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A Spreadsheet Solution of a System of Ordinary Differential Equations Using the Fourth-Order Runge-Kutta Method

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Abstract

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Abstract

Solving systems of ordinary differential equations (ODEs) by using the fourth-order Runge-Kutta (RK4) method in classroom or in examinations is quite tedious, tiring and boring since it involves many iterative calculations. Hence, there is a need to design a suitable tool in teaching and learning the numerical methods involved, especially those for solving systems of ODEs. Here, we present a new approach to solving systems of ODEs by the RK4 method through the use of an EXCEL spreadsheet to tackle these drawbacks. In doing so, we employ the concept of relative row, relative column and fixed column in the spreadsheet to obtain the solution of systems of ODEs by the RK4 method. With the appropriate differential function given by the user, it is found that the way suggested here is faster than applying a scientific calculator and the solution obtained is significantly more accurate. Besides, the concept presented here can be extended to solve systems of ODEs up to n equations using the spreadsheet. In addition to this, the students can gain a deep understanding of the iterative procedure involved in solving the systems of ODEs. Although the spreadsheet approach is not as good as other mathematical software, it does provide a step-by-step environment for teaching and learning the numerical methods. We conclude that the spreadsheet approach increases the interest of students in learning the RK4 method.

Keywords: Excel Spreadsheet, systems of ordinary differential equations, fourth-order Runge-Kutta methods

1. Introduction

Many engineering problems can be modelled as a system of ordinary differential equations (ODEs). There are several available numerical methods used to solve ODEs such as Euler's method, second-order Runge-Kutta (RK2) method and fourth-order Runge-Kutta (RK4) method. Solving systems of ODE manually by using the RK4 method is quite tedious, tiring and boring because of the repetitive calculations involved. To overcome these drawbacks, Tay [1] has proposed applying the Casio fx-570MS calculator in teaching and learning numerical methods including the RK4 method. A new model of scientific calculator, the Casio fx-570ES, has also been suggested as well, see e.g., Tay [2], where the method discussed is only for a single ODE and the systems of ODEs is not explored. Guerrero-García and Santos-Polomo

[3] presented a keystroke sequence for the numerical methods which is similar to what has been done by Tay [1]. Even though the Casio fx-570MS and Casio fx-570ES have been useful and informative in doing repetitive calculations with its CALC function, it is still boring and un- interesting if the iterations involved are too long.

A series of papers working on solving numerical methods in classroom and examination situations using Excel spreadsheet, which focus on systems of nonlinear and linear equations, approximation of interpolation, computing of eigenvalues, single ODE by the RK4 method and the Laplace equation, can be seen in [4-11]. However, literature works dealing with solving a system of ODEs by using the RK4 method in Excel have not been explored as yet. Therefore, in this paper, we present an alternative tool in solving systems of ODEs by using the RK4 method for the usage of Excel spreadsheet in classroom and examinations to tackle the drawbacks from the usage of scientific calculators. We use the concept of relative row, relative column and fixed column in EXCEL spreadsheet such that the solution of systems of ODEs by using the RK4 can be successfully obtained.

System of First-Order Differential Equations
Consider an initial-value problem of the first-order differential equation given below:

$$\frac{dx}{dt} = f(t, x, y)$$
$$\frac{dy}{dt} = g(t, x, y)$$

with $x(t_0) = x_0$, $y(t_0) = y_0$ and $t_0 \leq t \leq t_n$. This problem is a system of ODEs, which consists of a single pair of ordinary differential equations. The solution domain is discretized such that t_0, $t_1 = t_0 + h$, $t_2 = t_0 + 2h$, $t_n = t_0 + nh$, where h is the step size of t. The solution that is obtained by the RK4 method is given by

$$x_{i+1} = x_i + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4),$$

$$y_{i+1} = y_i + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4),$$

where

$$f_1 = hf(t_i, x_i, y_i), \qquad\qquad f_2 = hf\left(t_i + \frac{h}{2}, x_i + \frac{f_1}{2}, y_i + \frac{g_1}{2}\right),$$

$$f_3 = hf\left(t_i + \frac{h}{2}, x_i + \frac{f_2}{2}, y_i + \frac{g_2}{2}\right), \qquad f_4 = hf(t_i + h, x_i + f_3, y_i + g_3),$$

$$g_1 = hg(t_i, x_i, y_i), \qquad\qquad g_2 = hg\left(t_i + \frac{h}{2}, x_i + \frac{f_1}{2}, y_i + \frac{g_1}{2}\right),$$

$$g_3 = hg\left(t_i + \frac{h}{2}, x_i + \frac{f_2}{2}, y_i + \frac{g_2}{2}\right), \qquad g_4 = hg(t_i + h, x_i + f_3, y_i + g_3).$$

