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A fourth-order Runge-Kutta (RK4) Spreadsheet Calculator For Solving A System of Two First-Order Ordinary Differential Equations Using Visual Basic (VBA) Programming

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Keywords
Excel spreadsheet, system of ordinary differential equations, spreadsheet calculator, fourth-order Runge Kutta method

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1 Introduction

The system of ordinary differential equations (ODEs) is frequently used in the modelling of real-world problems. This system of ODEs presents a mathematical model and can be used to help engineers to understand the behaviour of real-world problems. In fact, solving the system of ODEs is an interesting task since the accuracy of the solution obtained could vary depending on different approaches used, and the meaning of the solution obtained might increase the understanding of real-world problems. Hence, it is necessary to design a tool to solve the system of ODEs easily.

Leaon et al [1] demonstrated teaching the numerical solutions of ODEs via the RK4 method using Excel 5.0. They used a macro function to solve a single first-order

ODE and subroutine macro to solve a system of two first-order ODEs using the RK4 method. The drawback for their method is that it only solves the particular case of single ODE or system of ODEs as given in their macro function or subroutine macro. It is not written to handle any single ODE or system of ODEs. For a new ODE or system of ODEs, users have to open macro function or subroutine macro to change to their desired new ODEs.

Tay et. al [2] demonstrated how to solve a system of two first-order ODEs via the RK4 method using Microsoft Excel features namely relative rows, relative columns, fix rows and fix columns in a standard spreadsheet interface without using VBA programming. Hence paper [2] is not a spreadsheet calculator. Therefore, each time a new system of ODEs needs to be solved, users have to retype all commands in a standard spreadsheet interface. Tips in [2] are suitable in a step-by step teaching and learning environment, where users can experience the implementation of RK4 algorithm themselves from scratch in a standard spreadsheet interface.

In our definition, a spreadsheet calculator is easy to use where users only need to input the required information to obtain an automatically calculated full solution. The work of spreadsheet calculator on bivariate approximation, Richardson's extrapolation and numerical differentiation can be seen in [3-6].

Thus, the aim of this paper is to develop a system of two first-order ODEs spreadsheet calculator by using the RK4 method via VBA programming in Microsoft Excel spreadsheet environment.

This paper is organized as follows. Section 2 provides the background of RK4 method for solving system of ODEs. A numerical example is given in Section 3. The spreadsheet calculator and its numerical solution are given in Section 4. Finally, a conclusion sums up the final part of this paper in Section 5.

2 Systems of Differential Equations

Consider an initial-value problem of a system of two first-order differential equations given below:

$$\frac{dx}{dt} = f(t, x, y)$$
$$\frac{dy}{dt} = g(t, x, y)$$

subject to the initial conditions, $x(t_0) = x_0, \quad y(t_0) = y_0$ on the interval, $t_0 \le t \le t_n$.

Here t is an independent variable, x and y are dependent variables, f and g are a system of the first-order DEs, t_0, x_0 and y_0 are initial values of t, x and y, t_n is ending value of t. The independent variable t is discretized such that $t_0,\ t_1 = t_0 + h, t_2 = t_0 + 2h, \ldots, t_n = t_0 + nh,$ where h is the step size of t, The approximate solution of the above system that is obtained by the RK4 method will be given by

$$x_{i+1} = x_i + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4),$$
$$y_{i+1} = y_i + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4),$$

where

$$f_1 = hf(t_i, x_i, y_i),$$
$$f_2 = hf\left(t_i + \frac{h}{2}, x_i + \frac{f_1}{2}, y_i + \frac{g_1}{2}\right),$$
$$f_3 = hf\left(t_i + \frac{h}{2}, x_i + \frac{f_2}{2}, y_i + \frac{g_2}{2}\right),$$
$$f_4 = hf(t_i + h, x_i + f_3, y_i + g_3),$$
$$g_1 = hg(t_i, x_i, y_i),$$
$$g_2 = hg\left(t_i + \frac{h}{2}, x_i + \frac{f_1}{2}, y_i + \frac{g_1}{2}\right),$$
$$g_3 = hg\left(t_i + \frac{h}{2}, x_i + \frac{f_2}{2}, y_i + \frac{g_2}{2}\right),$$
$$g_4 = hg(t_i + h, x_i + f_3, y_i + g_3).$$

3 Numerical Example

For illustration, a network circuit is considered in this section.

