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## Empowering Polynomial Theory Conjectures with Spreadsheets

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## **Abstract**

Polynomial functions and their properties are fundamental to algebra, calculus, and mathematical modeling. Students who do not have a strong understanding of the relationship between factoring and solving equations can have difficulty with optimization problems in calculus and solving application problems in any field. Understanding function transformations is important in trigonometry, the idea of the general antiderivative, and describing the geometry of a problem mathematically. This paper presents spreadsheet activities designed to bolster students' conceptualization of the factorization theorem for polynomials, complex zeros of polynomials, and function transformations. These activities were designed to use a constructivist approach involving student experimentation and conjectures.

## **Keywords**

experimentation with functions, function transformations, polynomial theory, factorization theorem for polynomials, conjugate pairs theorem, spreadsheets, active learning, inquiry-based learning

## **Cover Page Footnote**

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## Abstract

Polynomial functions and their properties are fundamental to algebra, calculus, and mathematical modeling. Students who do not have a strong understanding of the relationship between factoring and solving equations can have difficulty with optimization problems in calculus and solving application problems in any field. Understanding function transformations is important in trigonometry, the idea of the general antiderivative, and describing the geometry of a problem mathematically. This paper presents spreadsheet activities designed to bolster students' conceptualization of the factorization theorem for polynomials, complex zeros of polynomials, and function transformations. These activities were designed to use a constructivist approach involving student experimentation and conjectures.

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## 1 Introduction

Many active and exploratory learning activities use an approach based in constructivism; students start with a situation or problem, form the questions that would lead them to a solution, and develop an approach to explain or solve the problem. While it is not clear whether a constructivist approach in mathematics education significantly improves student learning outcomes [4], it is well-aligned with the scientific method and the pattern-finding and conjecturing approach that mathematicians use to develop theorems.

Rauff [8] discusses how students' beliefs about factoring influence their ability to understand and work with the factored form of a polynomial. Allowing students to experiment and form conjectures about polynomials rather than simply presenting them with theorems and rules should help them believe the statements of the theorems more completely. Connecting students' beliefs and ideas about the algebraic, geometric, and numerical properties of polynomial functions should help them understand such functions at a deeper level.

Since at least 1985, spreadsheets have been used in educational settings to aid student learning [5]. Spreadsheets are a valuable tool for instructors to

design activities since they can accomplish complex mathematical, statistical, and graphical tasks without requiring much programming knowledge. If students are required to use a common platform such as Excel, spreadsheets also avoid many issues of compatibility that may result from web-based tools or software that requires installation. In higher education, many universities provide Microsoft Office to students for free or at a discounted price, which helps break down the barrier of access to the technology. Spreadsheets are also widely used in workplace and laboratory environments, so the familiarity and skills that students gain through classroom spreadsheet activities is transferable to their future educational and work endeavors.

The work of Sadri [9] presented spreadsheet activities to help students understand polynomials and other functions. These activities were intended for secondary mathematics education, aligned with the Common Core Standards for Mathematics [3] and the earlier work by Alagic and Palenz [1]. Sadri included an activity that enabled students to experiment with the properties of quadratic functions and further activities that automated the calculations involved in applying the rational zeros theorem and the intermediate value theorem [7]. We aim to take further steps along this path with additional polynomial theory. Spreadsheets are used to expedite calculations, including computations with complex numbers that could not be accomplished on a standard graphing calculator. More importantly, the goal is to allow students to experiment with polynomial functions and receive instant symbolic and graphical feedback about how their choices affect the formula and graph of a function.

In this paper, we present activities intended for a precalculus or college algebra course at either the secondary or postsecondary level. These activities were designed in Microsoft Excel because of its wide availability. We did not use any macros, visual basic, or similar enhancements to Excel's basic functionality for ease of instructor implementation and to avoid issues of compatibility and security settings on students' computers.

## **2 Activity: Factors of a Quartic Polynomial**

This classroom activity uses a quartic polynomial to develop the factorization theorem for polynomials, the behavior of a polynomial near  $x$ -intercepts, and the conjugate pairs theorem [7]. This follows a constructivist approach where students choose coefficients, investigate the changes in the shape of

evaluate	=VALUE(IMPRODUCT(IMSUM(K6,C\$2),IMSUM(K6,E\$2),IMSUM(K6,G\$2),IMSUM(K6,I\$2)))
check precision	=IF(ABS(IMAGINARY(L6))<(10^-10),IMREAL(L6),L6)

Figure 1: Formula to evaluate  $f(x)$  with any four complex constants.

the polynomial, and develop conjectures about the behavior of polynomials in general. The repetition of the word “factor” in the instructions is intended to reinforce the concept for students who still have some uncertainty about the definition and process of factoring.

