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# Excel VBA-Based Solution to Pipe Flow Measurement Problem

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**Abstract**

Estimation of mean flow velocity in a pipe from pressure drop measurements using Darcy-Weisbach formula involves two nested loops of iteration. The outer loop iterates through the mean flow velocity while an inner loop is enclosed for calculating friction factor at a given flow velocity. The iterations are time-consuming and are somewhat difficult for hand calculations. MS Excel can be employed to solve this type of problem for saving time. This paper intends to show how it is possible to easily solve the mean flow velocity problem in a spreadsheet environment. Visual Basic for Applications (VBA) functions to be used with spreadsheet iterations were provided. Besides, solutions to variations of the above-mentioned problem such as calculation of roughness height were described. Both students and teachers can benefit from the procedures and electronic annex provided.

## 1. Introduction

One of the most frequent problems that field environmental engineers face during their daily work is the measurement of flow in pipes running with air, water, or any other fluids. The problem of pipe flow measurement can be overcome in a number of ways instrumentally by electronic flow meters which employ ultrasonic or thermal methods in addition to other methods for measuring fluid flow. On the other hand, measurement of flow in terms of either flowrate or flow velocity can be accomplished by simple differential pressure measurements especially in cases that no sophisticated devices are installed on the pipe and that quick, effortless measurements are required.

The flow velocity can be calculated as a function of pressure drop and pressure drop can be measured simply by a u-tube manometer. The pressure drop through a pipe is expressed by the phenomenological Darcy-Weisbach equation as follows:

$$\Delta P = \frac{1}{2}\rho \frac{fL}{D} V^2 \qquad (1)$$

where $\Delta P$ is the pressure drop (Pa), $\rho$ is the fluid's density (kg/m³), $L$ is the pipe length (m), $D$ is internal diameter of the pipe (m), $V$ is mean flow velocity (m/s), and $f$ is the friction factor (dimensionless). The friction factor ($f$) is a function of the pipe's roughness height and internal diameter as well as Reynolds number, and is calculated by the implicit Colebrook equation [1] as follows (only under turbulent conditions, that's, Re>4000):

$$\frac{1}{\sqrt{f}} = -2 \log_{10}\left[\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right] \qquad (2)$$

where $\varepsilon$ is roughness height of the pipe (m), and $Re$ is Reynolds number, which is a measure of the turbulence in the pipe and is calculated using the formula

$$Re = \frac{VD}{\upsilon} \qquad (3)$$

where $\upsilon$ is kinematic viscosity of the fluid (m²/s). For practical purposes, one can easily calculate the pressure drop if the roughness height, internal diameter, and length of the pipe, the density and kinematic viscosity of the fluid as well as the mean flow velocity in the pipe are known. In contrast, the solution to the problem of flow measurement, which involves calculating the velocity from pressure drop measurement, is not straightforward and one must use an iterative procedure since friction factor is a function of Reynolds number and Reynolds number, in turn, is a function of flow velocity.

The solution to the problem of calculating flow velocity from pressure drop involves two nested loops. The outer loop iterates through the flow velocity while the inner loop involves iterations for calculating friction factor at the current estimate of flow velocity. Therefore, calculations take time and use of spreadsheets offer advantages for saving time. MS Excel can be used for this iterative procedure. Today, MS Excel is one of the most commonly used tools for teaching and professional works, and a great number of research papers have been dedicated to the use of spreadsheets for teaching purposes [2-16].

The purpose of this "In the Classroom" paper is to present a spreadsheet-based method of solving the problem of calculating mean flow velocity in a full-flow pipe using pressure drop measurement. The design of an experimental section is also provided. The solution algorithm is prepared to minimize the requirement of students' programming skills and user-defined Visual Basic for Applications (VBA) functions for calculating friction factor iteratively by Colebrook equation is also provided. Finally, additional tasks and challenges for students are described related with the Darcy-Weisbach and Colebrook equations, and solutions for these additional tasks are given.