2. Numerical Examples

For illustration, a simple RL-circuit is considered, where we take a constant voltage in the first example and a sinusoidal voltage source in the second example.

2.1. RL circuit with a Constant Voltage

A simple RL-electrical circuit consisting of electrical current i (in amperes), resistance R (in ohms), inductance L (in henrys), and electromotive force $E(t)$ (in volts), is shown in the Figure 1.
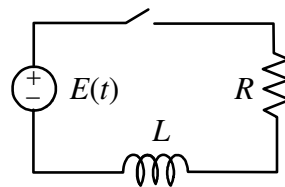
Figure 1: A simple RL circuit

We assume that R and L are constants. Also, the (defining) relation between the charge q (in coulombs) and the current i is

$$i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2q}{dt^2}. \tag{1}$$

According to Kirchhoff's Second Law, the charge q satisfies the differential equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = E(t). \tag{2}$$

As an example, we assume that $R = 15$ ohms, $L = 3$ henrys, $E(t) = 120$ volts and the initial conditions $q = 0$ and $i = 0$ when $t = 0$. We then want to find i and q for $0 \le t \le 5$ with $\Delta t = 0.25$ by using the RK4 method. The exact solutions for the charge q and the current i are given respectively, by

$$q = \frac{8}{5}(e^{-5t} - 1) + 8t \text{ and } i = 8(1 - e^{-5t}). \tag{3}$$

By comparing the solution obtained by the RK4 method with the exact solution obtained from equation (3), we can then find the absolute errors for i and q.

Solution

Step 1: Write the second-order differential equation (equation (2)) as a system of first-order differential equations by using equation (1).

$$\frac{dq}{dt} = i = f(t, q, i)$$

$$\frac{di}{dt} = \frac{E(t) - Ri}{L} = \frac{120 - 15i}{3} = 40 - 5i = g(t, q, i)$$

Step 2: Write the RK4 formula given below:

$$q_{k+1} = q_k + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4),$$

$$i_{k+1} = i_k + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4),$$

where

$$f_1 = 0.25\,i, \qquad\qquad f_2 = 0.25\left(i + \frac{g_1}{2}\right),$$

$$f_3 = 0.25\left(i + \frac{g_2}{2}\right), \qquad\qquad f_4 = 0.25(i + g_3),$$

$$g_1 = 0.25(40 - 5i), \qquad\qquad g_2 = 0.25\left[40 - 5\left(i + \frac{g_1}{2}\right)\right],$$

$$g_3 = 0.25\left[40 - 5\left(i + \frac{g_2}{2}\right)\right], \qquad\qquad g_4 = 0.25\left[40 - 5(i + g_3)\right].$$

with $h = 0.25$.