Question

The network circuit in Figure 1 below can be modelled by the following system of first-order differential equations:

$$\begin{pmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 100\sin(2t) \\ 40\sin(2t) \end{pmatrix}$$

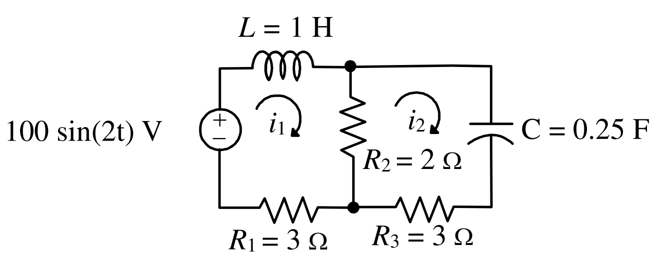


Figure 1: network circuit

Here, t is the time, i_1 and i_2 are the currents in the left and right loops of the network circuit respectively, $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ are the rate of change of currents in the left and right loops of the network circuit respectively. At the initial situation, $t=0$, $i_1=0$ and $i_2=0$. Approximate the currents i_1 and i_2 by using the RK4 method with $h=0.01$ until $t=0.05$. If the exact solution is

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \frac{16}{3}\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-4t} - \frac{8}{3}\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t} + \begin{pmatrix} 20 \\ 8 \end{pmatrix}\sin(2t) - \begin{pmatrix} 8 \\ 0 \end{pmatrix}\cos(2t),$$

find its absolute errors.

Solution

Step 1: The system of ODEs.

$$\frac{di_1}{dt} = -5i_1 + 2i_2 + 100\sin(2t) = f(t, i_1, i_2)$$
$$\frac{di_2}{dt} = -2i_1 + 40\sin(2t) = g(t, i_1, i_2)$$

Step 2: Write the RK4 formula given below:

$$i_{1,k+1} = i_{1,k} + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4)$$
$$i_{2,k+1} = i_{2,k} + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4)$$

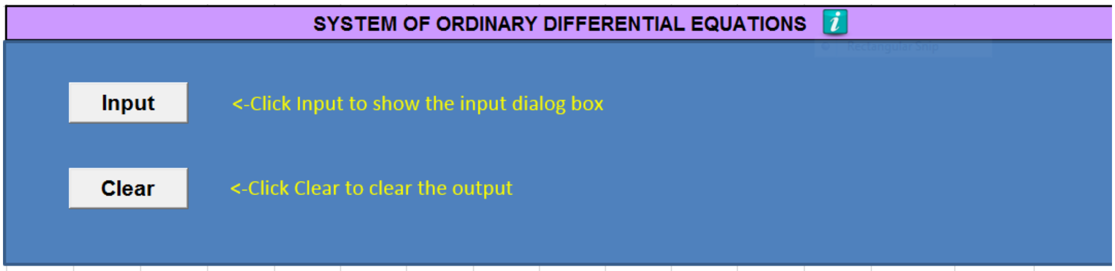
where

$$f_1 = -5i_1 + 2i_2 + 100\sin(2t),$$
$$f_2 = -5\left(i_1 + \frac{f_1}{2}\right) + 2\left(i_2 + \frac{g_1}{2}\right) + 100\sin\left(2\left(t + \frac{h}{2}\right)\right),$$
$$f_3 = -5\left(i_1 + \frac{f_2}{2}\right) + 2\left(i_2 + \frac{g_2}{2}\right) + 100\sin\left(2\left(t + \frac{h}{2}\right)\right),$$
$$f_4 = -5\left(i_1 + f_3\right) + 2\left(i_2 + g_3\right) + 100\sin\left(2\left(t + h\right)\right),$$
$$g_1 = -2i_1 + 40\sin(2t),$$
$$g_2 = -2\left(i_1 + \frac{f_1}{2}\right) + 40\sin\left(2\left(t + \frac{h}{2}\right)\right),$$
$$g_3 = -2\left(i_1 + \frac{f_2}{2}\right) + 40\sin\left(2\left(t + \frac{h}{2}\right)\right),$$
$$g_4 = -2\left(i_1 + f_3\right) + 40\sin\left(2\left(t + h\right)\right).$$

Step 3: The numerical solution for the currents i_1 and i_2 and their absolute errors are shown in Figure 6 in Section 4.