The spreadsheet allows students to select constants in the factored form of a fourth-degree polynomial and view the resulting graph and expanded form of the polynomial. We chose to have students input their constants to the form  $f(x) = (x + k_1)(x + k_2)(x + k_3)(x + k_4)$  rather than the standard factored form  $f(x) = (x - z_1)(x - z_2)(x - z_3)(x - z_4)$  to make students consciously think about how the signs of the zeros of  $f$  are the opposites of the signs of the constants that appear in the factored form.

In order to allow the most general case where any complex number can be a zero of the function, the IMSUM and IMPRODUCT functions were used rather than standard addition and multiplication. Because Excel can have issues with floating-point precision [6], complex terms do not always cancel as they should through multiplication and addition. This can result in values that should be real having a very small imaginary part. The IF and IMAGINARY functions were used to check for a small imaginary part, and the REAL function eliminates an imaginary part that was the result of a rounding error. The two steps in the formula (evaluating and then checking for floating-point error) appear in Figure 1. These could be combined in a single formula, but are kept separate in the figure for clarity.

If proficiency in Microsoft Excel commands is one of the learning outcomes of the course, the activity could begin by asking students to enter a formula to generate the function values and fill it down. Depending on the level of the students’ Excel knowledge, this could be done for the case of real constants only or for the more general case that handles complex constants.

The IMSUM and IMPRODUCT functions were also used to display the coefficients of the polynomial in expanded form, so students can make a visual connection between the factored form and expanded form, as well as being able to see when the coefficients have a nonzero imaginary part. Figure 2 shows the formulas for the coefficients.

term	coefficient
$x^4$	1
$x^3$	=IMSUM(C2,E2,G2,I2)
$x^2$	=IMSUM(IMPRODUCT(C2,E2),IMPRODUCT(C2,G2),IMPRODUCT(E2,G2),IMPRODUCT(C2,I2),IMPRODUCT(E2,I2),IMPRODUCT(G2,I2))
$x$	=IMSUM(IMPRODUCT(C2,E2,G2),IMPRODUCT(C2,E2,I2),IMPRODUCT(C2,G2,I2),IMPRODUCT(E2,G2,I2))
constant	=IMPRODUCT(C2,E2,G2,I2)

Figure 2: Coefficients for the expanded form of  $f(x)$ .

If an instructor wants students to generate Excel commands as part of the activity, the students could be asked to find the expanded form of the polynomial on paper, combine like terms, and then enter formulas for the coefficients. This could be done for real or complex constants.

In order to focus students on the shape and zeros of the function, the graph is generated automatically without requiring students to select an interval of  $x$ -values. The purpose of this graph is to visualize the zeros of the function. Since the students are free to set constants that could result in any possible values for the zeros of the function, the left and right bounds of the graph needed to be chosen dynamically based on the particular polynomial. For the case where the polynomial has real zeros, we used bounds based on the greatest and least zeros of the polynomial. To ensure that the edge of the graph does not stop exactly at one of the intercepts, we extended the graph at each end by the greater value of 1 unit or 10% of the greatest or least zero of the polynomial.

Ideally, we would show students a comprehensive graph that captures all zeros and all turning points (local extrema) of the function. When all the zeros of the function are real, all turning points of the graph occur between the  $x$ -intercepts, so including all of the  $x$ -intercepts is enough to produce a comprehensive graph. When  $f(x)$  has complex zeros, the graph may have critical points that are not between the  $x$ -intercepts, or no  $x$ -intercepts at all. The derivative of  $f(x)$  is a cubic polynomial, so the critical points of  $f$  could be found using the general solution of the cubic equation. However, the general solution of the cubic relies on complex terms canceling each other out, which does not reliably occur in Excel due to its issues with floating-point precision [6]. It would require a significant amount of coding to circumvent these precision errors, in addition to the complex formulas for the zeros themselves. Bounds on the zeros of  $f'(x)$  could be computed, however these bounds can be large relative to the actual zeros. With the high rate of change of a quartic function, using an overly wide interval often results in a graph with a very

left bound	=IF(OR(C4<>0,C5<>0), C4-MAX(0.1*ABS(C4),1), -CEILING(SQRT(MAX(IMABS(C2),IMABS(E2),IMABS(G2),IMABS(I2),1)),1))
increment	=IF(OR(C4<>0,C5<>0), (C5-C4+MAX(0.1*ABS(C5),1)+MAX(0.1*ABS(C4),1))/100, 0.02*CEILING(SQRT(MAX(IMABS(C2),IMABS(E2),IMABS(G2),IMABS(I2),1)),1))

Figure 3: Left bound and increment for  $x$ -values. Cells C4 and C5 contain the minimum and maximum real zeros of the polynomial, respectively.