Step 3: Compute the numerical solution as shown in Figure 2.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	System of ODEs by the RK4 Method															
2	h	0.25			E	120										
3																
4	k	t	q	i	f_1	g_1	f_2	g_2	f_3	g_3	f_4	g_4	Exact q	Exact i	|Error q|	|Error i|
5	0	0	0	0	0	10	1.25	3.7500	0.4688	7.6563	1.9141	0.4297	0	0	0	0
6	1	0.25	0.8919	5.5404	1.3851	3.0745	1.7694	1.1530	1.5292	2.3539	1.9736	0.1321	0.85841	5.7080	0.0335	0.1676
7	2	0.5	2.5512	7.2438	1.8109	0.9453	1.9291	0.3545	1.8553	0.7237	1.9919	0.0406	2.53134	7.3433	0.0199	0.0995
8	3	0.75	4.4465	7.7675	1.9419	0.2906	1.9782	0.1090	1.9555	0.2225	1.9975	0.0125	4.43763	7.8119	0.0089	0.0444
9	4	1	6.4143	7.9285	1.9821	0.0894	1.9933	0.0335	1.9863	0.0684	1.9992	0.0038	6.41078	7.9461	0.0035	0.0176
10	5	1.25	8.4044	7.9780	1.9945	0.0275	1.9979	0.0103	1.9958	0.0210	1.9998	0.0012	8.40309	7.9846	0.0013	0.0065
11	6	1.5	10.4014	7.9932	1.9983	0.0084	1.9994	0.0032	1.9987	0.0065	1.9999	0.0004	10.40088	7.9956	0.0005	0.0023
12	7	1.75	12.4004	7.9979	1.9995	0.0026	1.9998	0.0010	1.9996	0.0020	2.0000	0.0001	12.40025	7.9987	0.0002	0.0008
13	8	2	14.4001	7.9994	1.9998	0.0008	1.9999	0.0003	1.9999	0.0006	2.0000	0.0000	14.40007	7.9996	0.0001	0.0003
14	9	2.25	16.4000	7.9998	2.0000	0.0002	2.0000	0.0001	2.0000	0.0002	2.0000	0.0000	16.40002	7.9999	0.0000	0.0001
15	10	2.5	18.4000	7.9999	2.0000	0.0001	2.0000	0.0000	2.0000	0.0001	2.0000	0.0000	18.40001	8.0000	0.0000	0.0000
16	11	2.75	20.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	20.40000	8.0000	0.0000	0.0000
17	12	3	22.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	22.40000	8.0000	0.0000	0.0000
18	13	3.25	24.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	24.40000	8.0000	0.0000	0.0000
19	14	3.5	26.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	26.40000	8.0000	0.0000	0.0000
20	15	3.75	28.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	28.40000	8.0000	0.0000	0.0000
21	16	4	30.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	30.40000	8.0000	0.0000	0.0000
22	17	4.25	32.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	32.40000	8.0000	0.0000	0.0000
23	18	4.5	34.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	34.40000	8.0000	0.0000	0.0000
24	19	4.75	36.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	36.40000	8.0000	0.0000	0.0000
25	20	5	38.4000	8.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	2.0000	0.0000	38.40000	8.0000	0.0000	0.0000

Figure 2: Solution of a System of ODEs (Example 1) by the RK4 Method

Figure 3 shows the current i at a constant voltage of 120 volts for a RL series circuit as shown in Figure 1. Notice that, the current i reaches the steady state solution at $i = 8$ amperes as time t goes to infinity.
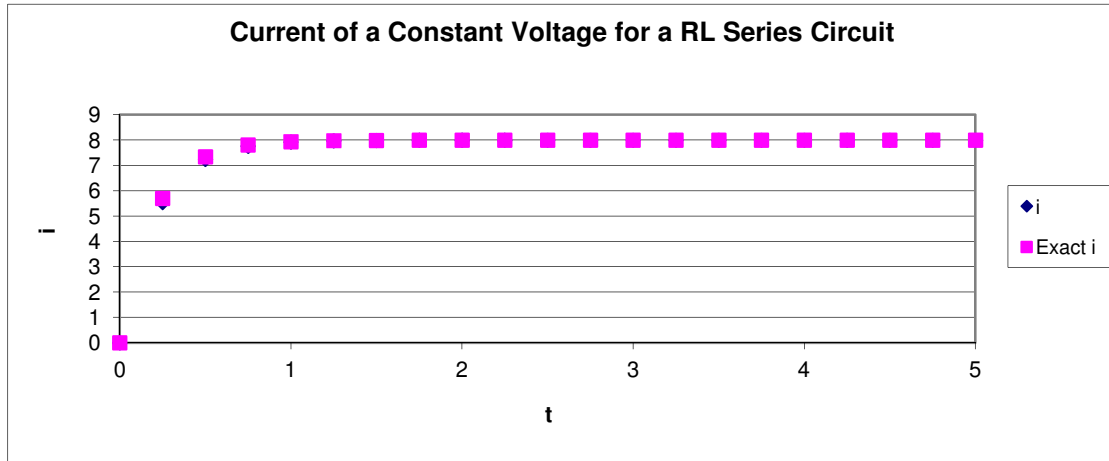

Figure 3: Current i of a Constant Voltage for a RL Series Circuit

It can be seen that the time constant, T, for a constant voltage for a RL circuit is $\dfrac{L}{R}$.

