46 by \$1.56.<sup>9</sup> To produce this result requires the assumption that a year has exactly 52 weeks. Under such an assumption, the 5.000% stated annual interest rate becomes a stated interest rate for a 52-week period that is subject to compounding in 26 weeks. Accordingly, the effective 26-week interest rate is  $1/2$  of 5.000%, which is 2.500%, and the corresponding effective weekly interest rate is  $(1 + 2.500\%)^{1/26} - 1 = 0.095017\%$ . The amortization of 25 years becomes that of  $25 \times 52 (= 1300)$  weeks. The use of a 0.095017% effective weekly interest rate and 1300 weekly repayments for equation (32) will lead to weekly repayments of \$536.02.

The above example reveals a practical issue. When a borrower of a 25-year mortgage loan at a 5.000% stated annual interest rate is given some repayment options, the effective semi-annual interest rate is supposed to be 2.500%, regardless which option the borrower chooses. However, it is not so, according to the online computations by HSBC and CMHC; when choosing weekly or bi-weekly repayment options (as opposed to semi-monthly or monthly options), the borrower of a 25-year mortgage loan will have to repay the loan in less than 25 years and also will incur a higher interest cost. A higher interest cost is because the 2.500% effective interest rate is for a shorter period of only 26 weeks. At times of low interest rates such as now, the corresponding increase in the interest cost to the borrower, for approximating a year by 52 weeks, may seem inconsequential. As students need to know how interest rate conversion can be performed properly, regardless of the prevailing interest rate levels, it is useful to have the above practical nuances examined, via various Excel examples, when the topic is taught.

As practical exercises, students can explore how weekly and bi-weekly repayment options are treated by various financial institutions in different countries. For example, the posted weekly repayment by the online calculator from National Australia Bank, for the same user input as above — which pertains to a 25-year \$400,000 mortgage loan at a 5.000% stated annual interest rate — is \$539. This posted result is greater than either of the above \$534.46 and \$536.02 figures.

It turns out that, although the stated annual interest rate is subject to the usual monthly compounding for monthly repayments, the weekly repayment option is based on weekly compounding instead. With a year assumed to have exactly 52 weeks, the corresponding effective

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<sup>9</sup>The electronic links to the online mortgage calculator from Canada Mortgage and Housing Corporation is [http://www.cmhc-schl.gc.ca/en/co/buho/buho\\_021.cfm](http://www.cmhc-schl.gc.ca/en/co/buho/buho_021.cfm).



weekly interest rate is  $1/52$  of 5.000%, which is 0.096154%. For a \$400,000 loan with this effective weekly interest rate and  $25 \times 52 (= 1300)$  weeks to fulfill the loan obligations, the use of equation (32) will lead to a weekly repayment of \$539.19, which can be rounded to \$539, the posted result by National Australia Bank. As compared to the monthly repayment option, both the weekly compounding here (instead of monthly compounding) for the same stated annual interest rate and the assumption of an exactly 52-week year are responsible for a higher borrowing cost.

## 5 Concluding Remarks

Interest rate conversion is part of the fundamental materials that are covered in introductory finance courses. Some students find the topic confusing, soon after it is introduced, because of the market convention that the stated annual interest rate does not represent the true annual interest rate. They are taught that the conversion of the stated annual interest rate to any intra-year per-period interest rates — which include effective daily, weekly, bi-weekly, semi-monthly, monthly, quarterly, semi-annual interest rates, and various others — requires the information of whether it is subject to daily, monthly, or semi-annual compounding. Conversely, given a per-period interest rate, the corresponding stated annual interest rate can be deduced, if such information is also known. The term *annual frequency of compounding* — which is often used to describe such information — can be confusing to some students, as they are also taught that, in a world of compound interests, compounding takes places all the time.

Further, as the task of interest rate conversion is algebraic in nature, students with inadequate mathematical preparedness may have difficulties in following the analytical materials involved. Some schools allow financial calculators to be used in examinations. Whether this is a good idea from a pedagogic perspective is debatable. Nevertheless, the use of such calculators is not supposed to be a substitute for actually learning the analytical materials involved. It is difficult for students who believe otherwise — and thus have no incentives to invest adequate time on learning the nuances of time value concepts — to build a good foundation in finance.

Adding to the challenge for some students is the need to convert interest rates over various discrete-time intervals to the corresponding continuously compounded interest rates, for use in continuous-time financial models. Very often, the topic of interest rate conversion is covered

only briefly when the time value of money is taught. Instructors may be unaware of the various difficulties that some students have encountered on the topic until the examination times. By then, the coverage of some other topics that also require the knowledge of interest rate conversion will be in progress or will have been completed.

At times of low interest rates such as now, although consequences of conversion mistakes, if any, tend to be inconsequential, students still need to know proper ways to convert interest rates, as history has taught us that times of low interest rates would not last forever. Our experience with teaching introductory finance courses has enabled us to recognize the importance of encouraging students to invest more efforts on the topic of interest rate conversion. By using Excel as a pedagogic tool, this paper is intended to improve the clarity and the effectiveness in the delivery of the topic and help students appreciate better some of the practical nuances involved.

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