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A Romberg Integral Spreadsheet Calculator

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Abstract

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1   Introduction

In Calculus, integration is referred to as anti-differentiation. For example the differentiation of sin(x) gives cos(x) and the integration of cos(x) gives sin (x). Integration has wide applications, for example it is useful for finding the area between the curve of a function on the *x*-axis or *y*-axis, area bounded by several curves, volume of solid, mass of solid etc.

Numerical integration is needed when analytical integration may be impossible or unfeasible. It is also required when integrating a tabulated data instead of a known function. There are several numerical integration methods such as trapezoidal, Simpson's 1/3 rule and Simpson's 3/8 rule. To obtain the desired accuracy of the numerical integration, we may use higher order formula, a smaller step size h or Romberg integral. Approximating the definite integral numerically by using the Romberg integration is straightforward, but its repetitive calculations are quite tedious. Hence, there is a need to develop a spreadsheet calculator to help students and educators who need its full solutions.

A series of papers working on solving numerical methods in classrooms and examination situations using spreadsheet which focuses on systems of nonlinear and linear equations, approximation of interpolation, computing of eigenvalues, ordinary differential equations (ODEs) by the Fourth-order Runge-Kutta (RK4) and the Laplace equation can be seen in [1-11]. Recently, a Richardson's extrapolation spreadsheet calculator up to level 4 was developed in [12] to approximate definite differentiation numerically. However, literature works dealing with spreadsheet calculator for approximating the definite integral using the Romberg integral in Excel

has not been explored as yet. Thus, in this paper, we developed a Romberg integral spreadsheet calculator to approximate the definite integral.

This paper is organized as follows. Section 2 provides the background of Romberg integration for definite integral. A numerical example is given in Section 3. The spreadsheet calculator, its numerical solution and Excel commands are discussed in Section 4. Finally, a conclusion is given in the final part of this paper, Section 5.

2 Romberg Integration

There are two ways to improve the accuracy of the numerical integration namely decreasing the step size h or using a higher order formula that employs more points. A third approach, based on Romberg integration which uses two integrals, gives a more accurate approximation

For a given a function of $f(x)$, approximations I, to the definite integral $\int_a^b f(x)\,dx$ can be approximated using the Romberg Integration as follows: [13]

$$I(J,K) = \frac{4^K I(J,K-1) - I(J-1,K-1)}{4^K - 1} + O(h^{2K+2}), \quad J = 1, 2, ..., \quad K = 1, 2, ... \tag{1}$$

$$I(J,0) = \frac{h}{2}\left[f_0 + f_n + 2\sum_{i=1}^{n-1} f_i \right], J = 0, 1, 2, ... \tag{2} \quad h_0 = h, h_1 = \frac{h}{2}, h_2 = \frac{h}{4}, ..., h_J = \frac{h}{2^J} \tag{3}$$

The iteration process (1) is repeated until $\left| I(J,J) - I(J,J-1) \right| < \varepsilon$ or $\left| I(J,J) - I(J-1,J-1) \right| < \varepsilon$ for a specified value of ε. The value of $I(J,J)$ then approximates $\int_a^b f(x)\,dx$ at this level.

Romberg Integration table for K from 1 to 4 is shown in Table 1.

Table 1: Romberg Integration Table for level K values from 1 to 4

J	h_J	K				
		0	1	2	3	4
		$I(J,0)$	$I(J,1)$	$I(J,2)$	$I(J,3)$	$I(J,4)$
0	h_0	$I(0,0)$				
1	h_1	$I(1,0)$	$I(1,1)$			
2	h_2	$I(2,0)$	$I(2,1)$	$I(2,2)$		
3	h_3	$I(3,0)$	$I(3,1)$	$I(3,2)$	$I(3,3)$	
4	h_4	$I(4,0)$	$I(4,1)$	$I(4,2)$	$I(4,3)$	$I(4,4)$

3 Numerical Example

In this section, a numerical example to be solved by Romberg integration is provided.

Question

The velocity of a falling parachutist is given by $v(t) = \dfrac{gm}{c}(1 - e^{-\frac{c}{m}t})$, where $g = 9.80665 \, \text{m/s}^2$, $m = 80 \, \text{kg}$ and $c = 10 \, \text{kg/s}$. At time $t = 0 \, \text{s}$, the initial velocity is 0 m/s at the origin. By using Romberg integration,

(a)   find the distance travelled by the parachutist from time $t = 0 \, \text{s}$ to $t = 10 \, \text{s}$.