The time required for the current flowing in the RL series circuit to reach its maximum steady state is equivalent to about 5 time constants or $5T$. In this example, $T = \dfrac{L}{R} = \dfrac{3}{15} = 0.2s$ and hence the time required for the current flowing in the RL series circuit to reach its maximum steady state is equivalent to about $5T = 5(0.2s) = 1s,$ which is clearly shown in Figure 3.

Figure 4 shows the voltage drop, V_L across the inductor L at a constant voltage for a RL series circuit as shown in Figure 1. Notice that, the voltage drop, V_L across the inductor L decays exponentially from its initial value to zero for the steady state solution. It can be proved that this voltage drop is $V_L = L\dfrac{di}{dt} = 3(40e^{-5t}) = 120e^{-5t}.$ Hence as time goes to infinity V_L decays to zero.
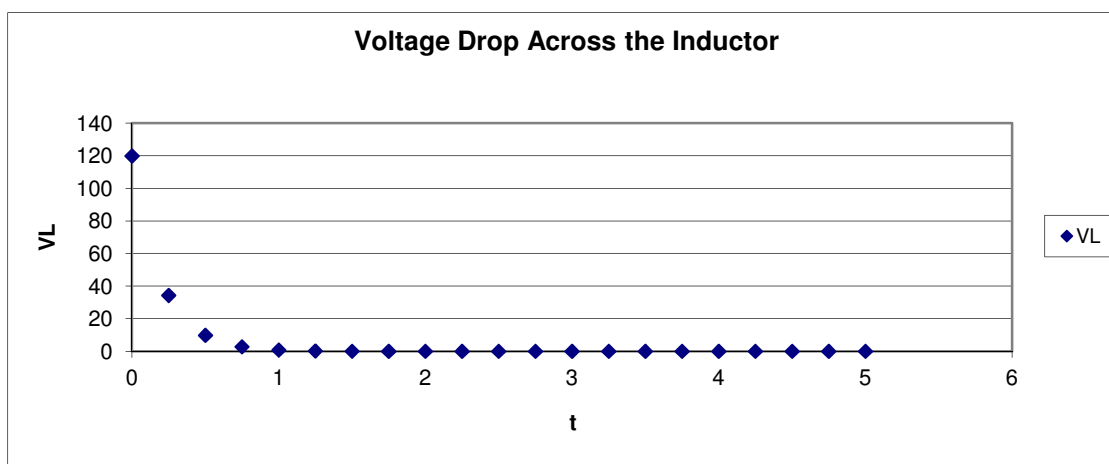
Figure 4: Voltage Drop, V_L, Across the Inductor of Constant Voltage for an LR Series Circuit

The numerical solution that has been obtained in Figure 2 results from the following steps and the respective Excel commands:

Step 1 In cell E5, type **=$B$2*$D5**.

Step 2 In cell F5, type =$B$2*(40-5*$D5).

Step 3 Copy cells E5:F5 to G5:H5 and modify the formulae in the target cells:
G5=$B$2*($D5+F5/2)
H5=$B$2*(40-5*($D5+F5/2))

Step 4 Copy cells G5:H5 to I5:J5. The formula will be automatically corrected as
I5=$B$2*($D5+H5/2)
J5=$B$2*(40-5*($D5+H5/2))

Step 5 Copy cells I5:J5 to K5:L5 and modify the formulae in the target cells:
K5=$B$2*($D5+J5)
L5=$B$2*(40-5*($D5+J5))

Note: The formulae that the user keys into spreadsheet are relative rows and relative columns. The symbol $ at the front of a column entry, for example $D5, fixes the column only, while $ at the front of the column entry and at the front of a row entry, for example $B$2, stands for a fixed column and a fixed row, i.e. the cell reference does not change.