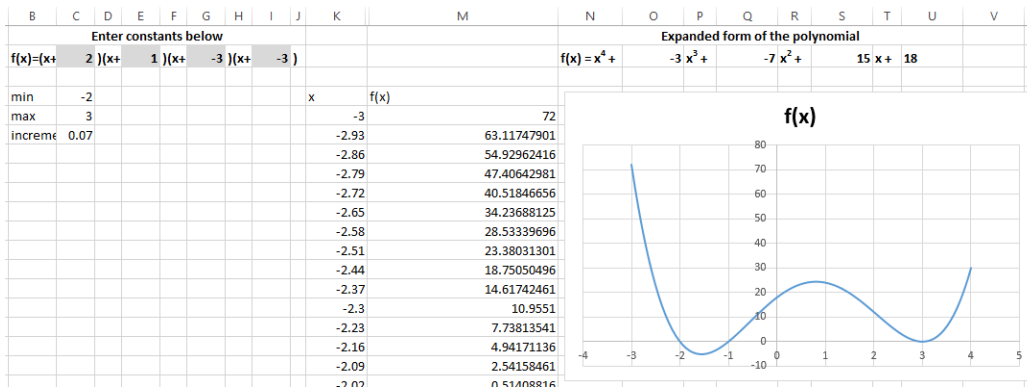


Figure 4: Example of Factors of a Quartic Polynomial spreadsheet with real-valued input.

large vertical scale that can make it difficult to visually determine whether the function has  $x$ -intercepts. Since the main goal of showing the graph in the complex case is to indicate there are no zeros, we used bounds that generated clear graphs in practice. The values used were  $\pm$  the greater of 1 or the square root of the magnitude of the largest constant entered, where the IMABS function was used to find the magnitudes of the constants. The formulas for the left bound of the graph and the increment used to generate one hundred  $x$ -values appear in Figure 3.

## 2.1 Real Coefficients

These exercises are meant to be completed one at a time, with students sharing their observations with the group after each exercise. The italicized questions are follow-up questions in case the student observations do not address particular behaviors of the polynomials. Figure 4 shows an example of the spreadsheet using real coefficients.

1.  $f(x)$  is a quartic, or fourth-degree, polynomial in factored form. Enter the same number for the constants in all four factors. What do you observe about the polynomial?

*Do you see any connection between the number you entered and the graph of the polynomial?*

2. Enter the same number for the constants in three of the factors and a different number for the fourth constant. What changed in the graph of the polynomial?

*Do you see the same relationship between the intercepts of the graph and the numbers you entered?*

3. Use two numbers again; this time, enter one number for the constants in two of the factors and a different number for the other two factors. What changed in the graph of the polynomial?

*Did the shape or behavior of the curve near the  $x$ -intercepts change?*

4. Now use three numbers; enter one number for the constants in two of the factors and different numbers for the two remaining factors. What changed in the graph? Try to come up with some conjectures, general statements about how the constants you choose affect the shape of the graph. How do the constants affect the  $x$ -intercepts of the graph?

*How does the number of times you use the same number (the multiplicity of the factor) affect the graph near the corresponding  $x$ -intercept?*

5. Enter four different numbers for the constants in the four factors. Does the graph appear to be consistent with the conjectures we made?

This activity could be followed by an introduction to the factorization theorem for polynomials, a more formal definition of multiplicity and a general discussion of the behavior of a polynomial near its  $x$ -intercepts.

## 2.2 Nonreal Complex Coefficients

As with the exercises regarding real zeros of a polynomial, these are intended to be completed one at a time and to have students state their observations at each step to develop an understanding of how their choices of zeros affect the polynomial. The italicized questions are follow-up questions. Figure 5 shows an example of the spreadsheet using nonreal complex coefficients.