## 2. Materials and methods
### 2.1. Solution by Newton-Raphson

Although a great number of explicit formulations have been proposed over years [17-30] (a detailed discussion of explicit formulae can be found in [31, 32]), Colebrook equation is the most widely used model for calculating friction factor under turbulent conditions. One major drawback of Colebrook equation is that the equation is defined implicitly in friction factor and one must calculate the friction factor iteratively.

In regard to the problem of calculating mean flow velocity from measured pressure drop, one can realize that the flow velocity is also implicitly defined in Eqn. (1) through (3), and iterations must be performed for calculating the flow velocity. Sophisticated methods like Newton-Raphson can be employed in iterations, however, the iterative procedure of calculating friction factor for a given flow velocity complicates the Newton-Raphson iterations because in each iteration, one must establish an inner loop for calculating friction factor.

Solution to the problem of calculating flow velocity by Newton-Raphson starts with re-arranging Eqn. (1) to obtain a function that has at least one real root as follows:

$$g(V) = \frac{1}{2}\rho\frac{fL}{D}V^2 - \Delta P \tag{4}$$

The root of the function $g$ is the desired value of mean flow velocity at which the pressure drop is measured. Applying Newton-Raphson to this function, following steps must be taken to calculate the flow velocity:

1.  Make an initial estimate of flow velocity ($V_0$), and set the iteration counter to zero ($i = 0$).

2.  Calculate Reynolds number ($Re_i$) for the $i^{\text{th}}$ estimate of flow velocity by Eqn. (3).

3.  Calculate friction factor ($f_i$) for the $i^{\text{th}}$ estimate of flow velocity by Eqn. (2). For this purpose, take the steps below.

3.1. Make an initial estimate of friction factor ($f_{i,0}$), and set the iteration counter to zero ($j = 0$).

3.2. Set the iteration counter as $j = j + 1$ and calculate next estimate of friction factor ($f_{i,j}$) by Eqn. (2).

3.3. Calculate percent change ($\xi$) between the last estimates of friction factor ($f_{i,j}$ and $f_{i,j-1}$) as follows:

$$\xi = \left|\frac{f_{i,j}-f_{i,j-1}}{f_{i,j}}\right| * 100\% \tag{5}$$

3.4. If the calculated value of percent change ($\xi$) is equal to or less than a predefined value of maximum percent change ($\xi_{max}$), assign the last estimate of friction factor as $f_i = f_{i,\,j}$ and leave the iteration. If not, return to Step 3.2.

4.  Set the iteration counter as $i = i + 1$ and calculate new estimate of flow velocity ($V_i$) as follows:

$$V_i = V_{i-1} - \frac{g(V_{i-1})}{g'(V_{i-1})} \tag{6.a}$$

The derivative term $g'(V_{i-1})$ can be approximated simply by a central finite difference as follows:

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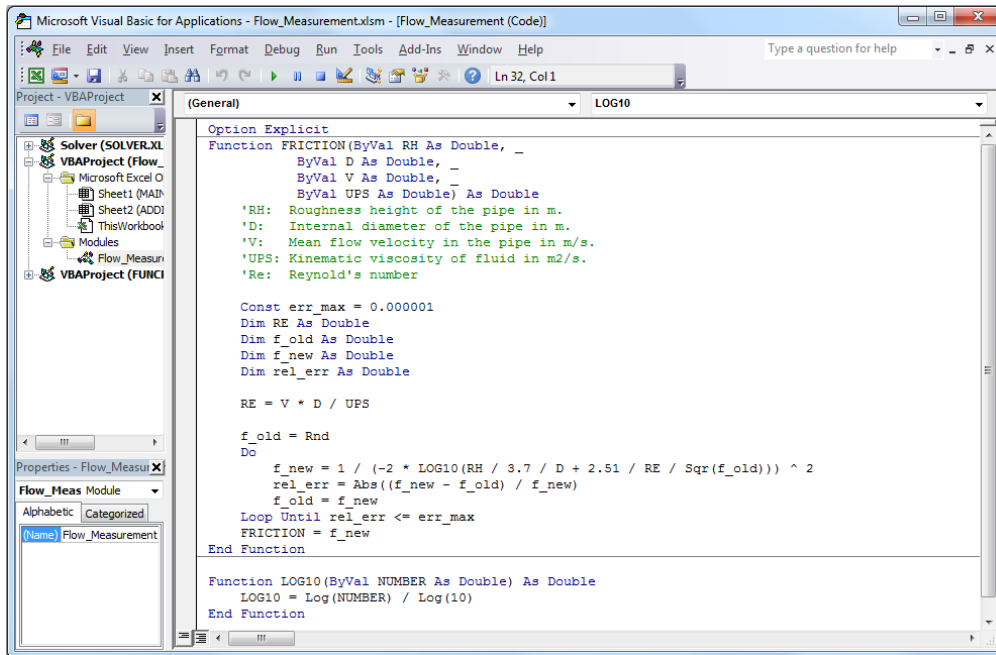$$g'(V_{i-1}) = \frac{g(V_{i-1}+h) - g(V_{i-1}-h)}{2h} \qquad (6.b)$$

where $h$ is assumed to be a small percent of the previous estimate of flow velocity ($V_{i-1}$), let's say $h = 0.001 V_{i-1}$.

5.  Calculate percent change ($\xi$) between the last estimates of flow velocity ($V_i$ and $V_{i-1}$) as follows:

$$\xi = \left| \frac{V_i - V_{i-1}}{V_i} \right| * 100\% \qquad (7)$$

6.  If the calculated value of percent change ($\xi$) is equal to or less than a predefined value of maximum percent change ($\xi_{max}$), assign the last estimate of flow velocity and leave the iteration. If not, return to Step 2.

The calculation of mean flow velocity from pressure drop involves two nested loops, and it cannot be solved by MS Excel spreadsheet iterations as shown in [11] for friction factor by Colebrook equation. In order to minimize required programming skills, a macro-enabled MS Excel file (Flow_Measurement.xlsm) was also provided to students. The file contains two user-defined functions in VBA named FRICTION and LOG10. The functions are used for calculating iteratively the friction factor at a given flow velocity by implicit Colebrook Equation. The code view is shown in Fig. 1. The functions are available on spreadsheet and one can use the function "FRICTION" by typing "=FRICTION(…;…;…;…)" in any cells.

Figure 1: The code view of user-defined Visual Basic for Applications functions

## 2.2. Experimental setup

An experimental setup was built for the laboratory section of numerical methods course (Fig. 2). The experimental setup consists of a plexiglas tube with 20 mm outside diameter and 12 mm internal diameter. Two manometer tappings were installed on the tube at a distance of 1 m from each other. Each manometer tapping consists of four pressure ports drilled at 90° angles around the perimeter of the tube in order to minimize the fluctuations in static pressure due to fluctuations in fluid flowrate and possible imperfections on the inner surface of the tube. Ambient air was used as the fluid for which an air compressor that is capable of providing air at desired flowrates was employed. A differential pressure transmitter (Honeywell DPTM1000D) was used for measuring the pressure drop.
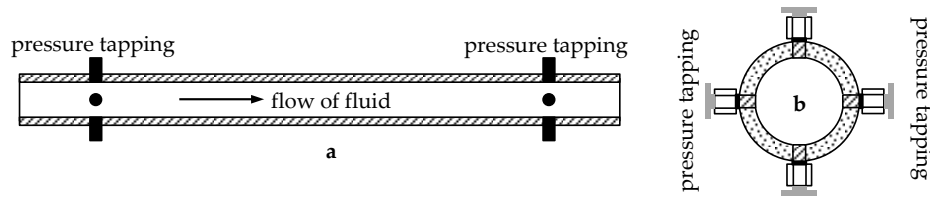
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Figure 2: Schematic view of the experimental setup. a. Longitudinal view, b. Cross-sectional view.

