Learn/Teach Analytical Geometry on a Spreadsheet.

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Abstract

For the last two years, we switched to online, physical, and hybrid teaching models during the pandemic and lockdown. We faced problems in teaching various modules in calculus and analytical geometry online. Considering the ongoing scenario, a graphical user interface (GUI) teaching tool has been designed to teach several analytical geometry modules to college and undergraduate students. These executable spreadsheets can help physics and mathematics students practice numerous principles of analytical geometry with few inputs and comprehensive solutions with a single click. Because spreadsheets are so widespread, they can be used to calculate equations of straight lines, planes, areas, and volumes in 3-dimensional geometry. Some online vector GUIs display the outcomes; however, this GUI illustrates the computation methods to get the results, making it easier to study the topic. The designed GUI also aids mentors in their lab work and tests by creating various papers with answer keys.

Keywords: Teaching through GUI, spreadsheet, analytical geometry,

1. Introduction

In recent years, the teaching of mathematics has received limited but increasing attention (Beswick & Goos, 2018). Indeed, undergraduate and college-level education generally has changed significantly due to online teaching and learning. Teaching techniques have evolved substantially because of technological advancements. Change happens quickly, travels quickly, and is favorably embraced in physics research, but the changes in physics education take time, spread slowly, and are usually met with resistance. Students may learn physics using simulations, an interactive and enjoyable way of teaching. Many scientists have recently used spreadsheets to model physics and other scientific content. In these lockdown situations, spreadsheets are helpful tools for teaching physics experiments online through simulation (Iqbal, Ahmed, Iqbal, & Uddin, 2020).

Excel is a collection of convenient tools. Students and teachers from many fields are familiar with Microsoft Office and Excel spreadsheets; they know how to use them for data analysis. Examples from physics and other sciences can be displayed and demonstrated on MS Excel or spreadsheets, which can be used to make up scenarios (Uddin, Ahsanuddin, & Khan, 2017). CLI (Command Line Interface) existed before the start of GUI (Graphical User Interface). No one imagined ordinary people could use a computer at the time. However, most people today own a computer and understand how to use it. GUI accomplished this. It did not require any additional information from the user. Instead, it made it easier for the user to start with the computer. The Information Technology industry exploded, with many work opportunities for creating and developing GUI. Future languages have evolved and are now employed to create the GUI, GUI will continue to improve and update itself into a more accessible and easier user interface. Viewing a visual and fully comprehending it is more appealing than reading theory. For example, if a student reads the word mango and does not see a picture of a mango, they will not be able to understand it effectively. So much flexibility is provided by technology that teachers are teaching with computer systems, and many materials are GUI-based applications rather than books.

Students are also following the educational process, and it is enjoyable for them to understand concepts by seeing animation, photos, or videos. Another example is where a student is learning the respiratory process. Instead of reading from a book, they can learn if taught by a GUI-based program such as a video presentation because human brains process visuals faster than sentences. Numerous GUI-based tools available on the internet can help the instructor create the ideal study lesson for students. One example is a PowerPoint presentation, which is very simple to use. GUIs help in acquiring information. Visual structures (widgets) that replicate physical elements such as 'switches' and 'buttons' aid learning by providing a natural way to communicate data to the computer. Almeida, Nogueira, & André (2013) highlighted the utilization of graphical user interfaces (GUIs) built with the MATLAB guide tool for university-level optical communications courses and research. Not only do, MATLAB-programmed graphical user interfaces help the learning process by making models easier to understand but they can also be updated and managed more effectively by students. MATLAB is already taught at many universities, Students can engage in a participatory experience in a variety of model settings. Scientists have created a variety of graphical user interfaces (GUIs) for time series analysis, DNA, and crystallography (Randhawa, Hill, & Kari, 2020; Rosadi, 2008; Toby, 2001). Von Dohlen (2020) evaluated several techniques for coding and graphical user interfaces (GUIs) in teaching an undergraduate numerical analysis class. Although MATLAB is a very useful software due to its framework and built-in features, some authors feel otherwise. First and foremost, the thing to look at is the level of teaching. The basics of analytic geometry are taught in high school courses like Geometry, second-year Algebra, and Pre-calculus in many countries. Some high schools used to provide a separate Analytic Geometry course, but the topic has been dispersed among various other classes during the last fifty years. MATLAB, Python, and other advanced software can be used in advanced courses and in research. Mojžišová & Pócsová (2019) developed a test finder for analytical geometry based on the MATLAB program. Excel, on the other side, is ubiquitous, inexpensive, and even available on smartphones. Secondly, making GUIs on Excel is far too easy for teaching purposes because its coding is simple. Authors found that when learning about numerical approaches, students must have practical experience applying the methods to situations where hand computations would be time-consuming. An instructor can employ different levels of coding depending on the computing proficiency of the intended students, among other things. As a result, they can employ various techniques, from pre-programmed graphical user interfaces (GUIs) to entire code generation and implementation.