2.2. RL circuit with a Sinusoidal Voltage

Now taking $E(t) = 120\sin 2t$ for the RL circuit in Figure 1. The exact solutions for the charge q and the current i are given, respectively, by

$$q(t) = 4 - \frac{16}{29}e^{-5t} - \frac{40}{29}\sin 2t - \frac{100}{29}\cos 2t \tag{4}$$

$$i(t) = \frac{80}{29}e^{-5t} + \frac{200}{29}\sin 2t - \frac{80}{29}\cos 2t \tag{5}$$

Solution
Step 1: Write the second-order differential equation (equation (2)) as a system of first-order differential equations by using equation (1).

$$\frac{dq}{dt} = i = f(t, q, i)$$

$$\frac{dq}{dt} = \frac{E(t) - Ri}{L} = \frac{120 \sin 2t - 15i}{3} = 40 \sin 2t - 5i = g(t, q, i)$$

Step 2: Write the RK4 formula given below:

$$q_{k+1} = q_k + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4),$$

$$i_{k+1} = i_k + \frac{1}{6}(g_1 + 2g_2 + 2g_3 + g_4),$$

where

$$f_1 = 0.25\,i, \qquad\qquad f_2 = 0.25\left(i + \frac{g_1}{2}\right),$$

$$f_3 = 0.25\left(i + \frac{g_2}{2}\right), \qquad\qquad f_4 = 0.25(i + g_3),$$

$$g_1 = 0.25(40 \sin 2t - 5i), \qquad g_2 = 0.25\left[40 \sin 2\left(t + \frac{h}{2}\right) - 5\left(i + \frac{g_1}{2}\right)\right],$$

$$g_3 = 0.25\left[40 \sin 2\left(t + \frac{h}{2}\right) - 5\left(i + \frac{g_2}{2}\right)\right],$$

$$g_4 = 0.25\left[40 \sin 2\left(t + h\right) - 5\left(i + g_3\right)\right].$$

Step 3: Compute the numerical solution as shown in Figure 5.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	System of ODEs by the RK4 method															
2	h	0.25			E		120 sin 2t									
3																
4	k	t	q	i	f_1	g_1	f_2	g_2	f_3	g_3	f_4	g_4	Exact q	Exact i	|Error q|	|Error i|
5	0	0	0	0	0	0	0	2.4740	0.3093	0.9278	0.2319	3.6345	0	0	0	0
6	1	0.25	0.1417	1.7397	0.4349	2.6196	0.7624	3.0045	0.8105	2.7640	1.1259	2.7851	0.1545	1.6758	0.0128	0.0639
7	2	0.5	0.9262	4.5633	1.1408	2.7106	1.4796	2.0916	1.4023	2.4785	1.7604	1.1727	0.9310	4.5392	0.0048	0.0241
8	3	0.75	2.3704	6.7339	1.6835	1.5576	1.8782	0.4490	1.7396	1.1419	1.9689	-0.7517	2.3672	6.7490	0.0031	0.0151
9	4	1	4.1850	7.3985	1.8496	-0.1551	1.8302	-1.3704	1.6783	-0.6109	1.6969	-2.4998	4.1771	7.4376	0.0079	0.0391
10	5	1.25	5.9456	6.2956	1.5739	-1.8847	1.3383	-2.8749	1.2145	-2.2560	1.0099	-3.6382	5.9360	6.3428	0.0096	0.0472
11	6	1.5	7.2272	3.6648	0.9162	-3.1698	0.5200	-3.6818	0.4560	-3.3618	0.0757	-3.8866	7.2188	3.7058	0.0084	0.0410
12	7	1.75	7.7178	0.1408	0.0352	-3.6839	-0.4253	-3.5892	-0.4134	-3.6484	-0.8769	-3.1836	7.7129	0.1646	0.0049	0.0237
13	8	2	7.2980	-3.4163	-0.8541	-3.2977	-1.2663	-2.6185	-1.1814	-3.0430	-1.6148	-1.7012	7.2978	-3.4160	0.0002	0.0002
14	9	2.25	6.0706	-6.1366	-1.5341	-2.1046	-1.7972	-1.0068	-1.6600	-1.6929	-1.9574	0.1976	6.0752	-6.1600	0.0046	0.0235
15	10	2.5	4.3363	-7.3543	-1.8386	-0.3963	-1.8881	0.8513	-1.7322	0.0715	-1.8207	2.0481	4.3445	-7.3958	0.0082	0.0414
16	11	2.75	2.5196	-6.7714	-1.6929	1.4089	-1.5167	2.5009	-1.3802	1.8184	-1.2383	3.3971	2.5295	-6.8207	0.0098	0.0493
17	12	3	1.0655	-4.5306	-1.1327	2.8692	-0.7740	3.5383	-0.6904	3.1201	-0.3526	3.9144	1.0745	-4.5757	0.0090	0.0451
18	13	3.25	0.3298	-1.1806	-0.2951	3.6269	0.1582	3.7093	0.1685	3.6578	0.6193	3.4733	0.3357	-1.2104	0.0060	0.0299
19	14	3.5	0.4927	2.4585	0.6146	3.4967	1.0517	2.9722	0.9862	3.3000	1.4396	2.1818	0.4941	2.4512	0.0014	0.0073
20	15	3.75	1.5144	5.4957	1.3739	2.5104	1.6877	1.5074	1.5623	2.1343	1.9075	0.3561	1.5109	5.5127	0.0035	0.0170
21	16	4	3.1446	7.1873	1.7968	0.9094	1.9105	-0.3265	1.7560	0.4460	1.9083	-1.5567	3.1371	7.2245	0.0075	0.0372
22	17	4.25	4.9843	7.1193	1.7798	-0.9142	1.6655	-2.0805	1.5198	-1.3515	1.4419	-3.0885	4.9745	7.1675	0.0098	0.0483
23	18	4.5	6.5831	5.3081	1.3270	-2.5140	1.0128	-3.3250	0.9114	-2.8181	0.6225	-3.8640	6.5734	5.3557	0.0097	0.0475
24	19	4.75	7.5494	2.1974	0.5494	-3.4983	0.1121	-3.7555	0.0799	-3.5948	-0.3493	-3.6935	7.5422	2.2325	0.0072	0.0351
25	20	5	7.6467	-1.4513	-0.3628	-3.6261	-0.8161	-3.2665	-0.7711	-3.4912	-1.2356	-2.6188	7.6437	-1.4372	0.0030	0.0141