In a laboratory section of numerical methods course, the students are given a brief summary of Darcy-Weisbach and Colebrook equations as well as the objective of the experiment as "measurement of mean flow velocity in the pipe using pressure drop." The students are asked to measure pressure drop experimentally and calculate the mean flow velocity using spreadsheet and the VBA functions provided.

### 2.3. Additional tasks

Although less frequent, engineers encounter two other problems related with Darcy-Weisbach formula for pressure drop calculations. The first one involves estimation of pipe's roughness height ($\varepsilon$) for calibration purposes given that pressure drop, internal diameter and length of the pipe, density and kinematic viscosity of fluid, and mean flow velocity. This problem, similar to the main objective of the experiment, involves an iterative procedure. For this purpose, following function $g$ can be written by re-arranging Eqn. (1).

$$g(\varepsilon) = \tfrac{1}{2}\rho \frac{fL}{D} V^2 - \Delta P \tag{8}$$

where the objective is to calculate the roughness height of the pipe ($\varepsilon$) iteratively using the following calculation procedure:

1. Calculate Reynolds number ($Re$) by Eqn. (3).

2. Make an initial estimate of roughness height ($\varepsilon_0$), and set the iteration counter to zero ($i = 0$).

3. Calculate friction factor ($f_i$) for the $i^{th}$ estimate of roughness height ($\varepsilon_i$) by Eqn. (2). For this purpose, take the steps 3.1 through 3.4 described previously.

4. Set the iteration counter as $i = i + 1$ and calculate new estimate of roughness height ($\varepsilon_i$) as follows:

$$\varepsilon_i = \varepsilon_{i-1} - \frac{g(\varepsilon_{i-1})}{g'(\varepsilon_{i-1})} \tag{9.a}$$

The derivative term $g'(\varepsilon_{i-1})$ can be approximated simply by a central finite difference as follows:

$$g'(\varepsilon_{i-1}) = \frac{g(\varepsilon_{i-1}+h)-g(\varepsilon_{i-1}-h)}{2h} \tag{9.b}$$

where $h$ is assumed to be a small percent of the previous estimate of roughness height ($\varepsilon_{i-1}$), let's say $h = 0.001\varepsilon_{i-1}$.

5. Calculate percent change ($\xi$) between the last estimates of roughness height ($\varepsilon_i$ and $\varepsilon_{i-1}$) as follows:

$$\xi = \left|\frac{\varepsilon_i-\varepsilon_{i-1}}{\varepsilon_i}\right| * 100\% \tag{10}$$

6. If the calculated value of percent change ($\xi$) is equal to or less than a predefined value of maximum percent change ($\xi_{max}$), assign the last estimate of roughness height and leave the iteration. If not, return to Step 2.

Another type of problem related with the topic of pressure drop and Darcy-Weisbach formula is that engineers sometimes need to estimate minimum pipe diameter that connect two fixed-head points such as process tanks in a water treatment plant. A similar iterative procedure can be applied to calculate minimum pipe diameter required. Eqn. (1) must be re-arranged as in Eqn. (11) for the solution to this type of problem. Iterative procedure is not given for this type of problem to test the students' ability for applying the method to similar problems and to limit number of pages of this paper. A sample spreadsheet is provided in electronic annex.

$$g(D) = \frac{1}{2}\rho\frac{fL}{D}V^2 - \Delta P \tag{11}$$

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A number of variations of the types of problems presented here can also be derived and these problems can be solved similarly by Newton-Raphson iterations.