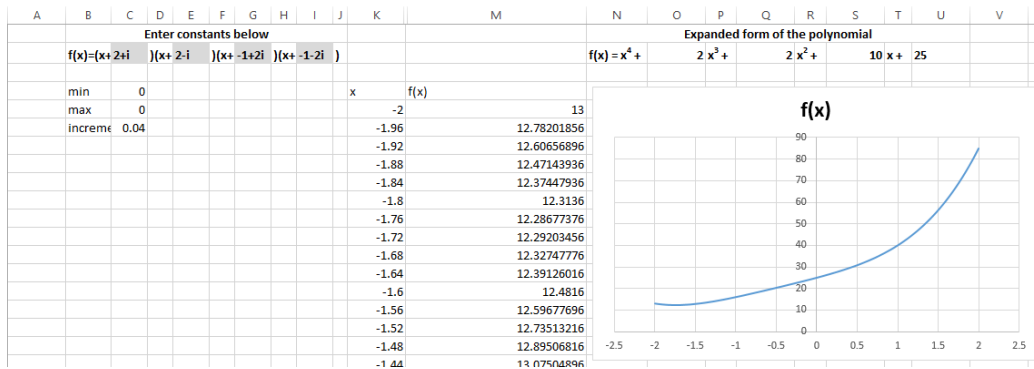


Figure 5: Example of Factors of a Quartic Polynomial spreadsheet with nonreal complex input.

1. Enter real numbers for the constants in three of the factors and a complex number in the form  $a + bi$  where  $b \neq 0$  for the fourth factor. What do you observe about the polynomial?

*Why do you think Excel didn't display a graph?*

2. Enter two real numbers in two of the factors and two nonreal complex numbers in the other two factors. Do you notice any difference compared to when the function had only one complex zero?
3. When you solve real quadratic equations with the quadratic formula, nonreal solutions always came in the form of conjugates  $a \pm bi$ . Enter two real numbers in two of the factors and two complex conjugates ( $a + bi$  and  $a - bi$ ) in the other two factors. What happens to the graph? What similarities and differences do you see between this graph and the graphs you generated using four real constants? Try to come up with some conjectures about how complex constants affect the shape of the graph.
4. Enter two pairs of complex conjugates ( $a \pm bi$  and  $c \pm di$ ). Does the graph appear to be consistent with the conjectures we made?

This activity could be followed by an introduction to the conjugate pairs theorem.

### 3 Activity: Function Transformations

This classroom activity uses quadratic, cubic, and quartic polynomials to help build an intuition for the various transformations that can be applied to the graph of a parent function. Here, students are supplied with the basic polynomial function and its graph in an Excel workbook. For each of the parent polynomials stated above, the associated workbook contains five sections:

1. **Constants Election** The top left-hand corner of the workbook contains  $f(x)$ ,  $p(x)$ , and the bound and increment cells. Here,  $f(x)$  represents the final transformed function to be generated off of the parent polynomial function  $p(x)$ . We use a conventional approach to denote our functional transformations where, given a parent polynomial  $p(x)$ , all the transformations which can be applied and still remain in the family of functions of  $p$  are captured by  $f(x) = a \cdot p(b \cdot (x - c)) + d$ . To this purpose, we have highlighted the cells of  $f$  which represent the constants  $a$ ,  $b$ ,  $c$ , and  $d$ . These highlighted cells are pre-populated with the entries  $a = 1$ ,  $b = 1$ ,  $c = 0$ , and  $d = 0$ , which the students will then change using a constructivist approach in the activity to build their understanding of the transformations that each of the constants can produce.
2. **Graph Window Parameters** Below the Constants Election section of the workbook is the parameterization of the Graph Window. This section is included so that the students can see how the plot of the graphs is chosen, respective of the transformations that have been applied. In particular, to choose the minimum  $x$ -value, we employ the MIN function in Excel to take the least value of either  $-2$  or  $\frac{-2}{b} + c$ . In a similar vein, to choose the maximum  $x$ -value, we employ the MAX function in Excel to take the greater value of either  $2$  or  $\frac{2}{b} + c$ . Both Excel functions are depicted in Figure 6. Finally, the cell beneath the “max”  $x$ -value is the increment width setting used to generate 100 partitioning  $x$ -values.
3. **Graph Window** Below the Graph Window Parameters is the Graph Window itself. Here, the students will see the graph of the parent polynomial function and each subsequent function transformation that they make. Since it is common practice to name a polynomial function

X		✓		fx		=MIN(-2,-2/E2+G2)	
B	C	D	E	F	G	H	I
Enter constants below							
f(x)=	1.00	(	1.00	(x-	0.00	) <sup>3</sup> +	0.00
p(x)=	x <sup>3</sup>						
min	-2						
max	2.0						
incremer	0.04						

X		✓		fx		=MAX(2,2/E2+G2)	
B	C	D	E	F	G	H	I
Enter constants below							
f(x)=	1.00	(	1.00	(x-	0.00	) <sup>3</sup> +	0.00
p(x)=	x <sup>3</sup>						
min	-2						
max	2.0						
incremer	0.04						

Figure 6: Parameterization of the Graph Window.