Figure 5: Solution of System of ODEs (Example 2) by the RK4 Method

Figure 6 shows the current i at a sinusoidal voltage for a RL series circuit as shown in Figure 1. Notice that, from the exact i given in equation. (5), the steady state solution is $i(t) = \frac{200}{29} \sin 2t - \frac{80}{29} \cos 2t = 7.4278 \sin(2t - 0.3805)$ as time t goes to infinity. This steady sinusoidal solution is shown in Figure 6, where frequency of this solution is as

same as the supplied voltage $E(t)$, with a phase shift, 0.3805, (Compare Figures 6 and 7).
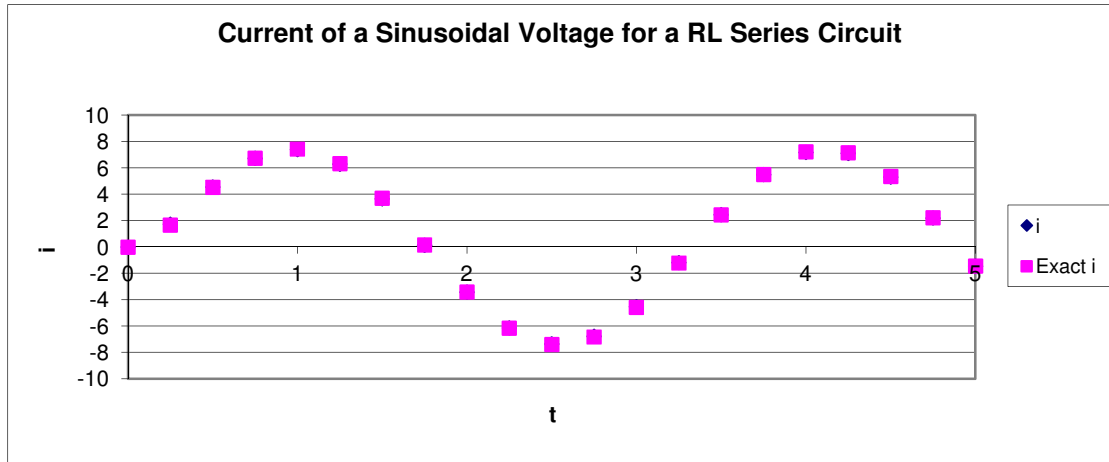