4 An RK4 Spreadsheet Calculator For Solving A System of Two First-Order ODEs

Figure 2 illustrates the RK4 System of ODEs spreadsheet calculator. Firstly, users can select Input button to input the required information needed to solve a system of ODEs or Clear button to clear previous output.



i	t	x	y	f1	g1	f2	g2	f3	g3	f4	g4	Exact x	Exact y	|Error x|	|Error y|
0	0	0	1	0	0	0.125	-0.0312	0.1543	-0.0288	0.3199	-0.1095				
1	0.5	0.1464	0.9618	0.3136	-0.1095	0.4917	-0.2091	0.5176	-0.1642	0.7308	-0.1784				
2	1	0.6569	0.7894												

Figure 2: The initial layout of the RK4 System of ODEs spreadsheet calculator

Once the Input button is clicked, a user form will pop up, and users can start to enter the independent variable, the first dependent variable and second dependent variable as shown in Figure 3. In the first portion of the input form, users can click Define Parameters button or Clear Variables button to delete the variables which have been typed.

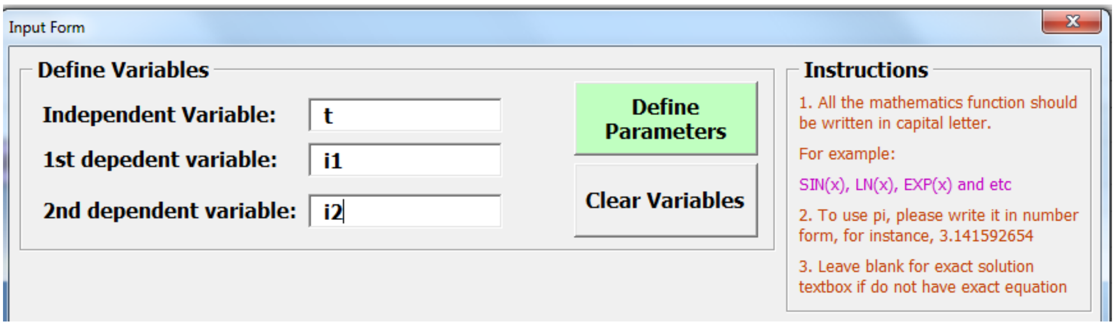


Figure 3: The first portion of the input form

When the Define Parameters button is clicked, users can enter the interval for the independent variable, the initial value of both dependent variables, the step size h and select the desired accuracy of one decimal place, two decimal places, up to the calculation of nine decimal places as seen in Figure 4. Again, users can select Define Equations button or Clear Parameters button to clear the values of the parameters in the second portion of the input form.

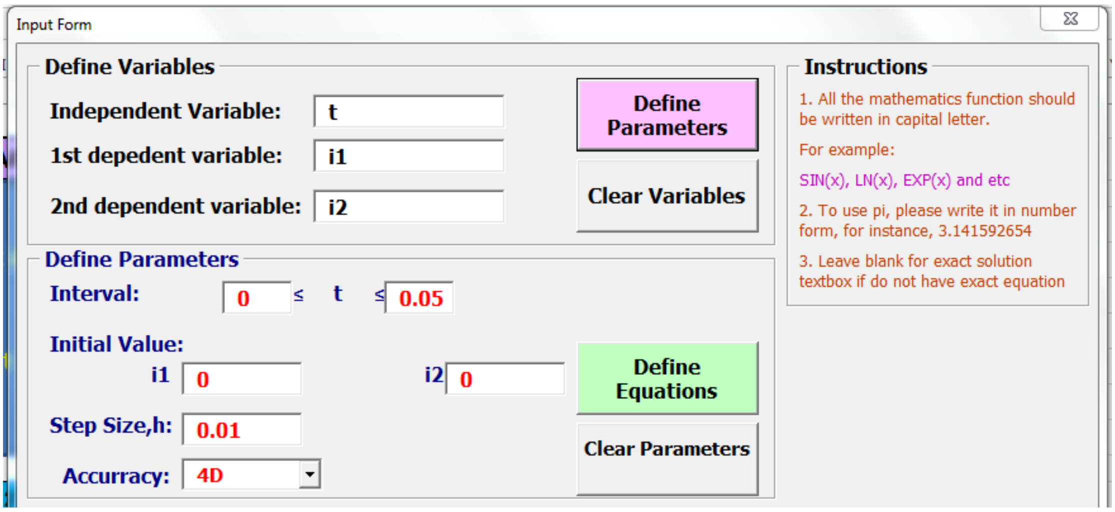


Figure 4: The second portion of the input form

After the Define Equations button is clicked, users can input the system of first-order ODEs via programming syntax. If the exact solutions for the system of first-order ODEs exist, then users can enter their exact solutions so that its absolute errors will be calculated as displayed in Figure 5. In the third portion of the input form, users can finally click Solve button to solve the system of ODEs up to the desired accuracy or click Clear Equations button to clear the system of ODEs and exact functions.