For the convenience of a broader readership, it is important to describe the difference between a school, college, and university student. In subcontinent countries, high schools and colleges are referred to by the same terms, i.e., 12 years of education. University teaching in these countries starts after completing high school and college. The undergraduate level of teaching in these countries comprises fourteen years of education. Students take direct admissions after high school or advanced level (A levels) in the USA, UK, Australia, and European countries. Analytical geometry, as described earlier, is important at the high school or college level, and is also a prerequisite for engineering. Students who intend to do bachelor's degree in mathematics and physics also need to take this course (Analytical Geometry) in the program's first year.

This article aims to design a GUI to make analytical geometry easily understandable and convenient for university students at high school, college, and undergraduate levels.

2. Spreadsheets in Mathematics

The computer spreadsheet has been around for about thirty years. Throughout its existence, the applications of this innovative model have expanded and spread way beyond its original application in financial domains into every aspect of human activity. It is now the key mathematical tool in the workplace. It is widely available on practically any computer, with Microsoft Excel being the most widely used version. It is also becoming more prevalent in mathematics instruction and communication at all levels. This work presents some academic insights into a wide range of spreadsheet applications in analytical geometry, a very important field of mathematics. The spreadsheet provides numerous benefits for learning mathematical concepts. It is a simple and creative tool with basic operations that most students and teachers are familiar with. Teachers can use spreadsheets to teach and reinforce mathematical ideas while constructing a spreadsheet model.

Furthermore, the spreadsheet design frequently enables students to successfully study a wide range of topics that would otherwise be regarded as too difficult for them. Its use also gives students excellent hands-on experience with a tool they will use in their future careers. Finally, the spreadsheet assists teachers and students in making mathematics learning enjoyable. Our approach to spreadsheet use includes an emphasis on creating good graphics to encourage the development of visualization abilities. To accomplish this, we give instructive examples of inventive methods to employ spreadsheets to create interactive animated images. Our samples are drawn from a variety of applications and fields. Each of these examples makes use of mathematics.

A spreadsheet can be used for a variety of educational purposes. For beginners, it allows instructors to easily design mathematical algorithms and models and build interactive graphs for use in student assignments and activities. This situation allows students to collaborate in groups on larger tasks. Secondly, it enables teachers to create unique and effective classroom demonstrations to help students understand mathematical concepts. Additionally, it enables teachers to develop visual models for most textbook topics, including algebra, calculus, statistics, numerical analysis, and linear algebra. Third, it can provide an avenue for educators' professional growth by allowing them to give professional presentations on novel methods of teaching and research. This is true not only of mathematics but of almost any other discipline. Finally, it is a good site for conducting continuing education courses and communicating with the general public and colleagues from various fields.

3. **Theory**

Equation of Straight Line in 3D Space

Consider a straight line passing through a given point $P_o(x_o, y_o, z_o)$ And having direction cosines l, m, n. Then ant point $p(x, y, z)$ on this line is given by

$$\vec{r} = \vec{r_o} + \overrightarrow{P_oP} \quad (1)$$

$$x\hat{\imath} + y\hat{\jmath} + z\hat{k} = x_o\hat{\imath} + y_o\hat{\jmath} + z_o\hat{k} + \lambda\hat{\imath} + \mu\hat{\jmath} + \nu\hat{k}$$

Where $\vec{r_o} = vector\ \overrightarrow{OP_o}$, $\vec{r} = vector\ \overrightarrow{OP}$

The equation of a straight line passing through some other point $P(x, y, z)$ and $P_o(x_o, y_o, z_o)$ can be given by rearrangement of equation (1)

$$\frac{x - x_1}{\lambda} = \frac{y - y_1}{\mu} = \frac{z - Z_1}{\nu} \quad (3)$$

or

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - Z_1}{n} \quad (4)$$

λ, μ, ν are direction ratios while l, m, n are direction cosines

Area of Parallelogram

A parallelogram is a quadrilateral plane whose opposite sides are parallel, and its area is given by

$$Area\ =\ Base\ \times\ Height \quad (5)$$

$$Area\ =\ a\, b \sin A =\ b\, a \sin B \quad (6)$$

This shows that the area is given by the determinant of $|AxB|$, ½ of $|AxB|$ gives the area of a triangle.