## 3. Solutions
### 3.1. Flow velocity problem

For solving the problem of calculating the mean flow velocity, students need to prepare a calculation table on spreadsheet view of MS Excel. A sample spreadsheet was prepared for calculations (Fig. 3). The spreadsheet includes cells for typing known parameters (roughness height, internal diameter, and length of the pipe as well as density and kinematic viscosity of fluid), measured parameter (pressure drop), and an initial estimate of mean flow velocity. For Newton-Raphson iterations, the cell D18 calculates the value of $g(V_0)$ using the following formula:

"= 0.5 * F$12 * F$11 / F$10 * B18 ^ 2 * FRICTION(F$9; F$10; B18; F$13) - F$8"

| OBJECTIVE: *To calculate mean flow velocity in a pipe from pressure drop* | | | | |
|---|---|---|---|---|
| | Do not change these cells | | | |
| | Measured parameter | | | |
| | Known parameters | | | |
| | Calculated values | | | |
| | Initial estimate of mean flow velocity | | | |
| Parameter | | Symbol | Unit | Value |
| Pressure drop | | $\Delta P$ | Pa | 120 |
| Roughness height | | $\varepsilon$ | m | 0.0000015 |
| Internal diameter | | $D$ | m | 0.012 |
| Length of the pipe | | $L$ | m | 1 |
| Density of fluid | | $\rho$ | kg.m$^{-3}$ | 1.2 |
| Kinematic viscosity of fluid | | $\upsilon$ | m$^2$.s$^{-1}$ | 0.000015 |

| CALCULATIONS: *Computational steps in Newton-Raphson iterations* | | | | | |
|---|---|---|---|---|---|
| Iteration no. | Flow velocity | Reynolds number ($Re$) | $g(V_i)$ | $g'(V_i)$ | Percent change |
| $i$ | $V_i$ (m.s$^{-1}$) | | | | $\xi$ |
| 0 | 10.0000 | 8000 | 44.808 | 28.555 | — |
| 1 | 8.4308 | 6745 | 2.688 | 25.129 | 18.61 |
| 2 | 8.3239 | 6659 | 0.014 | 24.890 | 1.29 |
| 3 | 8.3233 | 6659 | 0.000 | 24.890 | 0.01 |
| 4 | 8.3233 | 6659 | 0.000 | 24.887 | 0.00 |
| 5 | 8.3233 | 6659 | 0.000 | 24.889 | 0.00 |

Figure 3: Spreadsheet view of the solution to mean flow velocity problem.

The cell E18 calculates the value of $g'(V_0)$ using the following formula:

"=((0.5 * F$12 * F$11 / F$10 * (B18 * 1.001) ^ 2 * FRICTION(F$9; F$10; B18 * 1.001; F$13) - F$8) - (0.5 * F$12 * F$11 / F$10 * B18 ^ 2 * FRICTION(F$9; F$10; B18; F$13) -F$8)) / (B18 * 0.001)"

The next estimate of mean flow velocity is calculated in the cell B19 by Newton-Raphson method using the following formula:

"=B18 - D18 / E18"

The cells B19, D18, and E18 are extended to B23, D23, and E23, respectively, to apply the formulae to the whole range and execute the iterations. Finally, percent change in each iteration is calculated by typing the following formula into the cell F19 and extending the formula to F23.

"=ABS((B19 - B18) / B19) * 100"

A sample solution to the problem of calculating mean flow velocity is shown in Fig. 3, in which ambient air flows within the pipe and leads to a pressure drop of 120 Pa. For the given conditions (Fig. 3), the mean flow velocity was calculated as 8.3233 m/s on the fourth iteration with a relative error of less than 0.01%.

### 3.2. Roughness height problem

For solving the problem of calculating the roughness height, students need to prepare a calculation table on spreadsheet view of MS Excel. A sample spreadsheet was prepared for calculations (Fig. 4). The spreadsheet includes cells for typing known parameters (internal diameter and length of the pipe, mean flow velocity as well as density and kinematic viscosity of fluid), measured parameter (pressure drop), and an initial estimate of roughness height. For Newton-Raphson iterations, the cell C19 calculates the value of $g(\varepsilon_0)$ using the following formula:

"= 0.5 * E$12 * E$11 / E$10 * E$9 ^ 2 * FRICTION(B18; E$10; E$9; E$13) - E$8"