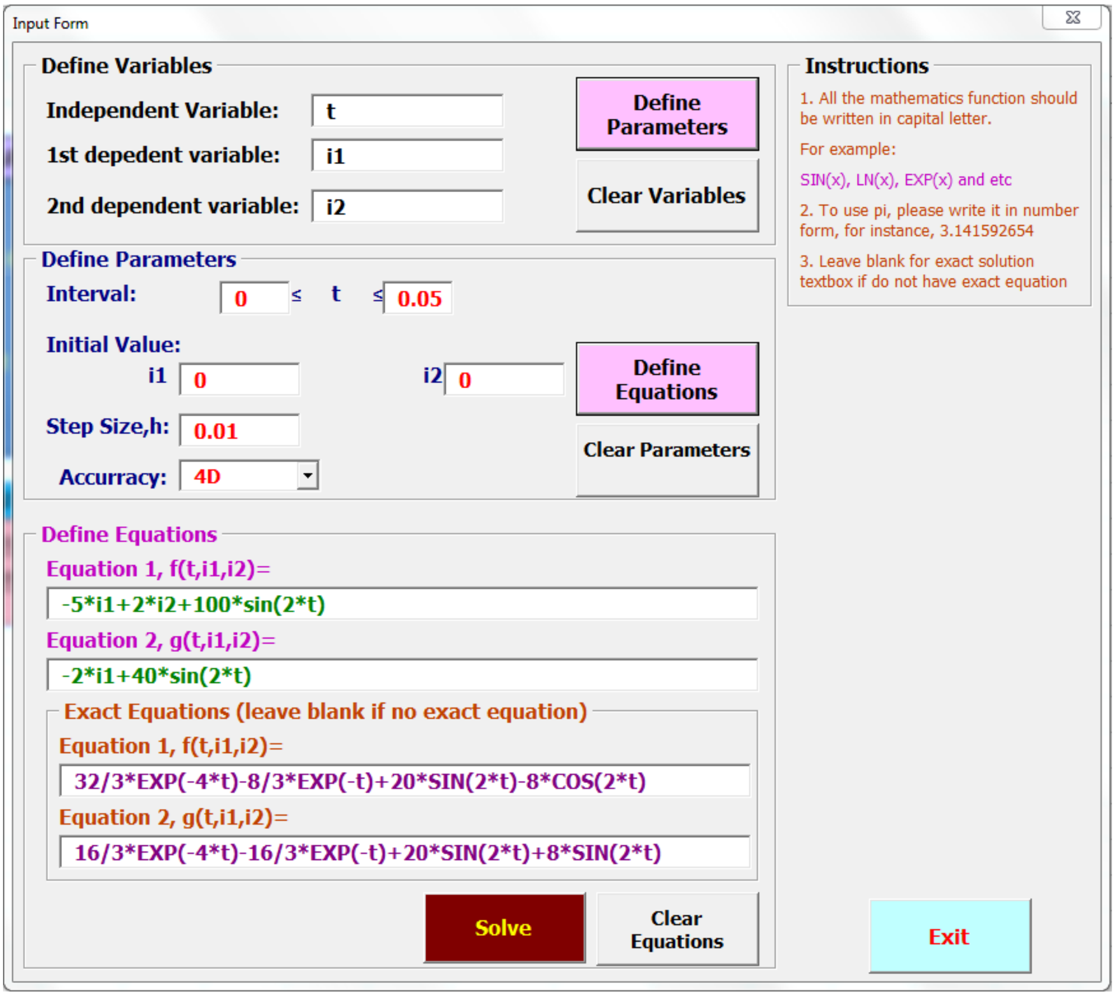
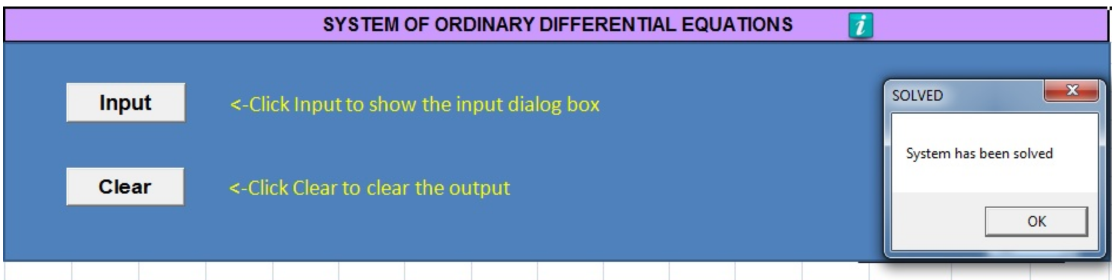