Enter constants below						Expanded form of the polynomial								
f(x)=	1.00	(	1.00	(x-	0.00	) <sup>3</sup> +	0.00	f(x)=1	x <sup>3</sup>	0	x <sup>2</sup>	0	x	0
p(x)=	x <sup>3</sup>													

Figure 7: Expansion of the Transformed Polynomial.

$p(x)$ , we chose to label the parent polynomial function  $p(x)$  for the double entendre and mnemonic where the dependent variable  $p$  represents both “polynomial” and “parent”. In a similar fashion, the final transformed function is billed as  $f(x)$  since it is the final function and it is in the family of functions of  $p$ .

- Polynomial Expansion** In the top right-hand corner of each workbook is the expanded form of the polynomial, based off of the Constants Election data that the students feed into the highlighted cells. Further exercises could be developed here to help tie together the concepts of the Function Transformations Activity and the Factors of Polynomials Activity workbooks. Since both of these workbooks include the expanded forms of the polynomials, relating the expansions would serve as a bridge between the two activities.
- Function Tables** Finally, below the Polynomial Expansion section are the 6 function tables that will produce the numerical values determined by the choices of  $a$ ,  $b$ ,  $c$ , and  $d$  as elected by the students. These tables are what we use to generate the graphs of each transformation and the parent function  $p$ .

The students are guided through a step-by-step process, outlined below, to transform  $p$  into  $f$ . Each function transformation is color coded to help identify the transformation to its graph, beginning with  $p$  being

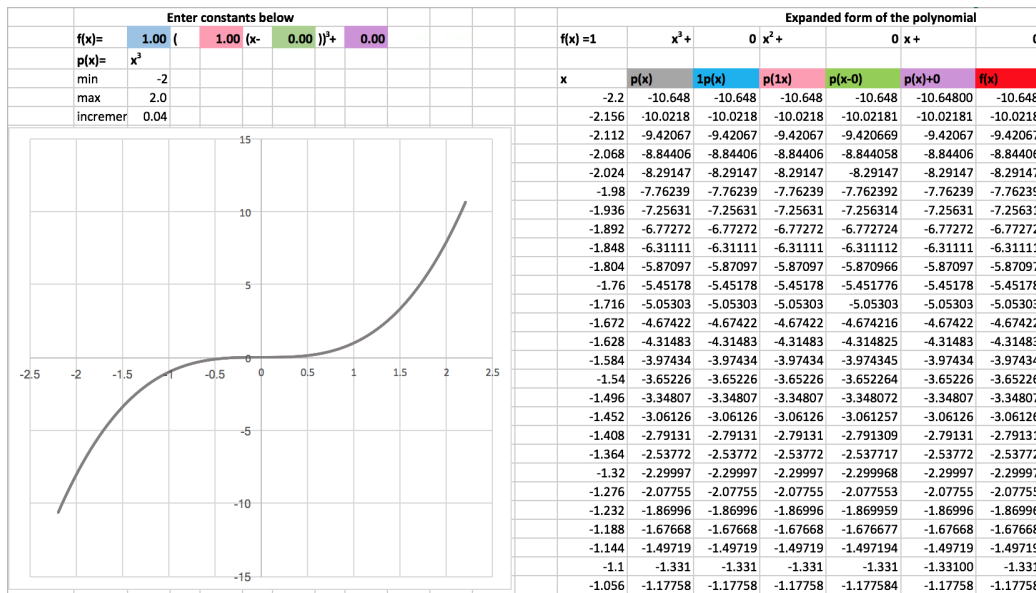


Figure 8: Cubic Parent Graph  $p(x) = x^3$ .

color coded grey, as depicted in Figure 8.

### 3.1 Cubic Transformations Activity

We outline the general ideas and processes for the transformations activities using the cubic polynomial workbook as our model. The basic principles are the same, and while the activities may feel redundant, it is deliberate that the questions we ask in the cases of the quadratic, cubic, and quartic polynomials remain the same. This is to affirm the roles that  $a$ ,  $b$ ,  $c$ , and  $d$  play in the transformations of a parent function, regardless of the function itself.

Before we describe the general process for this activity, we offer a brief discussion of the ranges that  $a$ ,  $b$ ,  $c$ , and  $d$  may assume.