The cell D19 calculates the value of $g'(\varepsilon_0)$ using the following formula:

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"=((0.5 * E\$12 * E\$11 / E\$10 * E\$9 ^ 2 * FRICTION(B18 * 1.001; E\$10; E\$9; E\$13) - E\$8) -(0.5 * E\$12 * E\$11 / E\$10 * E\$9 ^ 2 * FRICTION(B18; E\$10; E\$9; E\$13) - E\$8)) / (B18 * 0.001)"

The next estimate of roughness height is calculated in the cell B20 by Newton-Raphson method using the following formula:

"=B19 - C19 / D19"

| OBJECTIVE: To calculate roughness height of a pipe from pressure drop | | | |
|---|---|---|---|
| | Do not change these cells | | |
| | Measured parameter | | |
| | Known parameters | | |
| | Calculated values | | |
| | Initial estimate of mean flow velocity | | |
| Parameter | Symbol | Unit | Value |
| Pressure drop | $\Delta P$ | Pa | 120 |
| Mean flow velocity | $V$ | m.s$^{-1}$ | 8.3233 |
| Internal diameter | $D$ | m | 0.012 |
| Length of the pipe | $L$ | m | 1 |
| Density of fluid | $\rho$ | kg.m$^{-3}$ | 1.2 |
| Kinematic viscosity of fluid | $\upsilon$ | m$^2$.s$^{-1}$ | 0.000015 |
| Reynolds number | $Re$ | -- | 6659 |

| CALCULATIONS: Computational steps in Newton-Raphson iterations | | | | |
|---|---|---|---|---|
| Iteration no. | Rough. height | $g(\varepsilon_i)$ | $g'(\varepsilon_i)$ | Percent change |
| $i$ | $\varepsilon_i$ (m) | | | $\xi$ |
| 0 | 0.00000500 | 1.273 | 358856.920 | — |
| 1 | 0.00000145 | -0.018 | 353947.535 | 244.51 |
| 2 | 0.00000150 | 0.001 | 361191.901 | 3.40 |
| 3 | 0.00000150 | 0.000 | 378532.720 | 0.12 |
| 4 | 0.00000150 | 0.000 | 379137.089 | 0.00 |
| 5 | 0.00000150 | 0.000 | 377729.813 | 0.00 |

Figure 4: Spreadsheet view of the solution to roughness height problem.

The cells B20, C19, and D19 are extended to B24, C24, and D24, respectively, to apply the formulae to the whole range and execute the iterations. Finally, percent change in each iteration is calculated by typing the following formula into the cell E20 and extending the formula to E24.

"=ABS((B20 – B19) / B20) * 100"

A sample solution to the problem of calculating roughness height is shown in Fig. 4, in which ambient air flows within the pipe at a mean velocity of 8.3233 m/s and leads to a pressure drop of 120 Pa. For the given conditions (Fig. 4),

the roughness height was calculated as 0.0000015 m (0.0015 mm) on the fourth iteration with a relative error of less than 0.01%.

## 4. Conclusions

Calculation of mean flow velocity as well as other parameters from pressure drop measurements by Darcy-Weisbach formula and Colebrook equation involves iterations. Nested iterations need to be performed for solving these types of engineering problems, which is time consuming by hand calculations or by trial-and-error. Instead, MS Excel offers an easier solution to these types of problems both in spreadsheet and Visual Basic for Applications (VBA) environment. With MS Excel, students can learn fundamentals of iterative procedures by implementing simple spreadsheet programs. Besides, MS Excel offers field engineers a cost-effective and time-saving option for solving presented types of problems as well as many others in their daily work.

This paper presents solutions to the iterative problem of calculating mean flow velocity and roughness height from pressure drop measurement by implementing spreadsheet programs along with VBA functions. Students, teachers, and field engineers can benefit from these algorithms in their studies. A macro-enabled MS Excel file was also attached to this paper that involves VBA functions for calculating friction factor.

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