Figure 5: The third portion of the input form

Once the Solve button is clicked, the solution of the system of ODEs obtained via the RK4 method will be calculated automatically based on the desired accuracy. Solve button is associated with VBA programming that is written to compute the solution of the system of ODEs using the RK4 method. The full solution is shown in Figure 6.



i	t	i1	i2	f1	g1	f2	g2	f3	g3	f4	g4	Exact i1	Exact i2	|Error i1	|Error i2
0	0	0	0	0	0	0.01	0.004	0.0098	0.0039	0.0196	0.0078	0	0	0	0
1	0.01	0.0099	0.0039	0.0196	0.0078	0.0292	0.0116	0.029	0.0115	0.0384	0.0152	0.0099	0.0039	0	0
2	0.02	0.039	0.0154	0.0383	0.0152	0.0475	0.0188	0.0473	0.0187	0.0563	0.0223	0.0389	0.0155	0.0001	1E-04
3	0.03	0.0864	0.0342	0.0563	0.0223	0.0651	0.0257	0.0649	0.0256	0.0735	0.0289	0.0863	0.0342	0.0001	0
4	0.04	0.1514	0.0598	0.0735	0.0289	0.082	0.0322	0.0818	0.0321	0.09	0.0353	0.1513	0.0599	0.0001	0.0001
5	0.05	0.2332	0.0919									0.2332	0.092	0	0.0001

Figure 6: The full solution of the system of ODEs using the RK4 method

5 The Evaluation on RK4 Spreadsheet Calculator for Solving A System of Two First-Order ODEs

We have tested this RK4 spreadsheet calculator for a system of two first-order ODEs on 33 Master of Electrical Engineering students who took Advanced Engineering Mathematics (AEM) course in Universiti Tun Hussein Onn Malaysia in semester 1 2014/2015. One of the subtopics in the AEM course is System of ODEs which will be taught in week 8. Initially, the students have been taught how to solve a system of two first-order ODEs traditionally using calculators. After that they were taught how to solve the same system in standard Excel spreadsheet environment using the excel tips by Tay[2]. Lastly, the RK4 Spreadsheet Calculator was given to them and they were exposed on its usage and tested the same system for the third time. The RK4 Spreadsheet Calculator and questionnaire were given to them. They were asked to test the spreadsheet calculator for other systems of ODEs. The questionnaires were collected in week 14.

There are four aspects of evaluation being conducted for this spreadsheet calculator, namely content, design & presentation, teaching strategy design, and technical aspect using a questionnaire that consisted of 24 items. A 5-point Likert scale was used to rate the response of the participants towards the RK4 Spreadsheet Calculator. The participants were asked to answer each question by rating each item using the following scale: 1 = strongly disagree; 2 = disagree; 3 = neutral; 4 = agree and 5 = strongly agree.

The findings of the RK4 Spreadsheet Calculator are shown in Table 1. Findings indicated that the majority of the students agreed that the RK4 Spreadsheet Calculator is useful in terms of content (M=3.82, SD=1.20), design & presentation (M=3.79, SD=1.16), teaching strategy design (M=3.74, SD=1.11) and technical aspect (M=3.81, SD=1.15). According to the results shown in mode, the majority of the participants agreed that this RK4 Spreadsheet Calculator is good when it comes to teaching strategy design, followed by content and technical aspect. This can be proven by the statement given by the participants during the interview. One of the participants mentioned that "*the excel software is perfect program because everyone can use it easily without any problem and the users can get the results during one minute*." However, they are neutral with regards to the design and presentation of the RK4 Spreadsheet Calculator as stated by one of the participants "*Add in sound system for instruction to make it more user friendly*."

Table 1: Descriptive Analyses of RK4 Spreadsheet Calculator Evaluation

Descriptive Statistics	Content	Design & Presentation	Teaching Strategy Design	Technical Aspect
Mean	3.82	3.79	3.74	3.81
Std. Deviation	1.20	1.16	1.11	1.15
Mode	4.00	3.50	4.67	4.00

6 Conclusion

In this paper, a spreadsheet calculator, which applies the RK4 method for solving the system of two first-order ODEs was developed. We employed VBA programming to simplify the use of the calculator. It is very user friendly as users only need to enter relevant information to obtain the full solution of the system of ODEs which will later be displayed to users. However, this spreadsheet calculator is designed for a system of two first-order ODEs only. In future, we hope to increase its capacity to a system of more than two first-order ODEs. It can serve as a numerical tool for educators and students who require a full solution of the system of two first-order ODEs using the RK4 method.

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