While the ranges of each of these constants may appear to be limited, the values are not restricted to integers. The rationale in limiting the range on the constants is solely for the convenience of ensuring that the Graph Window remains within manageable tolerances. Indeed, given the rate at which the polynomials grow, an unrestricted choice of  $a$ ,  $b$ ,  $c$ , or  $d$  could quickly escalate out of control. For instance, simply relaxing the conditions

of either  $b$  or  $c$  to a range between  $-10$  and  $10$  could result in terms of the polynomials containing factors as large as magnitudes  $100,000$  in the case of the quartic. In fact, even magnitudes as large as  $1,000,000,000$  could arise if the conditions of both  $b$  and  $c$  are relaxed to a range of  $-10$  to  $10$  in the quartic! To this end, we have set the following ranges:

1.  $a \in [-10, 10]$
2.  $b \in [-5, 5]$
3.  $c \in [-5, 5]$
4.  $d \in [-10, 10]$

The tolerance on both  $a$  and  $d$  is due to their placement within the polynomials allowing for them to be more relaxed. Below, we describe this activity's process.

1. To begin, the students are asked to multiply  $p$  by a non-zero constant  $a$  greater than or equal to  $-10$  but less than or equal to  $10$ , and  $a$  should not be equal to  $1$ .

Once the students have chosen a value for  $a$  and entered it into the highlighted cell of the Constants Election section, a new graph will appear in the Graph Window. This new function is color coded blue in the Excel workbook to help the students identify what effect a scalar multiple of a function has on the graph of the function by comparing the new blue function to the original grey, exemplified in Figure 9.

2. Next, we ask the students to multiply a non-zero constant  $b$  greater than or equal to  $-5$  but less than or equal to  $5$  by the independent variable  $x$  in the parent function  $p$ . The graph and function associated with the effects of  $b$  is color coded pink. At this point, assuming reasonable and good choices for  $a$  and  $b$  have been made, four graphs should now be displayed: the original parent function  $p(x)$ , family members  $a \cdot p(x)$ , and  $p(b \cdot x)$ , and a new red graph. Such a graph can be witnessed in Figure 10. The red graph is the composite  $f(x) = a \cdot p(b \cdot x)$ . The goal is to have the students eventually formulate the conjecture that  $f(x)$  reacts to all of the transformations applied to  $p(x)$ .

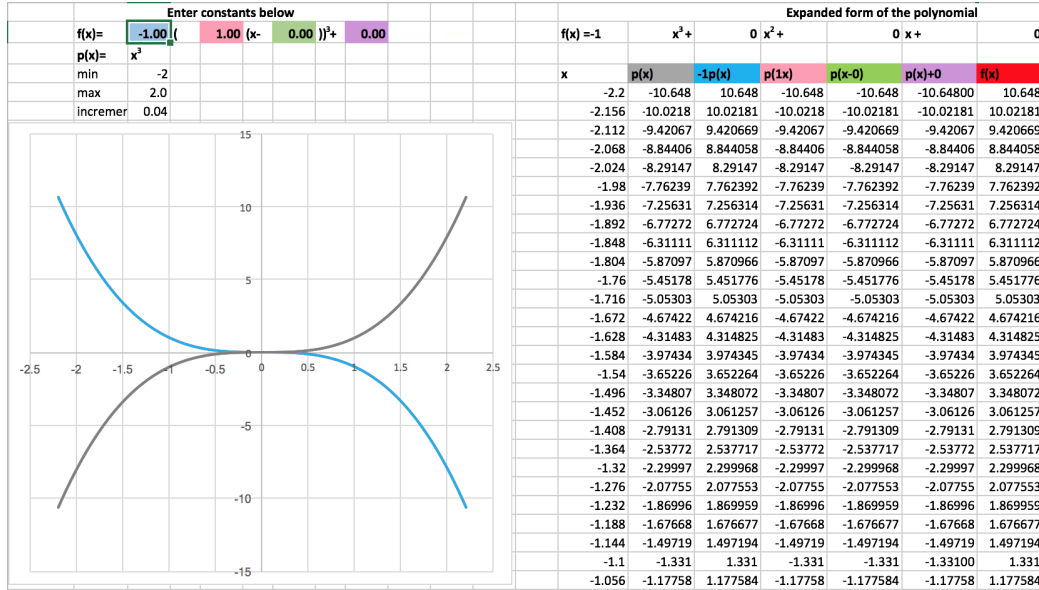


Figure 9: Scalar multiple of  $p(x)$ .