Equation of Plane

A plane is a two-dimensional surface described by a unit normal to the plane. Suppose P(x_o, y_o, z_o) is a fixed point of the plane and P(x, y, z) is the moving point, and a vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is perpendicular to the plane, then $\overrightarrow{P_oP} \bullet \left(a\hat{\imath} + b\hat{\jmath} + c\hat{k}\right) = 0$ or

$$(x - x_o)\hat{\imath} + (y - y_o)\hat{\jmath} + (z - z_o)\hat{k}] \bullet \left(a\hat{\imath} + b\hat{\jmath} + c\hat{k}\right) = 0$$

This given equation of the plane,

$$a(x - x_o) + b(y - y_o) + c(z - z_o) = 0 \quad (8)$$

The simplification gives $Ax + By + Cz + D = 0$

Volume of Tetrahedron

A tetrahedron is a 3D surface formed by four vertices A, B, C, and D, and it contains one of the points outside a triangular plane formed by the other three points; its volume is given by

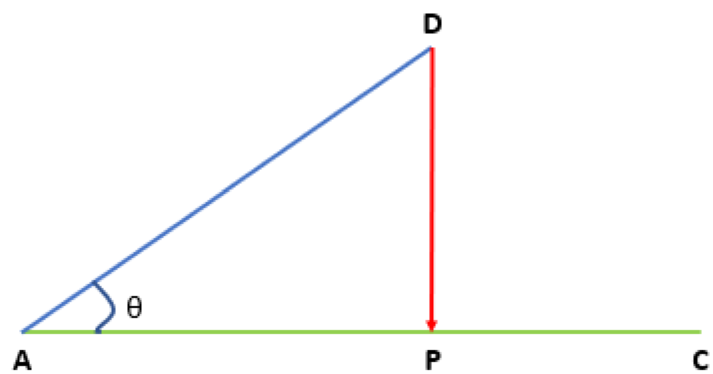
$$V = \frac{1}{3} \; of \; Volume \; of \; paralellopiped \quad (9)$$

Where the volume of the parallelepiped is given by $\overrightarrow{AD}.(\overrightarrow{AB}\text{x}\overrightarrow{AC})$

Distance of a Point from Line

Let $D(x_1, y_1, z_1)$ Is the point whose perpendicular distance from the line AC is to be found, the equation of line AC is given by

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \quad (10)$$



$$cos\theta = \frac{\overrightarrow{AD} \bullet (l\hat{\imath} + m\hat{\jmath} + n\hat{k})}{|AD|}$$

$$1erpendicular \; distance \; D \; from \; line \; AC \; = |DP| = |AD|sin\theta = |AD|(1 - cos^2\theta)$$

We can also find projection |AP| and then $|DP|^2 = |AD|^2 - |AP|^2$

Shortest Distance between Two Lines

Let λ_1, μ_1, ν_1 and λ_2, μ_2, ν_2 be direction cosines of two straight lines. Also, **u** is a vector perpendicular to both the lines given by

$$u = \begin{vmatrix} i & j & k \\ \lambda_1 & \mu_1 & \nu_1 \\ \lambda_2 & \mu_2 & \nu_2 \end{vmatrix} \tag{11}$$

The unit vector in the direction of the common perpendicular will be $\frac{u}{|u|}$, Now, if P_1 and P_2 are two points, one on each of the given lines, the projection of P_1P_2 on the common perpendicular will give the shortest distance

$$D = P1P2\,\frac{u}{|u|}$$

Though the information given in this section is available in textbooks, this spreadsheet program's pattern and its sequence require these equations. Readers can easily take this information to innovate this GUI.

4. Results and Discussion

There are two spreadsheets on which examples of the equation of a straight line in 3 Dimensions that contains two points, the area of the triangle, equation of a plane, and volume of the tetrahedron are shown. The spreadsheet is independent of other sheets. The following provides the detail of each sheet:

Sheet 1: Equation of a Straight Line in 3 Dimensions

This sheet shows an example of the problem of determining the equation of a straight line in 3 Dimensions that contains two points. The points in the 3-dimensional coordinates system are shown in cells B2, C2, D2, and B3, C3, and D3, respectively. The sheet contains a button; pressing the button instructs the macro program to calculate direction cosines, the equation of a straight line in 3 Dimensions that contains two points in general, and the parametric forms. The problem's solution will appear in the box on the right side of the button.