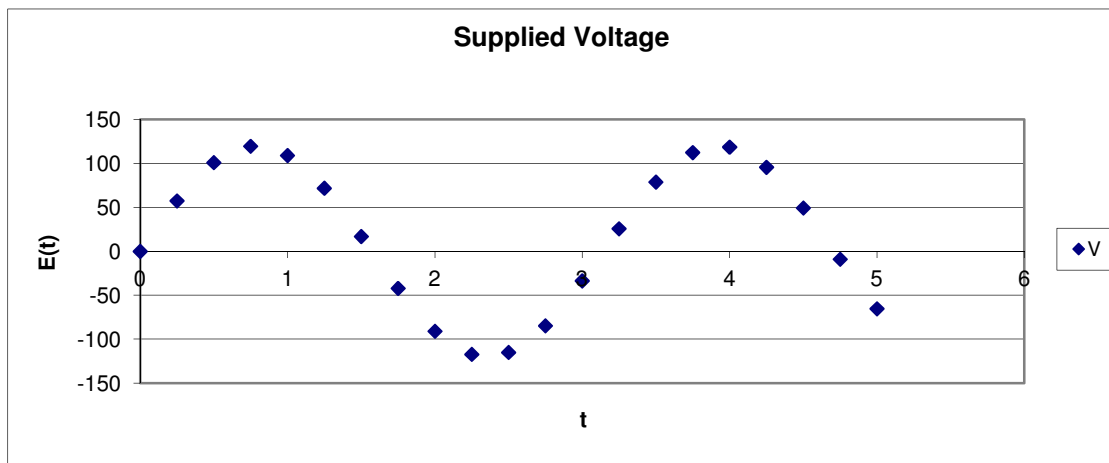Figure 6: Current i of a Sinusoidal Voltage for an LR Series Circuit

Figure 7: Supplied Voltage for a RL Series Circuit

The numerical solution that has been obtained in Figure 5 results from the following steps and the respective Excel commands:

Step 1 In cell E5, type **=$B$2*$D5**.

Step 2 In cell F5, type =$B$2*(40*SIN(2*$B5)-5*$D5).

Step 3 Copy cells E5:F5 to G5:H5 and modify the formulae in the target cells:
G5=$B$2*($D5+F5/2)
H5 =$B$2*(40*SIN(2*($B5+$B$2/2))-5*($D5+F5/2))

Step 4 Copy cells G5:H5 to I5:J5. The formula will be automatically corrected as
I5=$B$2*($D5+H5/2)
J5 =$B$2*(40*SIN(2*($B5+$B$2/2))-5*($D5+H5/2))

Step 5 Copy cells I5:J5 to K5:L5 and modify the formulae in the target cells:
K5=$B$2*($D5+J5)
L5 =$B$2*(40*SIN(2*($B5+$B$2))-5*($D5+J5))

3. Discussions

By applying relative row, relative column and fixed column concepts in Excel spreadsheet, we have shown that the system of ODEs can be solved easily, quickly and accurately in Excel spreadsheet by using the RK4 method. In this way, users only need to type the functions f_1 and g_1, and copy these functions for the functions given by f_2 and g_2, f_3 and g_3, f_4 and g_4, correspondingly in the respective target cell to get the correct formulae. Following from this, EXCEL will calculate the solutions. The numerical solutions can be obtained by dragging the formulae to all the solution domains. It is pointed out if the scientific calculators are used, each pair of the functions has to be calculated manually during every iteration steps.

4. Conclusion

We conclude that the full solutions obtained can be used by educators and students. Even though it is designed for a system of ODEs with two equations, the EXCEL commands involved are not very difficult if compared to the MATHCAD, MAPLE, MATLAB, or C Programming. Hence, the users can extend the concept presented here to solve a system of ODEs up to n equations or solve ODEs by other numerical methods. A limitation here is the tool we used is not fully generated automatically as yet. Thus, the users need to increase the number of iterations manually by selecting an appropriate iteration number. However, this manual option is useful in teaching and learning the numerical methods, where students can assess for themselves where to stop the iterations process. In our future research, we will explore the applications of EXCEL programming in teaching and learning numerical methods. So, as another function is given, users only need to change the function and all calculations will be automatically completed.

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