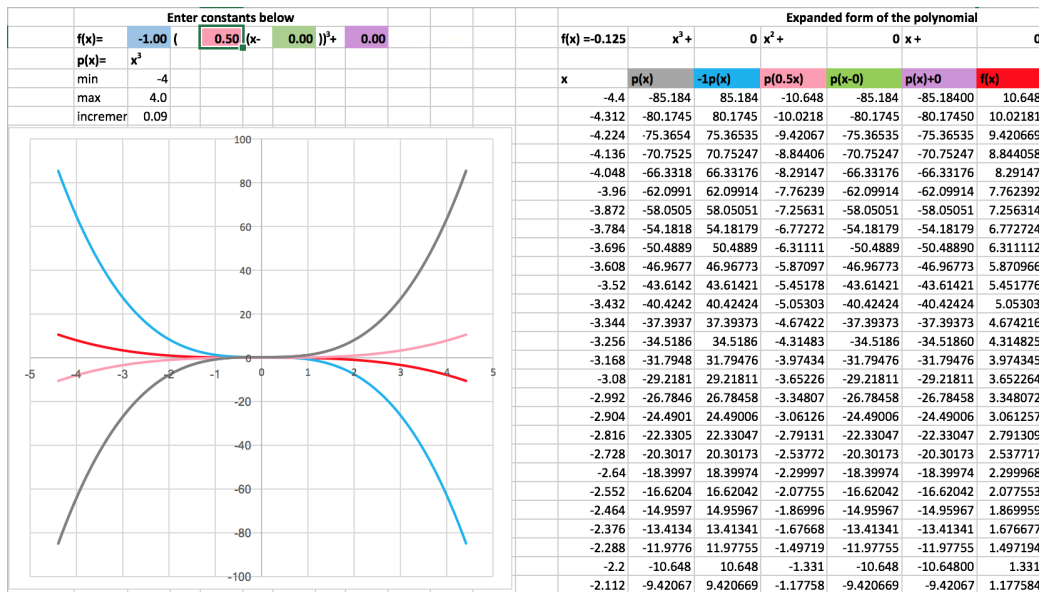


Figure 10: Scalar multiple of  $x$ .







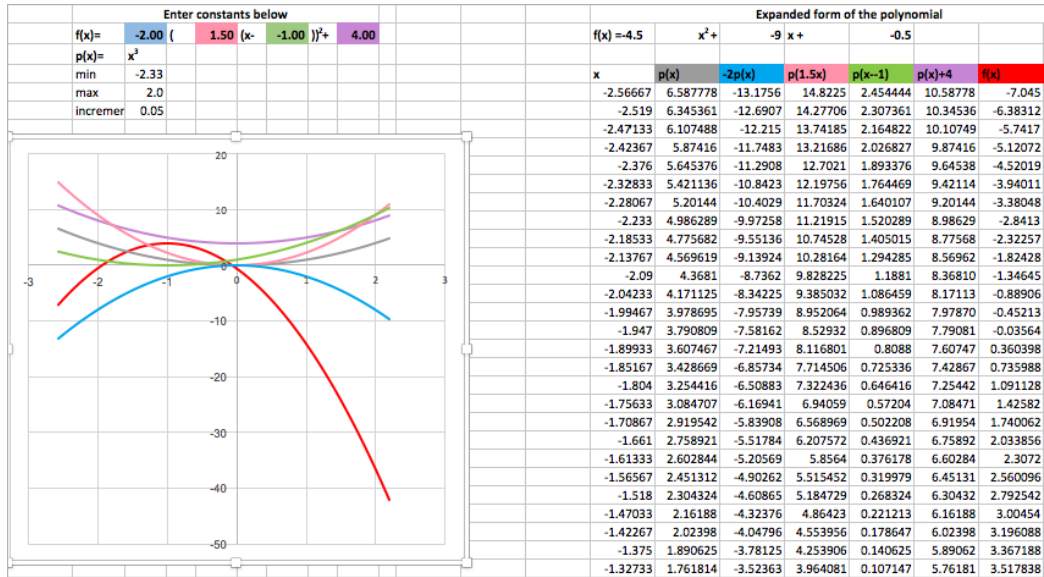


Figure 13: Example of a transforming the parent quadratic function  $p(x) = x^2$ .