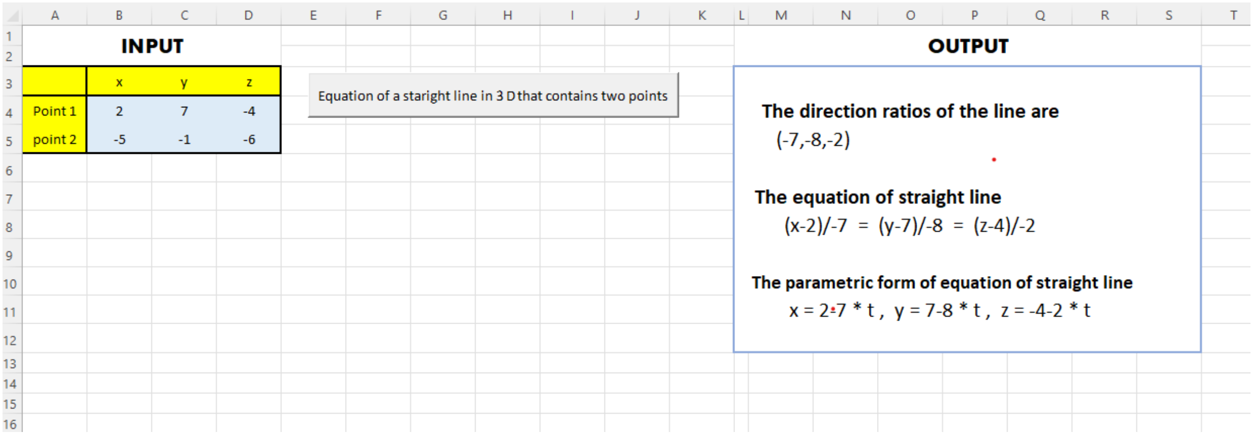


Figure 1: The screenshot of the equation of a straight line in 3-dimensions

Sheet 2: Area and Volumes in Analytical Geometry

The students can learn about important quantities like the triangle area formed by three points, the equation of a plane containing three points, and volumes of parallelopiped and tetrahedron formed by AB, AC, BC, and BD. The points A, B, C, and D are shown under yellow shade in cells B2:D5. Five command buttons appear on the screen that calculates the area of any triangle, the equation of a plane, the volume of parallelopiped and tetrahedron, the perpendicular distance of D from line AC, and the shortest distance between two lines AC and BD through corresponding buttons of the spreadsheet. The change in given points generates a new set of problems. The student can practice the same problem as many times as they wish.

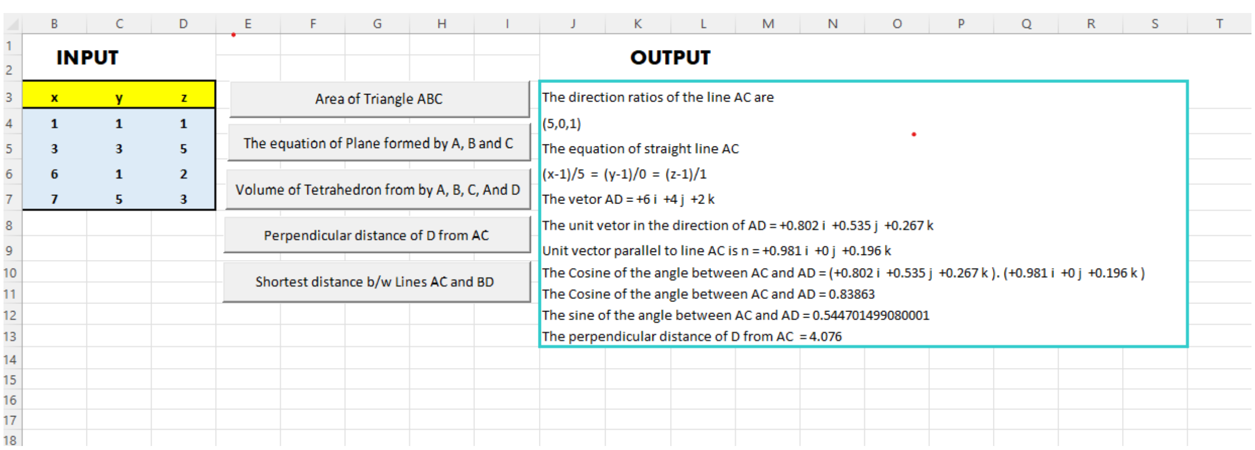


Figure 2: The screenshot of an area of triangles, Equation of plane, and Volumes

5. Conclusion

For the last two years, we switched to online, and hybrid teaching models during the pandemic and lockdown. We faced problems in teaching various modules in calculus and analytical geometry online. Considering that scenario, we developed various GUIs for different calculus and analytical geometry modules. This GUI is about some important examples from analytical 3-

dimensional geometry. The first sheet of this GUI contains finding the equation of the line passing through two given points in three dimensions. The second sheet contains five more examples: volume, angles, projections, the perpendicular distance from a line, and the shortest distance between two lines. The advantage of this GUI that is both students and teachers can make different questions with known answer keys. Students can learn by practicing different situations and can verify their solutions with answer keys. Viewers can also see a tutorial at the link https://drive.google.com/drive/u/0/my-drive. Moreover, an Excel program can be provided for better practice, but macros must be enabled before operating them.

6. Declaration of interests

The authors declare that they have no known financial or personal relationships that could have appeared to influence the work reported in this paper.

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