(b)   find the absolute error if the exact solution is $d(t) = \displaystyle\int v(t) = \dfrac{gm}{c}\left[t + \dfrac{m}{c} e^{-\frac{c}{m}t} \right]$,

Solution

(a)   The distance travelled by the parachutist from time $t = 0 \, \text{s}$ to $t = 10 \, \text{s}$ is given by $d(t) = \displaystyle\int_0^T \dfrac{gm}{c}(1 - e^{-\frac{c}{m}t}) \, dt$.

Given $g = 9.80665 \, \text{m/s}^2$, $m = 80 \, \text{kg}$ and $c = 10 \, \text{kg/s}$, then

$$d(t) = \int_0^{10} 9.80665(8)(1 - e^{-\frac{t}{8}}) \, dt.$$

The distance travelled can be approximated by Romberg integration as shown on top of Figure 7 in Section 4.

b)   The exact distance is given by $d(t) = \displaystyle\int v(t) = \dfrac{gm}{c}\left[t + \dfrac{m}{c} e^{-\frac{c}{m}t} \right]$. By substituting the numerical values of m, c and g and evaluate t from 0 to 10 into the given distance function, we get the exact distance

$$d(t) = \int_0^{10} v(t) \, dt = \dfrac{gm}{c}\left[t + \dfrac{m}{c} e^{-\frac{c}{m}t} \right]_0^{10} = 9.80665(8)[10 + 8e^{-\frac{5}{4}} - 8] = 336.7241450$$

which is the same as calculated at the bottom of Figure 7 in Section 4. The absolute error is also shown at the bottom of Figure 7 in Section 4.

4   Romberg Integration Spreadsheet Calculator

Figure 1 illustrates the initial layout of the Romberg integral spreadsheet calculator. The spreadsheet calculator is capable of computing the Romberg integral table up to any level.

Figure 1: Initial layout of Romberg Integral Spreadsheet Calculator

Once the Input button is clicked, it will open a parameter input form as given in Figure 2 to capture all required information to solve the integral by Romberg method. Users are only required to enter the variable in the integral, the integrand and the lower and upper limits of the integral, select desired accuracy of one decimal place, two decimal places, up to the calculation of nine decimal places from the drop down menu, provide exact function if it exists and lastly click the *Compute* button which is associated with VBA programming written to compute the Romberg integral table. If Compute button is clicked before any variable is input, an Error message box will be prompted on top of parameter input form as displayed in Figure 2.
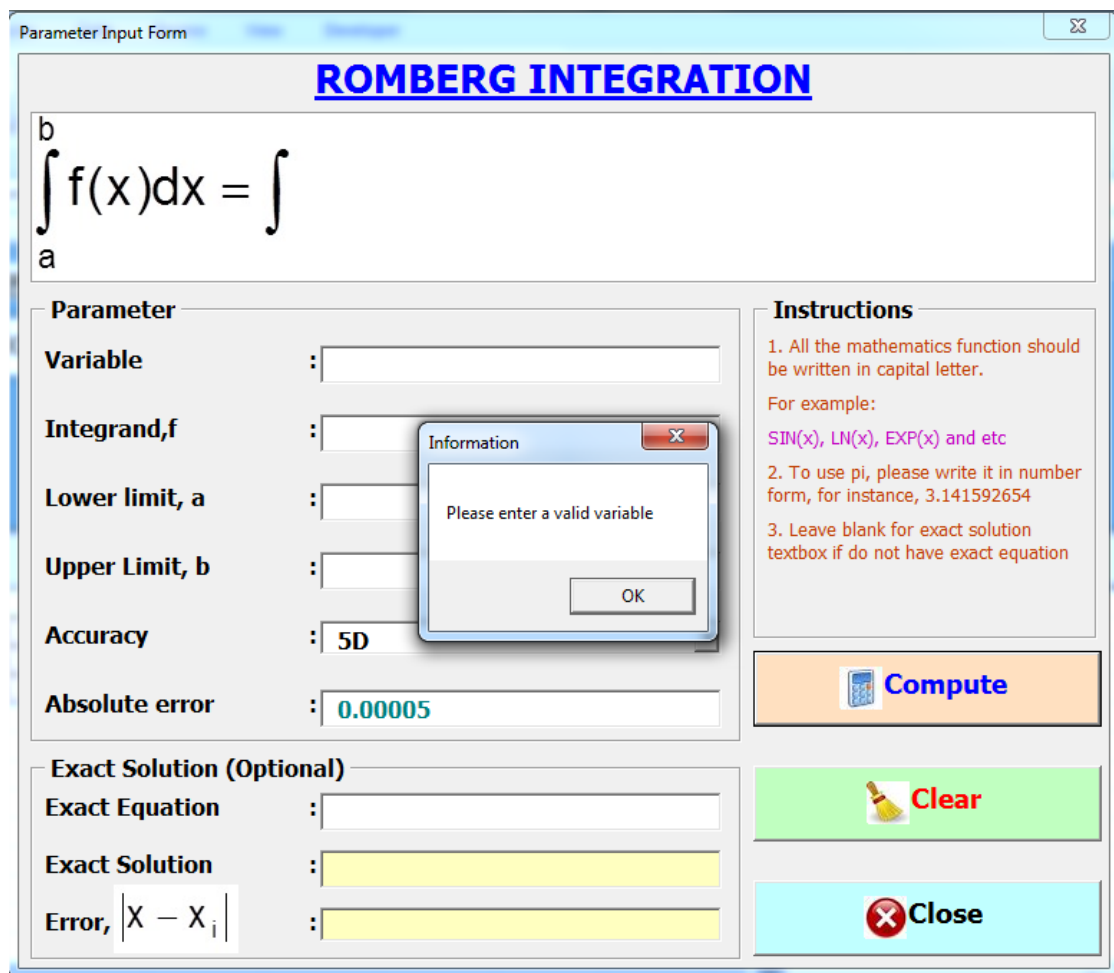
Figure 2: User input form and an error message box if no variable is provided