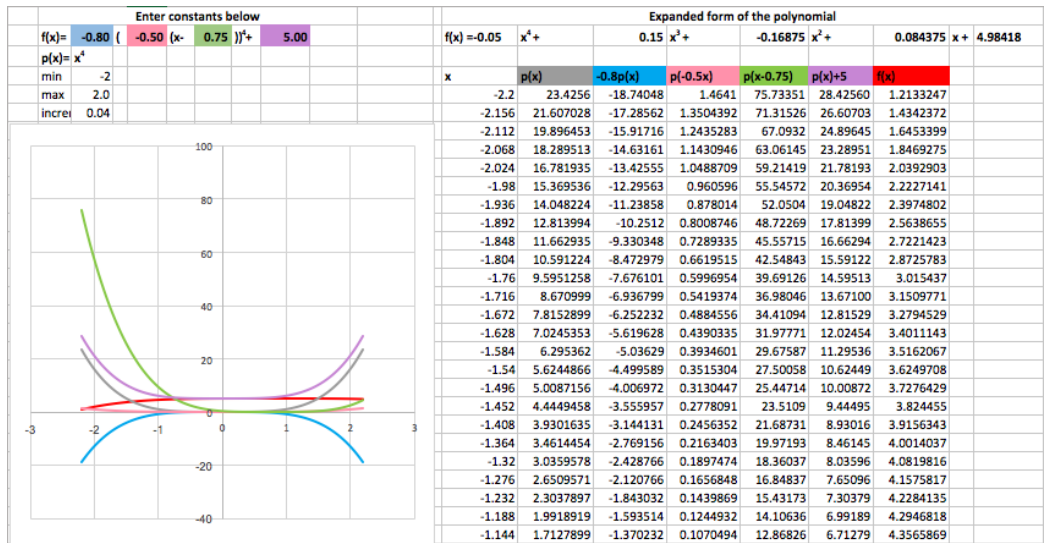


Figure 14: Example of a transforming the parent quartic function  $p(x) = x^4$ .

different values for  $a$ . Formulate a conjecture about how  $a$  transforms the parent function.

*If you have not already, try the following:*

- (a) *Enter a negative value for  $a$ . What happens to the new graph as compared to the parent polynomial?*
- (b) *Enter a value for  $a$  such that  $0 < |a| < 1$ . What happens to the new graph as compared to the parent polynomial?*

2. Enter a real non-zero number between  $-5$  and  $5$  for the scalar  $b$ . What do you observe about the resulting polynomial? How does it compare to the parent polynomial function? Repeat this step using different values for  $b$ . Formulate a conjecture about how  $b$  transforms the parent function.

*If you have not already, try the following:*

- (a) *Enter a negative value for  $b$ . What happens to the new polynomial graph as compared to the parent polynomial graph?*
- (b) *Enter a value for  $b$  such that  $0 < |b| < 1$ . What happens to the new polynomial graph as compared to the parent polynomial graph?*

3. Enter a real non-zero number between  $-5$  and  $5$  for the constant  $c$ . What do you observe about the resulting polynomial? How does it compare to the parent polynomial function? Repeat this step using different values for  $c$ . Formulate a conjecture about how  $c$  transforms the parent function. *If you have not already, try the following:*

- (a) *Reset the initial values for  $a$  and  $b$  to both be 1. Make note of the  $x$ -intercept(s) of the parent polynomial graph. Varying the values for  $c$  only, what happens to the  $x$ -intercepts of the new polynomial graph as compared to the parent polynomial graph?*
- (b) *The common approach to expressing a polynomial function transformation is  $f(x) = a \cdot p(b \cdot (x - c)) + d$ . Notice the sign preceding  $c$ . How does the resulting transformation caused by  $c$  differ from the sign preceding  $c$ ? Algebraically, what is happening?*

4. Enter a real non-zero number between  $-10$  and  $10$  for the scalar  $d$ . What do you observe about the resulting polynomial? How does it

compare to the parent polynomial function? Repeat this step using different values for  $d$ . Formulate a conjecture about how  $d$  transforms the parent function.

*If you have not already, try the following:*

- (a) *Reset the initial values for  $a$  and  $b$  to both be 1 and the initial value of  $c$  to be 0. Make note of the  $y$ -intercept of the parent polynomial graph. Varying the values for  $d$  only, what happens to the  $y$ -intercept of the new polynomial graph as compared to the parent polynomial graph?*

Once the students have verified that their conjectures are correct with the instructor, this activity can serve as the springboard into more generalized function transformation theory for non-polynomial functions including exponential, logarithmic, and trigonometric functions.

## 4 Conclusion

These spreadsheet activities were designed using a constructivist approach to help students build an algebraic, numerical, and graphical understanding of zeros and transformations of polynomial functions. Since Microsoft Excel is a common platform, the software should not be a barrier as long as students have their own computers or the activity occurs in a computer lab environment. Students who are familiar with Excel should have little difficulty approaching these activities. Learners sometimes have issues with the cognitive load of learning software and mathematics concurrently [2]. However, for these activities, the only input to the spreadsheet is numerical and the output is graphical and algebraic using standard notation. These activities provide a concrete hands-on introduction to theory and operations involving polynomials to enhance the learning process.

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