Once the Compute button is clicked after the variable is provided, but the integrand is not given, then another error message box will pop out as seen in Figure 3.

Figure 3: Error message box if no integrand is provided

Once *Compute* button is clicked after the variable and integrand are provided, but the lower limit of the integration is not given, the third error message will be displayed as shown in Figure 4.

Figure 4: Error message box if the lower limit is not provided

Once *Compute* button is clicked after the variable, integrand and the lower limit are provided, but the upper limit of the integration is not given, the fourth error message will be displayed as seen in Figure 5.

Figure 5: Error message box if the upper limit is not provided

Once all required parameters are entered as seen in Figure 6 and Compute button is clicked, the answer of the integral is displayed on top of the input form as seen in Figure 7, whereas the exact solution and its absolute error are displayed at the bottom of the exact equation as seen in Figure 7. On the other hand, the Romberg integral table will be displayed as seen in Figure 8. The Clear button will clear all data inputted, while the close button will close the input form.

Figure 6: All required parameters

Figure 7: The solution

Figure 8: The solution of Romberg integral table

If the variable typed does not tally with the variable in the integrand, then the following error message will be displayed as shown in Figure 9.
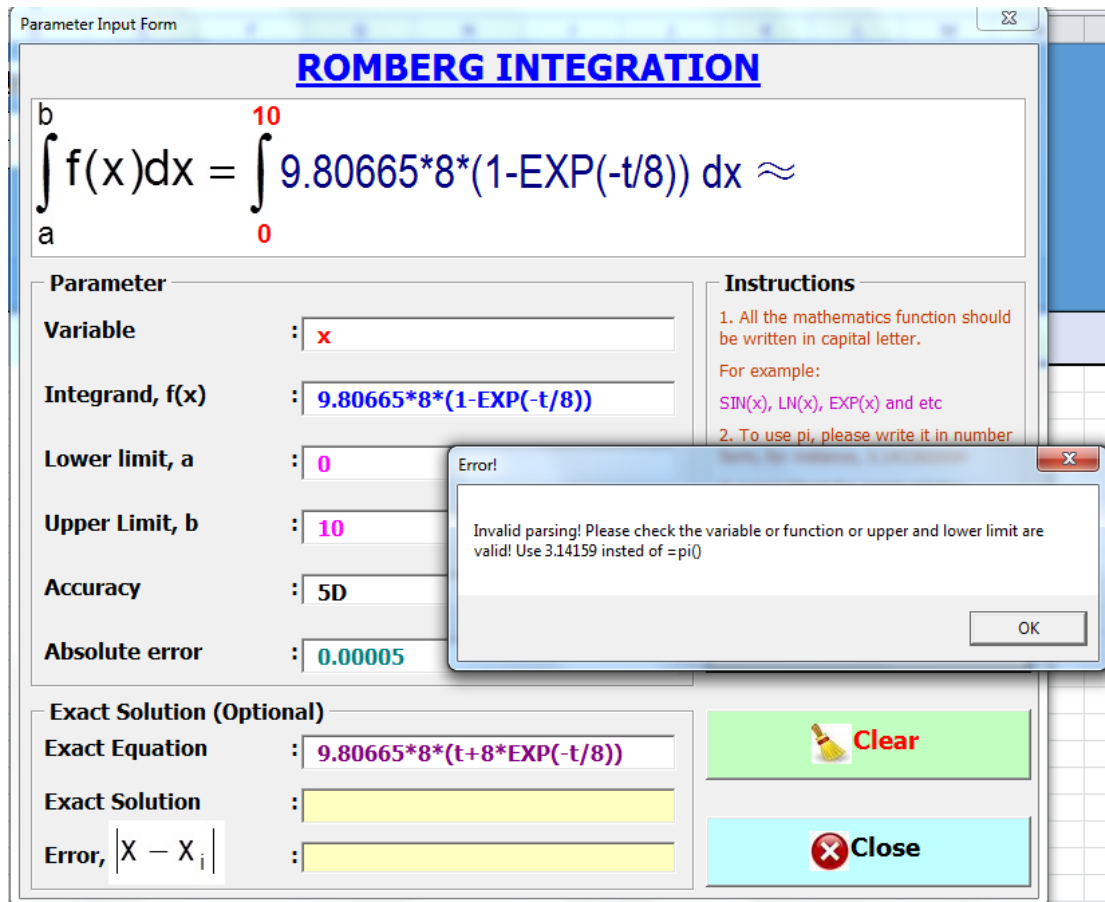
Figure 9: Error message if variable typed does not tally with integrand

5. The Evaluation of Romberg Spreadsheet Calculator

We have tested this Romberg spreadsheet calculator on 36 Master of Electrical Engineering students who took the Advanced Engineering Mathematics (AEM) course in Universiti Tun Hussein Onn Malaysia in semester 1 2014/2015. One of the sub topics in the AEM course is Romberg Integral which is taught in week 6. Initially, the students have been taught on how to solve Romberg integral traditionally using calculators. After that, they were taught how to solve Romberg integral in standard Excel spreadsheet environment. Lastly, the Romberg Spreadsheet Calculator was given to them and they were exposed on its usage. Students tested the same integral for the third time using this method. The Romberg Spreadsheet Calculator and questionnaires were later given to them. They were asked to test the spreadsheet calculator for other integrals. The questionnaires were collected in week 14.

There are four aspects of evaluation being conducted for this spreadsheet calculator, namely content, design & presentation, teaching strategy design, and technical aspect using a questionnaire that consisted of 24 items. A 5-point Likert scale was used to rate the response of the participants towards the RK4 Spreadsheet Calculator. The participants were asked to answer each question by rating each item using the

following scale: 1 = strongly disagree; 2 = disagree; 3 = neutral; 4 = agree and 5 = strongly agree.

The findings of the Romberg Spreadsheet Calculator are shown in Table 2. Findings indicated that the majority of the students agreed that the Romberg Spreadsheet Calculator is useful in terms of content (M=4.37, SD=0.42), design & presentation (M=4.35, SD=0.44), teaching strategy design (M=4.31, SD=0.54) and technical aspect (M=4.30, SD=0.54). According to the results shown in mode, the majority of the participants agreed that this Romberg Spreadsheet Calculator is good when it comes to teaching strategy design, followed by content and technical aspect. This can be proven by the statement given by the participants during the interview. Some of the participants mentioned that "*It is easier than Excel*" and "*I like it in terms of speed to get the result and the lack of complexity of it*" and "*It is clear, easy to use and facilitate my further understanding of Numerical Analysis concept*". However, they are neutral with regards to the design and presentation of the Romberg Spreadsheet Calculator as stated by one of the participants "*Add in audio for instruction to make it more user friendly.*"

Table 2: Descriptive Analyses of Romberg Spreadsheet Calculator Evaluation

Descriptive Statistics	Content	Design & Presentation	Teaching Strategy Design	Technical Aspect
Mean	4.37	4.35	4.31	4.30
Std. Deviation	0.42	0.44	0.54	0.54
Mode	4.67	4.33	4.67	4.50

6   Conclusion

A Romberg integral spreadsheet calculator with graphical user interface input form was developed to approximate the definite integral using VBA programming. The spreadsheet calculator is very user friendly. It provides an alternative tool for approximating the definite integral via Romberg integration. It can be used as a marking scheme for educators and students who need its full solutions. Last but not least, it effectively reduces the calculation time and increases the students' learning interest.

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